Kinematic artifacts in the extended subsurface offset domain
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Summary

We provide a kinematic analysis of the prestack image behaviour in the subsurface offset domain. When the medium properties are perfectly known, the image is expected to focus at the zero subsurface offset, where the incident and the scattering wavefields interact at a common point. However, kinematic artifacts are often observed in subsurface offset common-image gathers (CIGs) away from the zero offset trace, and artificially impairs the expected focusing. These artifacts emerge in relation with the acquisition geometry truncation at the boundaries of the seismic survey extent. We suggest a formation mechanism for the artifacts emergence by considering seismic migration as a superposition of subsurface offset extended impulse responses, contributed by individual data traces. The accumulation of the image, in a trace-by-trace manner, gives an insight to its fundamental building blocks that better explains the formation of the kinematic artifacts. We also discuss the defocusing of the subsurface offset image due to an erroneous migration velocity. In such case, the kinematic artifacts are formed by the same mechanism, while interfering with the essential defocusing information of the image away from the zero offset trace.
Introduction

Prestack migration operators are often described as the adjoint of extended Born-type modeling operators, after extending the definition of the reflectivity to depend on more degrees of freedom (Symes, 2008; Stolk et al., 2009). One conventional choice of extension is the horizontal subsurface offset (Claerbout, 1985; Sava and Vasconcelos, 2011). It is defined as the horizontal offset vector connecting the sunken shot and receiver in the subsurface, and involves an action at distance between the incident and scattered wavefields. For perfectly known velocity model, significant action takes place only at zero subsurface offset (i.e. the physical offset), where the image is expected to focus. Likewise, erroneous velocity defocuses the image and may produce a fake event at non-zero offset (i.e. non-physical offset). However, kinematic artifacts usually emerge in the subsurface offset extended image, away from the zero offset trace, regardless to the migration velocity assurance (Mulder, 2014; Almomin and Biondi, 2014).

In relation with seismic imaging, an impulse response refers to the migration of a single data trace (Yilmaz, 2001). By considering prestack migration in the subsurface offset domain as a superposition of extended impulse responses, the mentioned kinematic artifacts are explained in relation with an abrupt truncation of the acquisition geometry. The data traces acquired at the boundaries of this geometry leave some non-destructive signal in the image space while being imaged.

In the following, we suggest a formation mechanism for the appearance of these kinematic artifacts, while imaging a general dipping reflector in a homogenous medium. It is based on the formulation of the subsurface offset extended impulse response, proposed here as well.

Subsurface offset extended impulse response

We employ the following integral operator, as introduced by Stolk et al. (2009), to calculate the prestack image \( I(x, h) \), extended by the horizontal subsurface half-offset \( h \):

\[
I(x, h) = \int dx_s \int dx_r \int dt \frac{\partial^2}{\partial t^2} D(x_s, t; x_r) \int d\tau G(x + h, t - \tau; x_r) G(x - h, \tau; x_s) ,
\]

where \( G(x, t) \) is the Green’s function, \( D(x_r, t; x_s) \) stands for the seismic data, and \( \tau \) is the migration time. An extended impulse response in the subsurface offset domain is the outcome of applying the operator in equation 1 on a single data trace. In the following, the kinematic properties of this response are under study. Less importance is attributed to the amplitude behavior.

Representing the migrated data trace as a delta function, shifted to the reflection’s time \( t_{sr} \), and substituting accordingly the data term in equation 1 yields

\[
I(x, h) = \int dt \frac{\partial^2}{\partial t^2} \delta(t - t_{sr}) \int d\tau G(x - x_r + h, t - \tau) G(x - x_s - h, \tau) .
\]

Integration over time leads to the following expression:

\[
I(x, h) = \int d\tau G(x - x_r + h, t_{sr} - \tau) G(x - x_s - h, \tau) .
\]

Assuming a homogenous medium is under study (constant velocity \( V \)), we employ a whole space uniform velocity Green’s function in equation 3, and recast it respectively as

\[
I(x, h) = \int d\tau \frac{\delta\left( t_{sr} - \tau - \frac{|x - x_r + h|}{V} \right)}{\delta\left( \frac{|x - x_s - h|}{V} \right)} .
\]

Next, we substitute the shot-receiver surface coordinates \((x_r, x_s)\) with the midpoint and acquisition half-offset coordinates \((x_m, H)\) and integrate over the migration time \( \tau \):

\[
I(x, h) = \frac{\delta\left( t_{sr} - \frac{|(x - x_m) + (h - H)| + |(x - x_m) - (h - H)|}{V} \right)}{|(x - x_m) + (h - H)|| (x - x_m) - (h - H)|} .
\]
Hence, image is constructed according to the argument of the delta function in equation 5, which defines the kinematic imaging condition as:

$$t_{sr} = \frac{[(x-x_m) + (h-H)] + [(x-x_m) - (h-H)]}{V}.$$  \[(6)\]

In the general 3D case and for a given acquisition offset, this condition represents an ellipsoid in the extended image space of the form:

$$\frac{(x-x_m)^2}{(\frac{1}{2}V_{t,m})^2} + \frac{(y-y_m)^2}{(\frac{1}{2}V_{t,m})^2} + \frac{z^2}{(\frac{1}{2}V_{t,m})^2 - (h-H)^2} = 1.$$  \[(7)\]

We rewrite this equation according to the traveltime hyperbolic relation with the acquisition offset, and while taking into account migration velocity errors:

$$\frac{(x-x_m)^2}{\varepsilon^2(z_0^2 + H^2)} + \frac{(y-y_m)^2}{\varepsilon^2(z_0^2 + H^2)} + \frac{z^2}{\varepsilon^2(z_0^2 + H^2) - (h-H)^2} = 1,$$  \[(8)\]

where $z_0$ is the zero-dip imaging depth of the seismic event (or the bottommost point of the ellipsoid), and $\varepsilon$ is the ratio between the migration velocity and the true velocity. The ellipsoid represents an isochron surface of the constant traveltime $t_{sr}$, and considered here as a subsurface offset extended impulse response.

**Imaging by a superposition of subsurface offset extended impulse responses**

In this study, we consider prestack migration in the subsurface offset domain as a superposition of extended impulse responses, made by individual data traces. In the 2D case and according to equation 8, the extended impulse response has an elliptic form in $z-x$ image sections (constant $h$):

$$\frac{(x-x_m)^2}{\varepsilon^2(z_0^2 + H^2)} + \frac{z^2}{\varepsilon^2(z_0^2 + H^2) - (h-H)^2} = 1.$$  \[(9)\]

Note that the ellipse’s center is shifted by the data midpoint coordinate $x_m$, and that the focal distance is defined by the offsets difference $|h-H|$. Therefore, only when the subsurface offset is set to zero ($h=0$), the focal points represent the shot-receiver coordinates on the acquisition surface $(x_s, x_s)$. In any other non-zero case, the focal points are shifted.

Rearranging equation 9 to represent the $z-h$ image gather domain (constant $x$) yields another elliptic expression:

$$\frac{(h-H)^2}{\varepsilon^2(z_0^2 + H^2)} + \frac{z^2}{\varepsilon^2(z_0^2 + H^2) - \Delta x^2} = 1,$$  \[(10)\]

where $\Delta x=x-x_m$ is the imaging aperture, which represents the focal distance of the elliptic response in the gather domain. Also note that the ellipse is shifted on the gather axis by the acquisition offset $H$.

We exemplify our formulation so far by applying the extended migration operator in equation 1 to migrate a synthetic dataset, acquired above a homogenous 2D medium, consisting of a -5º dipping reflector. We demonstrate the extended impulse response behaviour by restricting the dataset to include two traces only, acquired with the offsets $H=0$m and $H=2500$m. The corresponding image results are presented in Figure 1 (true migration velocity was used). A couple of $z-x$ image sections are shown in Figure 1a, representing the subsurface offsets: $h=0$m and $h=1000$m (top and bottom respectively). Two elliptic responses are recognized in each of these image sections with regards to the two data traces. They perfectly follow the appropriate elliptic curves, calculated by equation 9 after setting $\varepsilon=1$, which are illustrated to the right. Note that only in the upper image section, where $h=0$m, both ellipses come in phase tangent to the subsurface reflector position (marked with green), and contribute constructively to its formation.

Figure 1b presents the image results in the $z-h$ image gather domain (i.e. subsurface offset CIG), at the location $x=4$km (marked with red in Figure 1a). Two elliptic-shaped responses are clearly shown. They match the analytic curves, calculated by equation 10 after setting $\varepsilon=1$, which are illustrated to the right. In the gather domain, a focused image of the reflector is constructed at the zero offset trace.
where all the extended impulse responses share a common intersection point (imaging depth of 2km in this example).

**Data truncation kinematic artifacts in the subsurface offset domain**

As the migration operator accumulates the response of more and more data traces, a constructive interference occurs at the zero offset trace of the subsurface offset CIG. All the elliptic impulse responses intersect the same imaging depth where the reflector’s image is constructed. Moreover, out of phase destructive interference takes place elsewhere. As a result, the final image becomes focused at zero subsurface offset. However, since the acquisition geometry is always bounded by a finite maximum offset \( H_{\text{max}} \), this destructive interference away from the zero subsurface offset leaves some remnant non-destructive part of energy that contaminates the image. This arouses an intriguing relation between the acquisition and subsurface offsets: The subsurface offset CIGs are contaminated with kinematic artifacts, due to the truncation of the seismic data by a maximum acquisition offset.

We demonstrate this by migrating the same dataset, collected above the -5º dipping reflector, but without any restrictions on the input traces. Note that the dataset was acquired by a ‘split-spread’ geometry, bounded by the maximum acquisition offset \( H_{\text{max}}=\pm2500\text{m} \). Figure 2a presents the resulting image. It shows the zero subsurface offset image section to the left of a subsurface offset CIG, calculated at the marked position \( x=4\text{km} \). The image is focused at zero offset, although some energy clearly leaks to non-zero offsets. This ‘leakage’ is linked to the truncation of the acquisition geometry by \( H_{\text{max}} \). The illustration on the right side of the figure simulates the formation of the image. Elliptic curves, calculated according to equation 10 after setting \( \varepsilon=1 \), are accumulated in such a way that they all intersect the same depth with the vertical axis. The mentioned artifact in the image follows the elliptic curve contributed by the data traces acquired with this maximum offset \( H_{\text{max}} \) (highlighted in the illustration), and therefore considered as data truncation kinematic artifact.

While imaging by an erroneous migration velocity, the resulting image in the subsurface offset domain is defocused. We demonstrate this defocusing by the CIGs in Figures 2b and 2c, calculated at the same location \( x=4\text{km} \), by using 10% too-high and too-low migration velocity respectively. The defocusing of the image is explained by the illustrations on the right side of the figures. They show a superposition of elliptic curves according to equation 10, after setting \( \varepsilon \) to 1.1 and 0.9 respectively, to account for the velocity error. When the velocity is wrong, the ellipses do not share a common intersection point at zero subsurface offset. The defocusing is formed by the envelope of all elliptic curves. It has a concave shape curving, up or down, with respect to the velocity error. The appearance of the kinematic artifact is deformed in the image due to the velocity error. However, its formation mechanism is still related to \( H_{\text{max}} \), as emphasized by the highlighted curves in the illustrations.

![Figure 1](image-url)  
*Figure 1* Subsurface offset extended impulse response. (a) \( z-x \) image sections extracted at the subsurface offsets 0m (top) and 1000m (bottom). (b) Subsurface offset CIG, calculated at \( x=4\text{km} \).
Figure 2 Imaging of a -5° dipping reflector in the subsurface offset domain, by using (a) true, (b) 10% too-high, and (c) 10% too-low migration velocity.

Conclusions

Extending the migration operator to the horizontal subsurface offset domain allows exploiting the redundancy of the seismic data by utilizing subsurface offset CIGs. Although a focused image at zero subsurface offset is prominently formed when the medium properties are known, some non-destructive signal leaks and contaminates the extended image with kinematic artifacts. These artifacts commonly emerge as an edge effect related to the seismic survey design. It is a consequence of an abrupt truncation of the acquisition geometry at the edges of the survey extent. We suggested a formation mechanism for the mentioned artifacts by considering seismic migration as a superposition of subsurface offset extended impulse responses, contributed by individual data traces. The same mechanism was suggested to the case when the medium properties are unknown. In such a case, the kinematic artifacts interfere with the essential defocusing information of the image, away from the zero offset trace. Understanding the artifacts origin, by tracking down the fundamental building blocks of the image, is the first step in the vital attempt of their elimination.

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References