Reducing the cost of extended waveform inversion by multiscale adaptive methods

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The Rice Inversion Project (TRIP)

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Overview

Objective

Recover Earth model by extended waveform inversion

Problems

Computational cost

Solution

Multiscale method

Adaptive approach
Extended modeling concept

Abstract setting for forward map \( \mathcal{F} : \mathcal{M} \rightarrow \mathcal{D} \)

\[
\mathcal{F}[m] = d
\]

\( F \): forward modeling operator
\( m \): model \((v, r)\)
\( d \): sampled pressure data at receivers

Extended forward map \( \bar{\mathcal{F}} : \bar{\mathcal{M}} \rightarrow \mathcal{D} \) [Symes, 2008]

\[
\bar{\mathcal{F}}[\bar{m}] = d
\]

\( \bar{F} \): extended forward modeling operator
\( \bar{m} \): extended model \((v(x), \bar{r}(x, h), ...)\)
\( d \): sampled pressure data at receivers
Extended linearized acoustic modeling $\bar{F}[v] \bar{r}$

$u$ - reference (incident) pressure field

$$\left( \frac{1}{v^2(x)} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) u(t, x; x_s) = w(t) \delta(x - x_s)$$

$\delta u$ - scattered (perturbation) pressure field

$$\left( \frac{1}{v^2(x)} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \delta u(t, x; x_s) = \frac{\partial^2}{\partial t^2} \int dh \frac{2\delta \bar{v}(x, h)}{v(x + h)v^2(x - h)} u(t, x; x_s)$$

$w(t)$: source function
$v$: velocity of seismic waves
$x$: position in earth model
$x_s$: source location
$
\bar{r}(x, h) = \frac{\delta \bar{v}(x, h)}{v(x)}$: extended reflectivity ($\delta \bar{v}(x, h)$: extended velocity perturbation)
Why $\bar{F}[\nu] \bar{r}$ expensive?

Express the solution of equation (2) as:

$$
\bar{F}_H[\nu] \delta \bar{v}(t, x_r; x_s) = \frac{\partial^2}{\partial t^2} \int dx \int_{-H}^H dh \int d\tau \frac{2\delta \bar{v}(x, h)}{v(x + h)v^2(x - h)}
G(t - \tau, x + h; x_r)G(\tau, x - h; x_s) \ast w(\tau)
$$

Computational cost

Increase with number of grid points in $h \ (N_h = \frac{2H}{dh})$

$N_h \downarrow \iff dh \uparrow$ coarse grid

$N_h \downarrow \iff H \downarrow \iff$ velocity error $\downarrow$
Extended full waveform inversion (EFWI)

Objective function:

\[
\min_{v, \bar{r}} J[v, \bar{r}] = \frac{1}{2} \| \bar{F}[v] \bar{r} - d \|^2 + \frac{\alpha^2}{2} \| A \bar{r} \|^2
\]

\( A \): annihilator, differential semblance operator, subsurface offset \( h \).

Separable least-squares, solved with variable projection method [Golub and Pereyra, 1973]. The inverse problem is solved by a nested optimization approach:

- **Inner loop**, optimize \( J[v, \bar{r}] \) over \( \bar{r} \).
- **Outer loop**, optimize reduced objective function \( J[v, \bar{r}[v]] \) over \( v \).
Inner loop, optimize $J$ over $\bar{r}$

Gradient of the objective function $J[v, \bar{r}]$ with respective to $\bar{r}$:

$$\nabla_{\bar{r}} J[v, \bar{r}] = \bar{F}[v]^*(\bar{F}[v]\bar{r} - d) + \alpha^2 A^* A \bar{r}$$

where $*$ denotes adjoint.

Setting the gradient function to zero, least-squares extended reverse time migration (LSERTM), solved by a linear iterative method, e.g. conjugate gradient method.

$$\bar{N}[v]\bar{r} = \bar{F}[v]^* d$$

where $\bar{N}[v] = \bar{F}[v]^* \bar{F}[v] + \alpha^2 A^* A$
Outer loop, update $v$

The gradient of the reduced objective function $J[v, \bar{r}[v]]$ respect to $v$:

$$\nabla_v J[v, \bar{r}[v]] = \Lambda^{-2s} D\bar{F}[v]^T (\bar{r}[v], \bar{F}[v] \bar{r} - d)$$

(3)

where $\Lambda^{-2s}$ is a smoothing operator for positive $s$ [Symes and Kern, 1994].
\[ \vec{r} = \tilde{N}[v]^{-1} \tilde{F}[v]^* d \] at different \( v_{mig} \)

**Figure:** (a) \( v_{mig} = 0.9v \), (b) \( 0.8v \), (c) \( 0.7v \), (d) \( 1.1v \), (e) \( 1.2v \), (f) \( 1.3v \)
Multiscale adaptive method for determining $h$

1: loop
2: // Outer loop
3: $d_{obs}, w \leftarrow \text{low-pass}(f_{\text{min}} - f)$ on $d_{obs}$ and $w$
4: $\bar{r} \leftarrow \bar{N}_H[v]\bar{r} = \bar{F}_H[v]*d$
5: // Inner loop
6: $\Delta d_{H/2}, \Delta d_H \leftarrow \bar{F}_{H/2}[v]\bar{r} - d_{obs}, \bar{F}_H[v]\bar{r} - d_{obs}$
7: if $\Delta d_H < X$ and $\Delta d_{H/2} < X$ then
8: $dh \leftarrow dh/2, H \leftarrow H/2, dx \leftarrow dx/2, f \leftarrow 2f, dt \leftarrow dt/2$
9: else if $\Delta d_H < X$ and $\Delta d_{H/2} \geq X$ then
10: exit
11: else
12: $H \leftarrow 2H$, go to 4 // $\Delta d_{H/2} \geq \Delta d_H \geq X$
13: $\Delta v \leftarrow \nabla_v = \Lambda^{-2s}D\bar{F}[v]^T (\bar{r}[v], \bar{F}[v]\bar{r} - d)$
14: $v \leftarrow v + \Delta v$
15: end loop
Figure: a) velocity model, (b) the original source wavelet \((3 - 24 \text{ Hz})\), (c) source wavelet filtered \(3 - 8 \text{ Hz}\), (d) source wavelet filtered \(3 - 16 \text{ Hz}\)
Step 1: $v_{mig} = 2.4 \text{ km/s}$, start with $H = 320 \text{ m}$

**Figure**: Step 1: $dh = dx = dz = 40 \text{ m}$, $f : 3 - 8 \text{ Hz}$, $dt = 6 \text{ ms}$ (a) extended RTM image, (b) 20 iteration of LSERTM with slower velocity
Step 1: \( v_{mig} = 2.4 \text{ km/s} \)

\[
\Delta d_{160} \geq \Delta d_{320} \geq X, \ H \leftarrow 2H = 640 \text{ m}
\]

\[
\Delta d_{640} < X \text{ and } \Delta d_{320} \geq X, \text{ exit}
\]

**Figure**: Step 1: \( dh = dx = dz = 40 \text{ m}, f : 3 - 8 \text{ Hz}, dt = 6 \text{ ms} \) (a) LSERTM result of 20 CG iterations, (b) the relative data residual
Step 2: \( v_{mig} = 2.6 \text{ km/s} \)

\[ \Delta d_{640} < X \text{ and } \Delta d_{320} < X, \quad dh \leftarrow dh/2, \quad H \leftarrow H/2, \quad dx \leftarrow dx/2, \quad f \leftarrow 2f, \quad dt \leftarrow dt/2 \]

\[ \Delta d_{320} < X \text{ and } \Delta d_{160} \geq X, \text{ exit} \]

Figure: Step 2: \( dh = dx = dz = 20 \text{ m}, \ f : 3 - 16 \text{ Hz}, \ dt = 3 \text{ ms} \), (a) LSERTM result of 25 CG iterations, (b) the relative data residual
Step 3: $v_{mig} = 2.8 \text{ km/s}$

$\Delta d_{320} < X$ and $\Delta d_{160} < X$, $dh \leftarrow dh/2$, $H \leftarrow H/2$, $dx \leftarrow dx/2$, $f \leftarrow 2f$, $dt \leftarrow dt/2$

$\Delta d_{160} < X$ and $\Delta d_{80} \geq X$, exit

**Figure**: Step 3: $dh = dx = dz = 10 \text{ m}$, $dt = 1.5 \text{ ms}$, the original data and source function (a) LSERTM result of 40 CG iterations, (b) the relative data residual
Summary

Reduce computational cost of EFWI

- Step 1: \( \left( \frac{1}{4} \right)^3 \approx 0.39\% \)
- Step 2: \( \left( \frac{1}{2} \right)^3 = 12.5\% \)

Methods:

- Multiscale method
- Adaptive approach

Future work

- Good preconditioner
- Implementation in IWave
Acknowledgement

Thanks to
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