Matched Source Waveform Inversion: Volume Extension

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Outline

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Overview of Source-based WI

Src-recv extension: $\bar{f} = \bar{f}(\mathbf{x}_r, \mathbf{x}_s, t) \in \mathbb{R}^3 (\text{or } \mathbb{R}^5)$

- Works for single arrival (traveltime tomography).
- Fail if multi-arrivals exist.
  (1). Ambiguity when fitting data from different branches;
  (2). Slope of traveltime is lost (single trace fit).
  (3). $G(\mathbf{x}_r, \mathbf{x}_s, t) * f_{sr}(t) = d(\mathbf{x}_r, \mathbf{x}_s, t)$ is NOT solvable in L_2 sense.

Space-time extension: $\bar{f} = \bar{f}(\mathbf{x}, \mathbf{x}_s, t) \in \mathbb{R}^4 (\text{or } \mathbb{R}^6)$

- Solve the problem (1)-(2), but no guarantee for (3).
- Limitation to 3D Helmholtz eqn solver.
- Huge storage requirement of $\bar{f}(\mathbf{x}, \mathbf{x}_s, t) \in \mathbb{R}^6$ in 3D.

Any other choices of source extn that can solve all the problems (1)-(3) and w/o limitation like space-time extn?

Outline

Extended Modeling & Annihilator

Extended Modeling:

$\bar{f}(\mathbf{x};\mathbf{x}_s)$: extended model of $\delta(\mathbf{x}-\mathbf{x}_s)$

Extended modeling operator $\bar{S}\bar{f}=\bar{u}$:

$$\frac{1}{v^2}\frac{\partial^2 \bar{u}}{\partial t^2}-\Delta\bar{u}=\bar{f}(\mathbf{x},\mathbf{x}_s)\delta(t).$$

Presume that the recorded data is deconvolved by wavelet $f(t)$.

Annihilator:

$A=|\mathbf{x}-\mathbf{x}_s|$: Penalize non-focusing energy around src position $\mathbf{x}_s$.

Matched Source Waveform Inversion

Extended waveform inversion:

$$J_\alpha[v] = \frac{1}{2\alpha} \sum_{\mathbf{x}_r, \mathbf{x}_s} \int |\bar{S}[v]\bar{f} - d|^2 dt + \frac{1}{2} \sum_{\mathbf{x}_s} \int |A\bar{f}|^2 d\mathbf{x}$$

$$s.t. \qquad (\bar{S}^T \bar{S} + \alpha A^T A)\bar{f} = \bar{S}^T d.$$

Key feature: data fitting via $\bar{f}$ $\Rightarrow$ no cycle skipping problem!

Outline

Property of $\bar{S}$ and $\bar{S}^T\bar{S}$

Lemma (FIO)

Under some mild assumption of velocity and there is no grazing rays, the extended source forward modeling operator $\bar{S}$ is fourier integral operator.

Lemma (ΨDO)

The extended normal operator $\bar{S}^T\bar{S}$ is ΨDO of order -2,

$$\bar{S}^T\bar{S} \in \mathrm{OPS}^{-2}$$

Furthermore, we have

$$\bar{S}^T\bar{S}\bar{f} = \frac{1}{(2\pi)^2}\int \frac{1}{|\mathbf{k}|^2}\frac{e^{i\mathbf{k}\cdot(\mathbf{y}-\mathbf{x})}}{4\frac{\cos\alpha_r}{v_r}}\bar{f}(\mathbf{y})d\mathbf{k}d\mathbf{y}$$

ΨDO Verification of $\bar{S}^T \bar{S}$

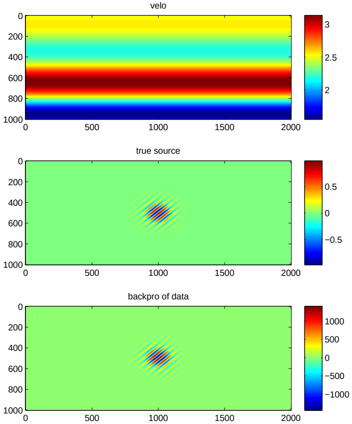


Figure: True velo, true source $\bar{f}$, and backpropagation field $\bar{S}^T \bar{S} \bar{f}$

Like the pair of migration and demigration operator!

Smooth Objective Functional

The direct consequence of these two lemmas yields the following important conclusion,

Theorem

The volume based MSWI objective functional $J_\alpha[v]$ is smooth function in velocity v independent of data spectrum.

Note that the objective function admits the bilinear form,

$$J_\alpha[v] = \frac{1}{2\alpha}\langle (I - \bar{S}N_\alpha^{-1}\bar{S}^T)d, d\rangle$$

See C. Stolk and W. Symes (IP-2000) for general argument.

Relation with Stereotomography

Theorem

The Hessian of MSWI function at the consistent data is equivalent to stereotomography,

$$\delta^2 J_\alpha[v^*] \approx C\|\frac{\partial}{\partial \theta_s}\delta\tau(\mathbf{x}_r,\mathbf{x}_s)\|^2 + O(\alpha).$$

where C is frequency independent constant.

NOTE:
$\delta\tau(\mathbf{x}_r,\mathbf{x}_s) = 0$ and $\frac{\partial}{\partial \mathbf{x}_r}\delta\tau(\mathbf{x}_r,\mathbf{x}_s) = 0$ is satisfied automatically by backpropagation.

See H. Chauris etc. (2002) for similar discussions.

Outline

Model

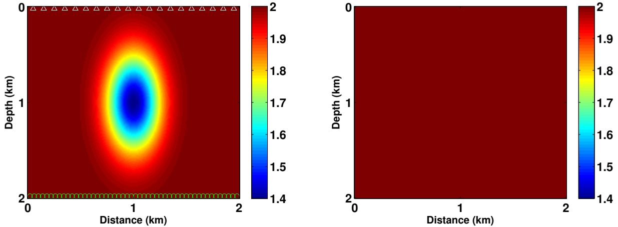


Figure: Transmission configuration: true model and initial model

Data

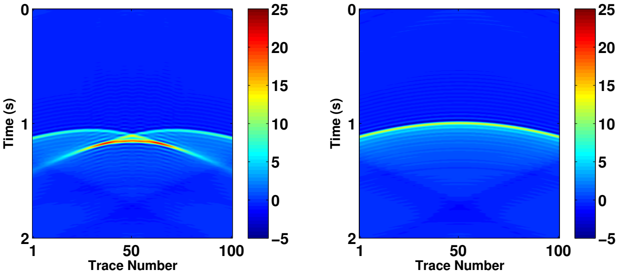


Figure: Recorded data and simulated data with initial model at center shot $x_s = 1$ km

Inverted Velocity

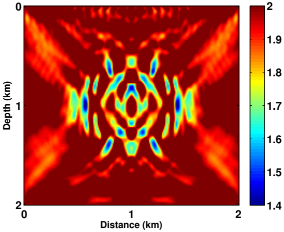


Figure: Inverted velocity by FWI and volume-based MSWI with 9-20 Hz data

Layer Salt Model

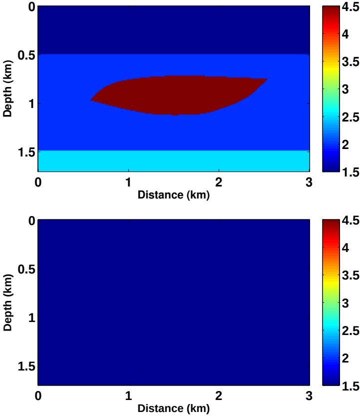
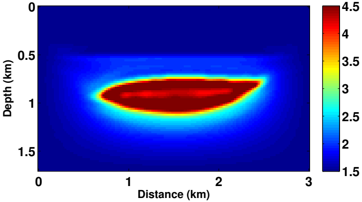


Figure: True model and constant initial model

Comparison of Results



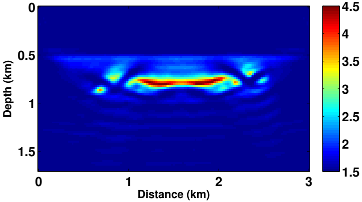


Figure: Inverted velocity by MSWI and FWI method with 6-12 Hz data

Marmousi

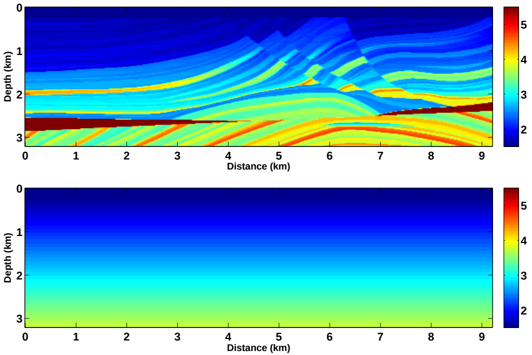


Figure: Marmousi model and 1D initial model

Marmousi

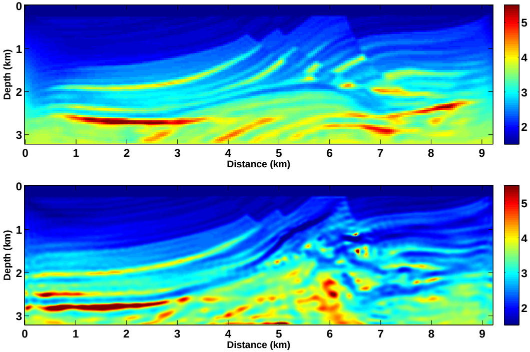


Figure: MSWI result (6-10 Hz data) and FWI result (4-8 Hz data)

SEG/EAGE 2D Salt Model

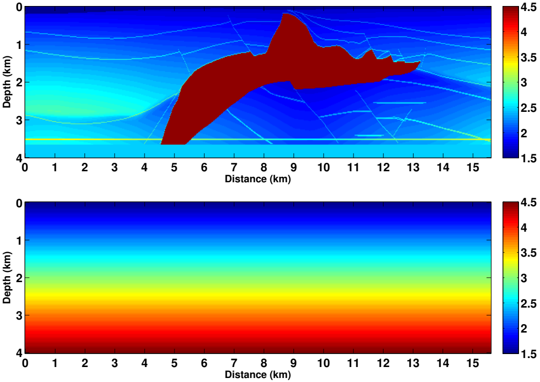


Figure: True model and 1D initial model

Inverted Results

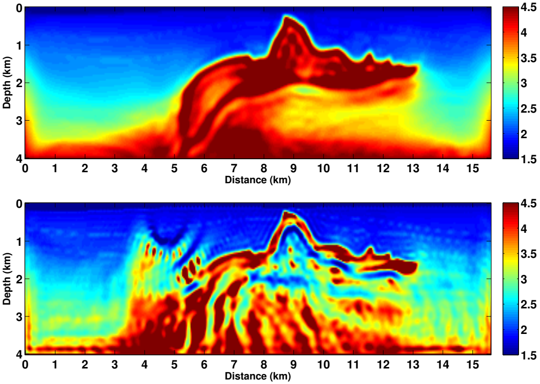


Figure: MSWI result and FWI result (3-6 Hz data)

Slice of SEAM Phase I Model

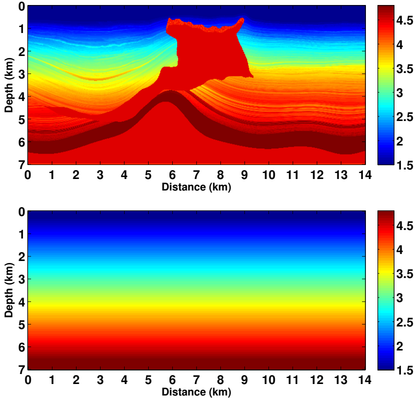


Figure: True model and 1D initial model

Inverted Result

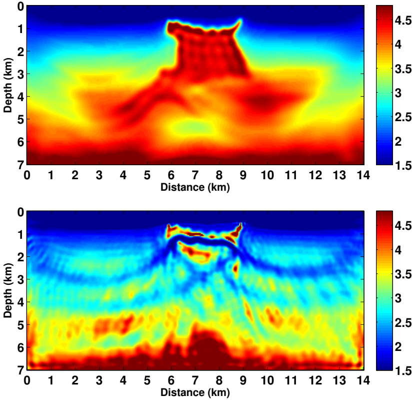


Figure: MSWI result (3-6 Hz data) and FWI result (2-5 Hz data)

Slice of BP Model

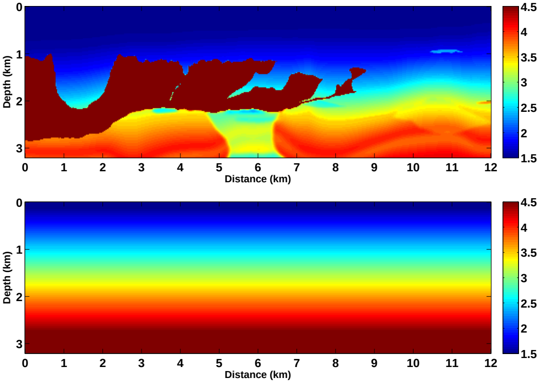


Figure: True model and 1D initial model

Inverted Results

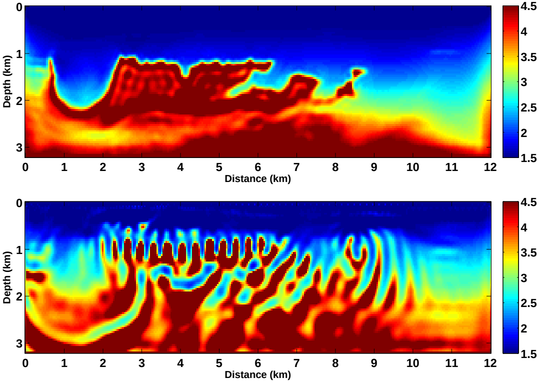


Figure: MSWI result (3-8 Hz data) and FWI result (2-5 Hz data)

Conclusion

- Nonlinear extended waveform inversion can handle reflection/refraction wave.
- No cycle skipping problem and insensitive to the frequency content and initial model.
- Equivalent to traveltime tomography under high frequency for transmission wave.
- Potential application in salt body reconstruction.
- How does it work for reflection wave?

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