Fast extended waveform inversion using Morozov's discrepancy principle

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Extended waveform inversion (EWI)

Object function:

$$J_\alpha[m,r] = \frac{1}{2}\|F[m]r - d\|^2 + \frac{\alpha}{2}\|Ar\|^2$$

Role of α: controls amount of penalty applied for model extension.

$\alpha \to 0$,	weak constraint on r,	good data fit
$\alpha \to \infty$,	strong	poor

m - nonlinear model

α - weighting

r - linear extended model

A - annihilator

Variable projection method

$$J_\alpha[m, r] = \frac{1}{2}\|F[m]r - d\|^2 + \frac{\alpha}{2}\|Ar\|^2$$

Classified as separable least-squares inverse problems, solved with variable projection method.
[Golub and Pereyra, 1973, Golub and Pereyra, 2003]

Nested optimization:

- inner loop, given m, find r optimizes $J_\alpha[m, r]$.
- outer loop, find m optimizes reduced objective function $J_\alpha[m, r[m]]$.

Inner loop, optimize J over r

$\nabla_r J_\alpha[m, r] = 0 \Rightarrow$ normal equation:

$$(F^T F + \alpha A^T A) r = F^T d$$

Solved by linear iterative method, e.g., conjugate gradient (CG).

Solution depends on α

$$r_\alpha \approx (F^T F + \alpha A^T A)^{-1} F^T d$$

Outer loop, update m

Gradient of reduced objective function $J_\alpha[m, r_\alpha[m]]$ respect to m:

$$\nabla_m J_\alpha[m, r[m]] = DF^T\,(r_\alpha, F[m]r_\alpha - d)$$

DF^T tomographic or WEMVA operator.

DF, bilinear operator, linear in dm and r.

Weight α

Rewrite data misfit and penalty terms

$$\begin{aligned} e(\alpha) &= \frac{1}{2}\|Fr(\alpha) - d\|^2 \\ p(\alpha) &= \frac{1}{2}\|Ar(\alpha)\|^2 \end{aligned}$$

Differentiate e and p

$$2p \geq \frac{de}{d\alpha} \geq 0$$

$$\frac{dp}{d\alpha} \leq 0$$

Weight α

Basic bound:

$$\frac{de}{d\alpha} \leq 2p$$

Suppose current weight α_c, updated weight α_+

$$\alpha_+ \gtrapprox \alpha_c + \frac{e(\alpha_+) - e(\alpha_c)}{2p(\alpha_c)}$$

Morozov's discrepancy principle

Target data misfit X, $X_- < X < X_+$.

Adjust α *to keep* $e(\alpha)$ *between* $\frac{1}{2}X_-^2$ *and* $\frac{1}{2}X_+^2$*, i.e. "near"* $\frac{1}{2}X^2$*.*

Application

Suppose $e(\alpha_c) < \frac{1}{2}X_-^2$, e.g. m is updated, adjust α to keep

$e(\alpha) \in [\frac{1}{2}X_-^2, \frac{1}{2}X_+^2]$

$$\alpha_+^{\text{est}} = \alpha_c + \frac{\frac{1}{2}X_+^2 - e(\alpha_c)}{2p(\alpha_c)}$$

Algorithm to update α

1. Initial m, calculate $e(\alpha = 0)$ to estimate $X_- = 0.8 * e(0)$ and $X_+ = 1.2 * e(0)$

   $e(0) \neq 0$, determined by accuracy of solving normal equation

2. Update α

$$\alpha_+^{\text{est}} = \alpha_c + \frac{\frac{1}{2}X_+^2 - e(\alpha_c)}{2p(\alpha_c)}$$

3. If $e(\alpha_+^{\text{est}})$ is not in an acceptable range,

   $e(\alpha_+) < X_-, \alpha_+ * = 2;$

   $e(\alpha_+) > X+, \alpha_+ / = 1.5;$

Overthrust model

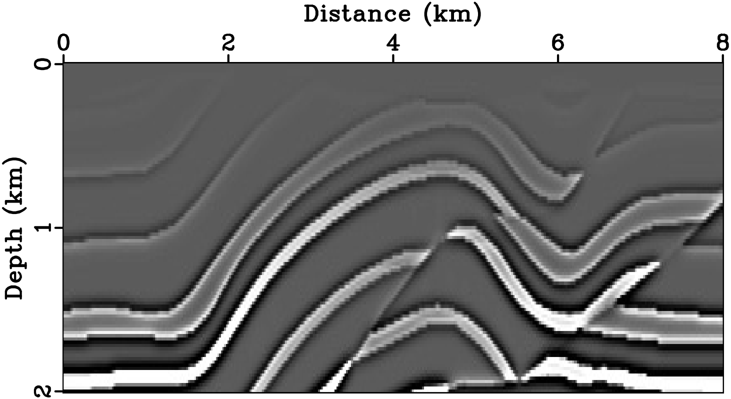
2D constant density acoustic 2-8 order finite difference code

Subsurface offset extended Born approximation.

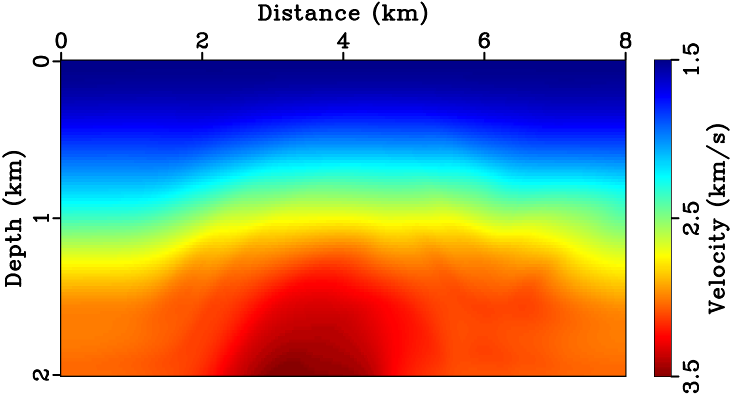
Adaptive multiscale method (2 refinement stages)

Parameter	Measurements
Source wavelet	bandpass $5 - 20\,Hz$
Initial velocity	$v = 1.5\,km/s$
Max iter inner loop	20
Source position $\mathbf{x_s}$	$x : 1 - 7\,km$ every $40\,m$, $z = 20\,m$
Receiver position $\mathbf{x_r}$	$x : 0 - 8\,km$ every $40\,m$, $z = 0\,m$
Space and time	$x = 8\,km$, $z = 2\,km$, $t = 3\,s$
Grid size	$dx = dh = dz = 20\,m$, $dt = 2\,ms$

r at $h = 0\,m$



True background velocity



Initial background velocity v_0

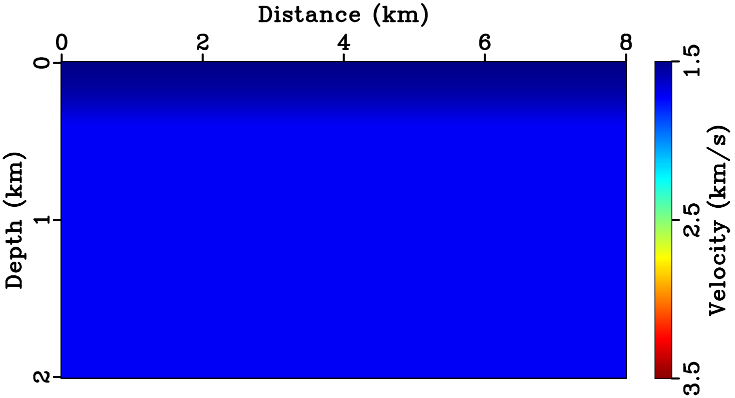
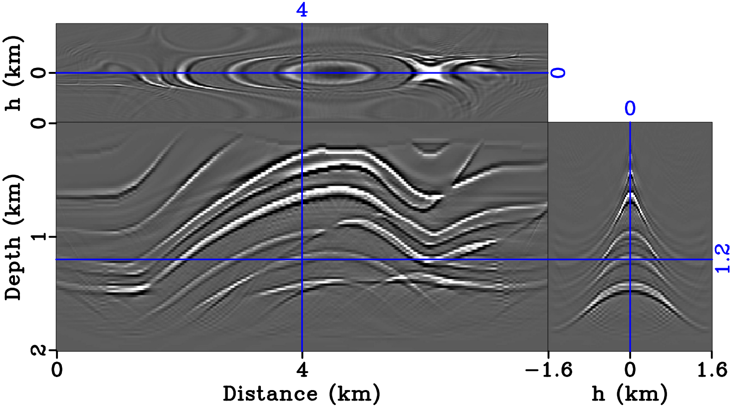


Image r_0 with initial velocity v_0



Inverted background velocity v_{20}

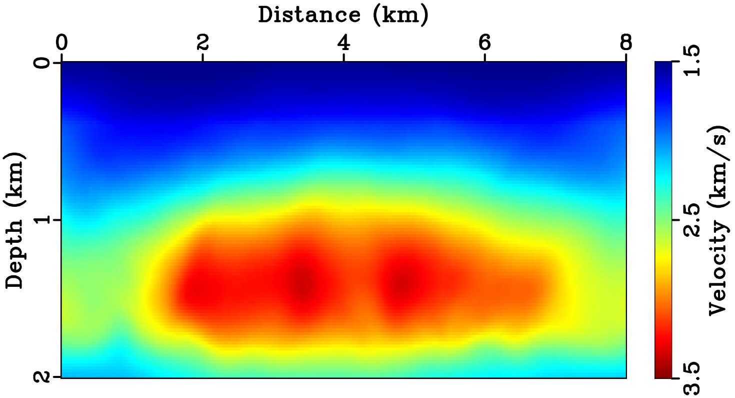
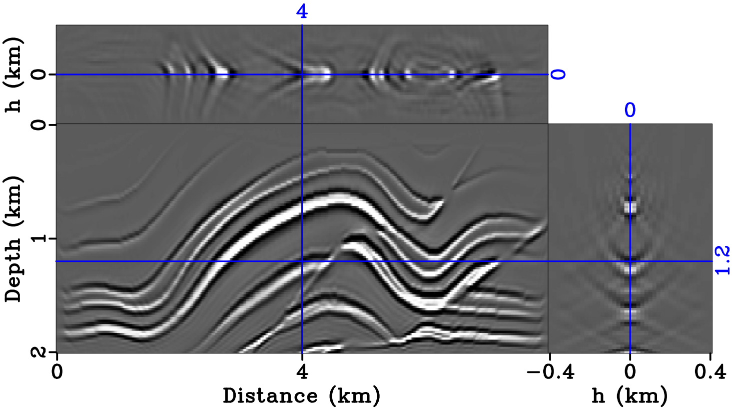
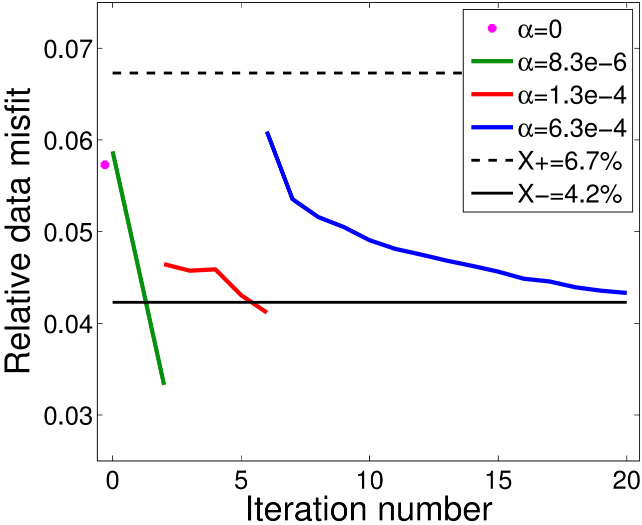


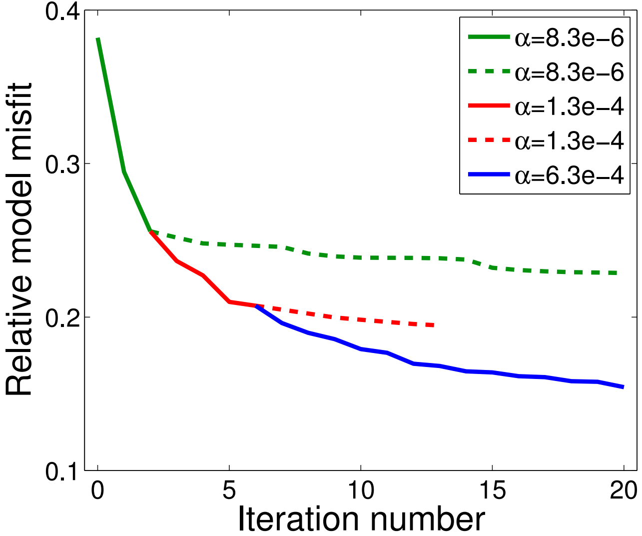
Image r_{20}



Relative data misfit



Relative model misfit



Overthrust model, scan test

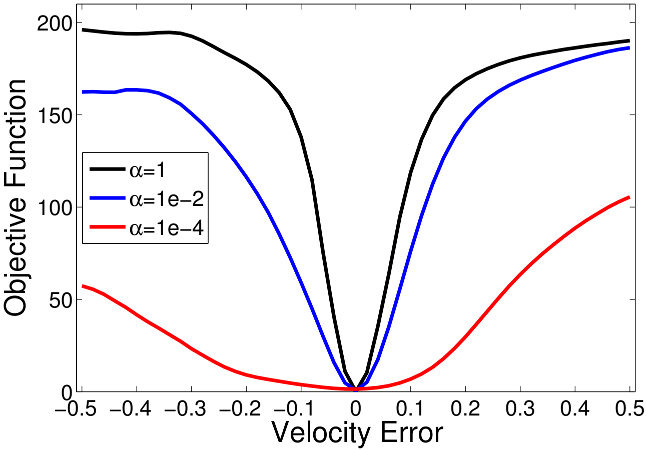


Figure : Scan test: objective function with different values of α. Background velocity error from -50% to $+50\%$

Summary

1. Update rule for α, significantly improves convergence rate.

$$\alpha_{+}^{\text{est}} = \alpha_c + \frac{\frac{1}{2}X_{+}^{2} - e(\alpha_c)}{2p(\alpha_c)}$$

2. Morozov discrepancy principle: adjust α to keep

$$e(\alpha) \in [\frac{1}{2}X_{-}^{2}, \frac{1}{2}X_{+}^{2}]$$

Future work

Consider physics errors, explore with field data.

Use variable density acoustics.

Accelerate inner loop and outer loop (preconditioner, optimization methods)

Acknowledgments

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Golub, G., and V. Pereyra, 1973, The differentiation of pseudoinverses and nonlinear least squares problems whose variables separate: SIAM Journal on Numerical Analysis, **10**, 413–432.

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