Matched Source Waveform Inversion: Volume Extension

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Outline

1. Overview of Source-based WI
2. MSWI: Volume Extension
3. Analysis of Transmission Problem
4. Numerical Examples
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Overview of Source-based WI

**Src-recev extension:** $\bar{f} = \bar{f}(x_r, x_s, t) \in \mathbb{R}^3$ (or $\mathbb{R}^5$)

- Works for single arrival (traveltime tomography).
- Fail if multi-arrivals exist.
  1. Ambiguity when fitting data from different branches;  
  2. Slope of traveltime is lost (single trace fit).  
  3. $G(x_r, x_s, t) \ast f_{sr}(t) = d(x_r, x_s, t)$ is NOT solvable in $L_2$ sense.

**Space-time extension:** $\bar{f} = \bar{f}(x, x_s, t) \in \mathbb{R}^4$ (or $\mathbb{R}^6$)

- Solve the problem (1)-(2), but no guarantee for (3).
- Limitation to 3D Helmholtz eqn solver.
- Huge storage requirement of $\bar{f}(x, x_s, t) \in \mathbb{R}^6$ in 3D.

Any other choices of source extn that can solve all the problems (1)-(3) and w/o limitation like space-time extn?
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Extended Modeling:

\[ \tilde{f}(x; x_s): \text{extended model of } \delta(x - x_s) \]

Extended modeling operator \( \tilde{S} \tilde{f} = \tilde{u} \):

\[
\frac{1}{v^2} \frac{\partial^2 \tilde{u}}{\partial t^2} - \Delta \tilde{u} = \tilde{f}(x, x_s) \delta(t).
\]

Presume that the recorded data is deconvolved by wavelet \( f(t) \).

Annihilator:

\[ A = |x - x_s|: \text{Penalize non-focusing energy around src position } x_s. \]
Matched Source Waveform Inversion

Extended waveform inversion:

\[
J_\alpha[v] = \frac{1}{2\alpha} \sum_{x_r,x_s} \int |\bar{S}[v] \bar{f} - d|^2 dt + \frac{1}{2} \sum_{x_s} \int |A \bar{f}|^2 dx
\]

s.t. \quad (\bar{S}^T \bar{S} + \alpha A^T A) \bar{f} = \bar{S}^T d.

Key feature: data fitting via \( \bar{f} \) \( \Rightarrow \) no cycle skipping problem!
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Property of $\tilde{S}$ and $\tilde{S}^T \tilde{S}$

**Lemma (FIO)**

*Under some mild assumption of velocity and there is no grazing rays, the extended source forward modeling operator $\tilde{S}$ is fourier integral operator.*

**Lemma ($\Psi DO$)**

*The extended normal operator $\tilde{S}^T \tilde{S}$ is $\Psi DO$ of order $-2$, \hspace{1cm} \tilde{S}^T \tilde{S} \in OPS^{-2}$.*

*Furthermore, we have*  

$$\tilde{S}^T \tilde{S} \bar{f} = \frac{1}{(2\pi)^2} \int \frac{1}{|k|^2} \frac{e^{ik \cdot (y-x)}}{4\cos \alpha_r \nu_r} \bar{f}(y) dk dy$$
ΨDO Verification of $\bar{S}^T \bar{S}$

Figure: True velo, true source $\bar{f}$, and backpropagation field $\bar{S}^T \bar{S} \bar{f}$

Like the pair of migration and demigration operator!
Smooth Objective Functional

The direct consequence of these two lemmas yields the following important conclusion,

**Theorem**

>The volume based MSWI objective functional \( J_\alpha[v] \) is smooth function in velocity \( v \) independent of data spectrum.

Note that the objective function admits the bilinear form,

\[
J_\alpha[v] = \frac{1}{2\alpha} \left\langle (I - \bar{S}N_\alpha^{-1}\bar{S}^T)d, d \right\rangle
\]

Relation with Stereotomography

**Theorem**

The Hessian of MSWI function at the consistent data is equivalent to stereotomography,

\[ \delta^2 J_\alpha[v^*] \approx C \| \frac{\partial}{\partial \theta_s} \delta \tau(x_r, x_s) \|^2 + O(\alpha). \]

where \( C \) is frequency independent constant.

**NOTE:**
\( \delta \tau(x_r, x_s) = 0 \) and \( \frac{\partial}{\partial x_r} \delta \tau(x_r, x_s) = 0 \) is satisfied automatically by backpropagation.

See H. Chauris etc. (2002) for similar discussions.
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   - Layer Salt Model
   - Marmousi Model
   - SEG/EAGE 2D Salt Model
   - SEAM Phase I Model
   - Slice of BP Model
Overview of Source-based WI

MSWI: Volume Extension

Analysis of Transmission Problem

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Figure: Transmission configuration: true model and initial model
Data

Figure: Recorded data and simulated data with initial model at center shot $x_s = 1$ km
Inverted Velocity

Figure: Inverted velocity by FWI and volume-based MSWI with 9-20 Hz data
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Layer Salt Model

Figure: True model and constant initial model
Comparison of Results

![Inverted velocity by MSWI and FWI method with 6-12 Hz data](image_url)

**Figure:** Inverted velocity by MSWI and FWI method with 6-12 Hz data
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Marmousi

**Figure:** Marmousi model and 1D initial model
Figure: MSWI result (6-10 Hz data) and FWI result (4-8 Hz data)
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Figure: True model and 1D initial model
Inverted Results

Figure: MSWI result and FWI result (3-6 Hz data)
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Slice of SEAM Phase I Model

Figure: True model and 1D initial model
Inverted Result

Figure: MSWI result (3-6 Hz data) and FWI result (2-5 Hz data)
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Figure: True model and 1D initial model
Inverted Results

Figure: MSWI result (3-8 Hz data) and FWI result (2-5 Hz data)
Conclusion

- Nonlinear extended waveform inversion can handle reflection/refraction wave.
- No cycle skipping problem and insensitive to the frequency content and initial model.
- Equivalent to traveltime tomography under high frequency for transmission wave.
- Potential application in salt body reconstruction.
- How does it work for reflection wave?
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