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- EDUCATION

  PhD student, Rice University — 2011 - present
  Geophysics

  MS, University of Utah — 2009 - 2011
  Electromagnetic Modeling and Inversion

  BS, University of Science and Technology of China — 2005 - 2009
  Geophysics, seismology

- RESEARCH

  Subsurface offset extended seismic full waveform inversion

  Reducing the cost of extended waveform inversion by multiscale adaptive methods

  Marine 3-D seismic data of Galicia rifted margin

Reducing the cost of extended waveform inversion by multiscale adaptive methods

Lei Fu, William W. Symes

The Rice Inversion Project (TRIP)

May 1, 2015

Overview

Objective

Recover Earth model by waveform inversion with extension

Problems

Computational cost

Solution

Multiscale method

Adaptive approach

Extended modeling concept

Abstract setting for forward map $\mathcal{F} : \mathcal{M} \to \mathcal{D}$

$$F[m] = d$$

F: forward modeling operator
m: model $(v,\, r)$
d: sampled pressure data at receivers

In order to fit the data, thus avoiding cycle-skipping.

Extended forward map $\bar{\mathcal{F}} : \bar{\mathcal{M}} \to \mathcal{D}$

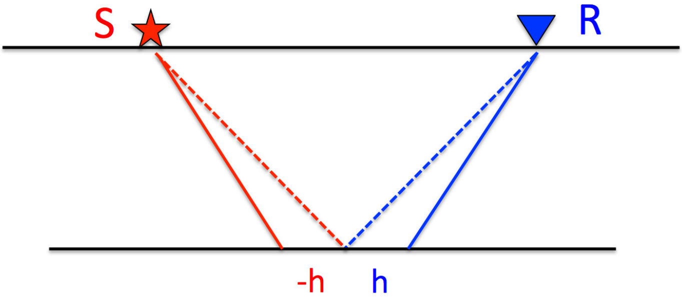
$$\bar{F}[\bar{m}] = d$$

$\bar{F}$: extended forward modeling operator
$\bar{m}$: extended model $(v(\mathbf{x}),\, \bar{r}(\mathbf{x}, \mathbf{h}),\, ...)$

Subsurface offset extension

Subsurface Extension: 2h = distance between subsurface scattering points (subsurface offset)

Physical meaning: action at a distance



Subsurface offset extended linearized acoustic modeling $\bar{F}_H[v]\bar{r}$

u - reference (incident) pressure field

$$\left(\frac{1}{v^2(\mathbf{x})}\frac{\partial^2}{\partial t^2} - \nabla^2\right) u(t,\mathbf{x};\mathbf{x_s}) = w(t)\delta(\mathbf{x}-\mathbf{x_s}) \quad (1)$$

δu - scattered (perturbation) pressure field

$$\left(\frac{1}{v^2(\mathbf{x})}\frac{\partial^2}{\partial t^2} - \nabla^2\right) \delta u(t,\mathbf{x};\mathbf{x_s}) = \frac{\partial^2}{\partial t^2}\int_{-H}^{H} d\mathbf{h}\frac{2\bar{r}(\mathbf{x},\mathbf{h})v(\mathbf{x})}{v(\mathbf{x}+\mathbf{h})v^2(\mathbf{x}-\mathbf{h})}u(t,\mathbf{x};\mathbf{x_s}) \quad (2)$$

v: P-wave velocity
$w(t)$: source function, t: time
$\mathbf{h}$: horizontal subsurface offset
$\mathbf{x}$: position
$\mathbf{x}_s$: source location
H: limit of $\mathbf{h}$

$\bar{r}(\mathbf{x},\mathbf{h}) = \dfrac{\delta\bar{v}(\mathbf{x},\mathbf{h})}{v(\mathbf{x})}$: extended reflectivity ($\delta\bar{v}(\mathbf{x},\mathbf{h})$: extended velocity perturbation)

Extended full waveform inversion (EFWI)

Objective function:

$$min_{v,\bar{r}} J[v,\bar{r}] = \frac{1}{2}\|\bar{F}_H[v]\bar{r} - d\|^2 + \frac{\alpha^2}{2}\|A\bar{r}\|^2$$

A: annihilator, differential semblance operator, multiplication by h. $\alpha = 0$, can fit data with any v; $\alpha > 0$, penalty for non-focus.

Separable least-squares, solved with variable projection method [Golub and Pereyra, 1973]. The inverse problem is solved by a nested optimization approach:

Inner loop, optimize $J[v,\bar{r}]$ over $\bar{r}$.

Outer loop, optimize reduced objective function $J[v,\bar{r}[v]]$ over v.

Inner loop, optimize J over $\bar{r}$

Gradient of the objective function $J[v, \bar{r}]$ with respective to $\bar{r}$:

$$\nabla_{\bar{r}} J[v, \bar{r}] = \bar{F}_H[v]^*(\bar{F}_H[v]\bar{r} - d) + \alpha^2 A^* A \bar{r}$$

where $*$ denotes adjoint.

Setting the gradient function to zero, least-squares extended reverse time migration (LSERTM) is solved by a linear iterative method, e.g. conjugate gradient method.

$$(\bar{F}_H[v]^* \bar{F}_H[v] + \alpha^2 A^* A)\bar{r} = \bar{F}_H[v]^* d$$

Outer loop, update v

The gradient of the reduced objective function $J[v, \bar{r}[v]]$ respect to v:

$$\nabla_v J[v, \bar{r}[v]] = \Lambda^{-2s} D\bar{F}_H[v]^T \left(\bar{r}[v], \bar{F}_H[v]\bar{r}[v] - d\right) \tag{3}$$

where Λ^{-2s} is a smoothing operator for positive s [Symes and Kern, 1994].

Why $\bar{F}_H[v]\bar{r}$ expensive?

Recall equation 2:

$$\left(\frac{1}{v^2(\mathbf{x})}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\delta u(t,\mathbf{x};\mathbf{x_s}) = \frac{\partial^2}{\partial t^2}\int_{-H}^{H} d\mathbf{h}\frac{2\bar{r}(\mathbf{x},\mathbf{h})v(\mathbf{x})}{v(\mathbf{x}+\mathbf{h})v^2(\mathbf{x}-\mathbf{h})}u(t,\mathbf{x};\mathbf{x_s})$$

Computational cost

Increase with number of grid points in h ($N_h = \dfrac{2H}{dh}$)

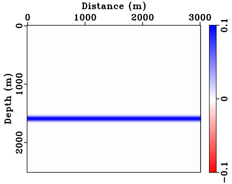
$N_h \downarrow \Leftarrow dh \uparrow$ coarse grid $\Leftarrow$ multiscale method

$N_h \downarrow \Leftarrow H \downarrow$ How to choose and decrease H?

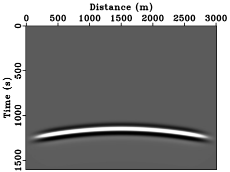
Single reflector example

2D constant density acoustic 2-8 order finite difference code

Source	Ricker wavelet $f_{peak} = 15\,Hz$
Source position $\mathbf{x_s}$	$x : 300 - 2700\,m$ every $40\,m$, $z = 0\,m$
Receiver position $\mathbf{x_r}$	$x : 0 - 3000\,m$ every $20\,m$, $z = 0\,m$
Subsurface offset h	$-1500\,m \leqslant h \leqslant 1500\,m$
Space and time	$x = 3000\,m,\ z = 2500\,m,\ t = 1.6\,s$
Grid size	$dx = dh = dz = 20\,m,\ dt = 2\,ms$
Background velocity	$v_{true} = 3.0\,km/s$



(a)



(b)

Figure : (a) Extended reflectivity $\bar{r}$ at $h = 0\,m$ (b) data at shot 31 at the center

Single reflector example

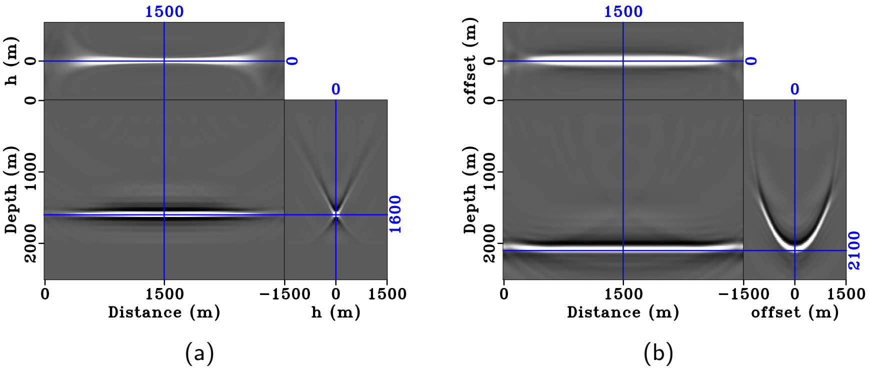


Figure : Inverted $\bar{r}$, 20 iterations of CG, $\alpha = 0$. (a) $v = v_{true}$ (b) $v = 1.3 v_{true}$

Inverted $\bar{r}$ at $x = 1500\,m$ for different v

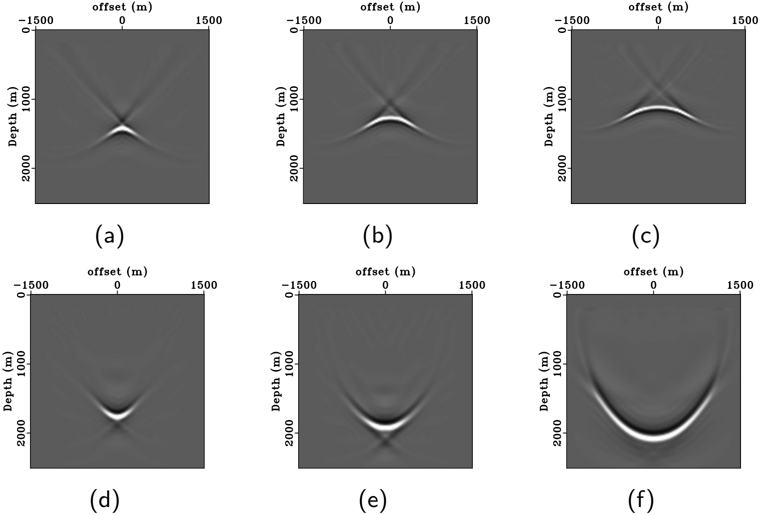


Figure : 20 iterations of CG (a) $v = 0.9v_{true}$, (b) $0.8v_{true}$, (c) $0.7v_{true}$, (d) $1.1v_{true}$, (e) $1.2v_{true}$, (f) $1.3v_{true}$

Relationship between H and $\Delta d_H = \|\bar{F}_H[v]\bar{r} - d\|$

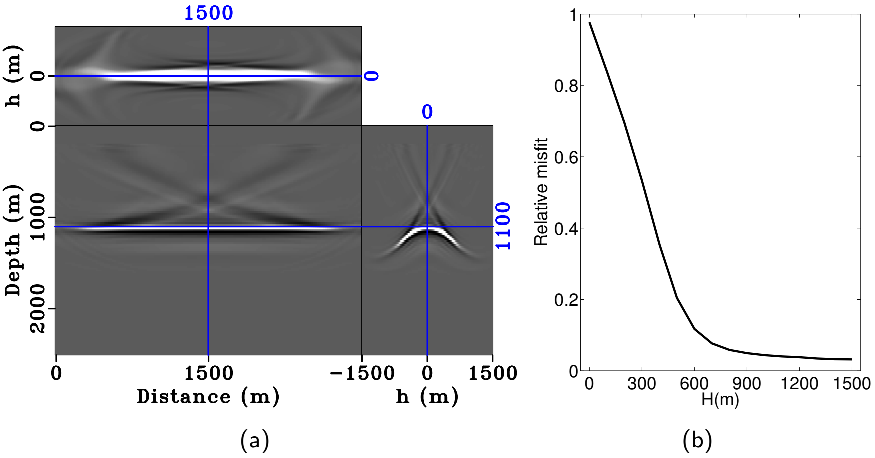


Figure : 20 iterations of CG, $v = 0.7v_{true}$ (a) inverted $\bar{r}$ (b) H vs relative Δd_H

Relationship between H and Δd_H at different v

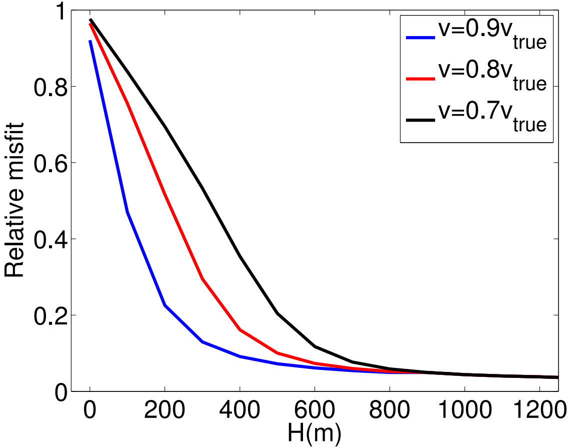


Figure : Inverted $\bar{r}$, 20 iterations of CG

Multiscale adaptive method for determining H

1.Given d, w, and v, choose:

lowest frequency, filter

dx, dh and dz

CFL condition, dt

initial H

2. Solve LSERTM by CG, $(\bar{F}_H[v]^* \bar{F}_H[v] + \alpha^2 A^* A)\bar{r} = \bar{F}_H[v]^* d$

3. Compute $\Delta d_H = \|\bar{F}_H[v]\bar{r} - d\|$ and $\Delta d_{H/2} = \|\bar{F}_{H/2}[v]\bar{r} - d\|$

Multiscale adaptive method for determining H

1. $d_{obs}, w \leftarrow$ low-pass($f_{min} - f$) on d_{obs} and w
2. $\bar{r} \leftarrow$ Solve LSERTM
3. $\Delta d_{H/2}, \Delta d_H \leftarrow \bar{r}$

if $\Delta d_H < X$ **and** $\Delta d_{H/2} < X$ **then**

$(dh, H, d\mathbf{x}, dt) \leftarrow (dh/2, H/2, d\mathbf{x}/2, dt/2)$, $f \leftarrow 2f$, go to 1

else if $\Delta d_H < X$ **and** $\Delta d_{H/2} \geqslant X$ **then**

exit

else

$H \leftarrow 2H$, go to 2 $\quad$ // $\Delta d_{H/2} \geqslant \Delta d_H \geqslant X$

end if

Layer model

2D constant density acoustic 2-8 order finite difference code

Source wavelet	bandpass $3-30\,Hz$
Source position $\mathbf{x_s}$	$x: 0-3\,km$ every $0.1\,km$, $z=0\,m$
Receiver position $\mathbf{x_r}$	$x: 0-3\,km$ every $12.5\,m$, $z=0\,m$
Space and time	$x=3\,km$, $z=2.5\,km$, $t=2.4\,s$
Grid size	$dx=dh=dz=12.5\,m$, $dt=2\,ms$
Background velocity	$v_{true}=3.0\,km/s$

Layer model

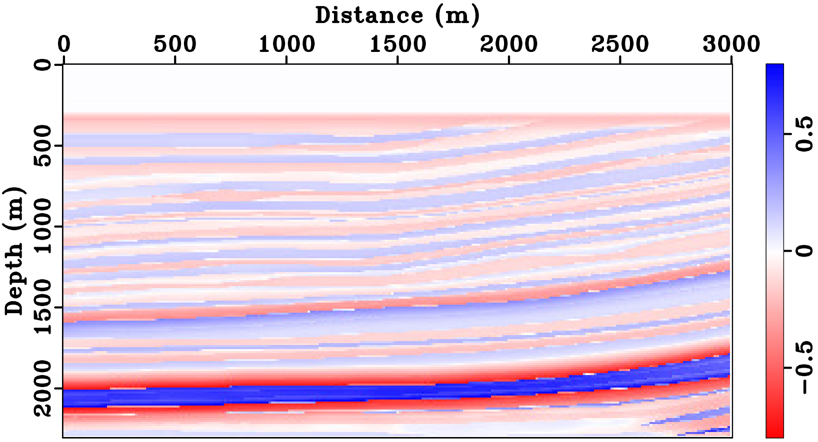
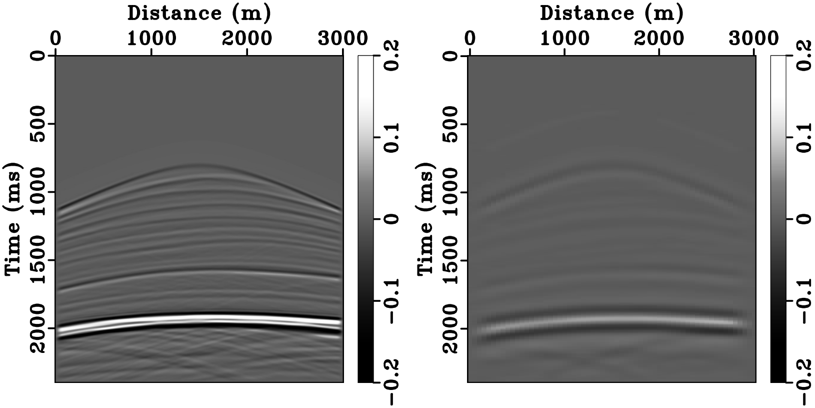


Figure : Extended reflectivity $\bar{r}$ at $h = 0\,m$

Data



(a) (b)

Figure : (a) original data (b) filtered data ($f : 3 - 8\ Hz$)

Step 1: $v = 2.4\,km/s$

$\Delta d_{250} \geqslant \Delta d_{500} \geqslant X$, $H \leftarrow 2H = 1000\,m$
$\Delta d_{1000} < X$ and $\Delta d_{500} \geqslant X$, exit

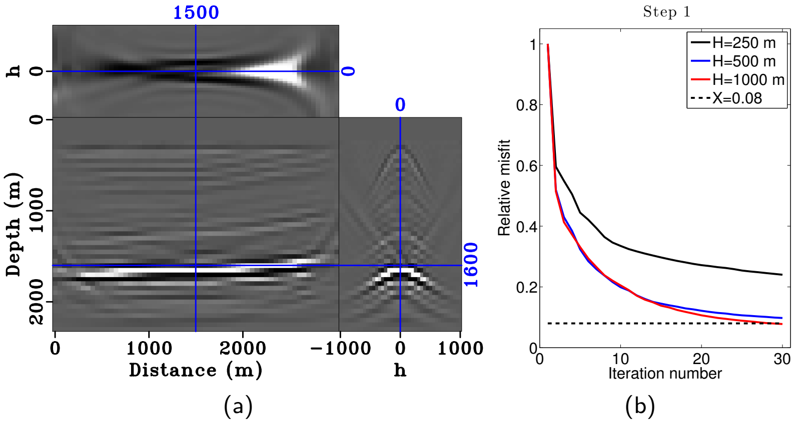


Figure : Step 1: $dh = dx = dz = 50\,m$, $f : 3 - 8\,Hz$, $dt = 8\,ms$ (a) LSERTM result of 30 CG iterations, (b) the relative data residual

Step 2: $v = 2.8\,km/s$

$\Delta d_{1000} < X$ and $\Delta d_{500} < X$, $(dh, H, d\mathbf{x}, dt) \leftarrow (dh/2, H/2, d\mathbf{x}/2, dt/2)$, $f \leftarrow 2f$
$\Delta d_{500} < X$ and $\Delta d_{250} \geqslant X$, exit

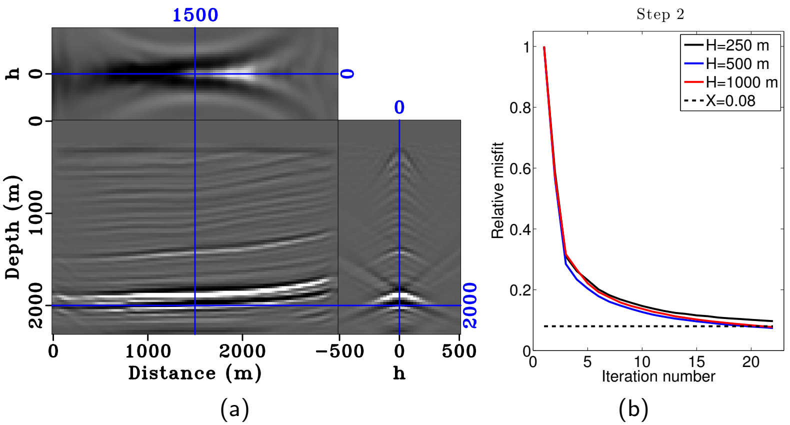


Figure : Step 2: $dh = dx = dz = 25\,m$, $f : 3 - 16\,Hz$, $dt = 4\,ms$, (a) LSERTM result of 22 CG iterations, (b) the relative data residual

Step 3: $v = 2.9\, km/s$

$\Delta d_{500} < X$ and $\Delta d_{250} < X$, $(dh, H, d\mathbf{x}, dt) \leftarrow (dh/2, H/2, d\mathbf{x}/2, dt/2)$, $f \leftarrow 2f$
$\Delta d_{250} < X$ and $\Delta d_{125} \geqslant X$, exit

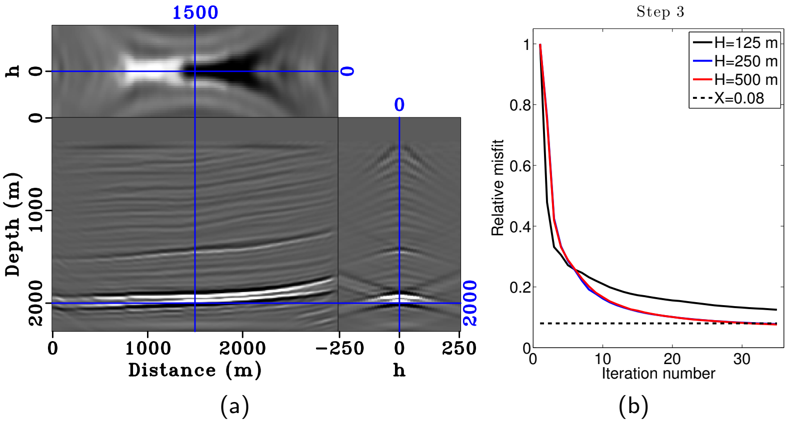


Figure : Step 3: $dh = dx = dz = 12.5\, m$, $dt = 2\, ms$, the original data and source function (a) LSERTM result of 35 CG iterations, (b) the relative data residual

Summary

Reduce computational cost

Nonadaptive algorithm with largest H and finest dh, dt cost:

$$N_{outer} * N_{inner} * Cost_{inner}, \; Cost_{inner} \propto N_h * N_x * N_z * N_t$$

Our method cost:

$$(1 \cdot \frac{1}{4^4} + \frac{1}{2} \cdot \frac{1}{2^4} + \frac{1}{4} \cdot 1)/3 \approx 8\%$$

Methods:

- Use data residual to determine sufficient H
- Multiscale method
- Adaptive approach

Future work

- Good preconditioner
- Update v
- Implementation in IWAVE

Outer loop, update v

The gradient of the reduced objective function $J[v, \bar{r}[v]]$ respect to v:

$$\nabla_v J[v, \bar{r}[v]] = \Lambda^{-2s} D\bar{F}_H[v]^T \left(\bar{r}[v], \bar{F}_H[v]\bar{r}[v] - d\right)$$

where Λ^{-2s} is a smoothing operator for positive s [Symes and Kern, 1994].

Source	Ricker wavelet $f_{peak} = 15\,Hz$
Source position $\mathbf{x_s}$	$x : 0 - 6\,km$ every $60\,m$, $z = 0\,m$
Receiver position $\mathbf{x_r}$	$x : 0 - 6\,km$ every $20\,m$, $z = 0\,m$
Subsurface offset h	$-500\,m \leqslant h \leqslant 500\,m$
Space and time	$x = 6\,km$, $z = 2\,km$, $t = 2.4\,s$
Grid size	$dx = dh = dz = 20\,m$, $dt = 3\,ms$
Initial velocity	$v = 3.0\,km/s$
Number of iteration inner loop	30

Table : Lens model

Lens model

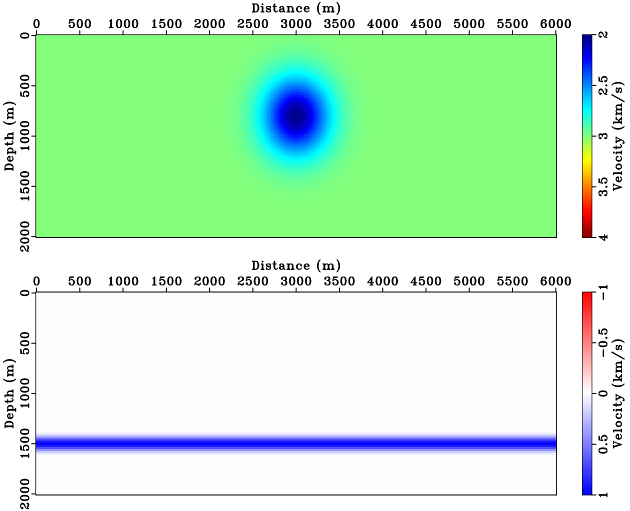


Figure : (a) background velocity (b) extended reflectivity at $h = 0\,m$

Data

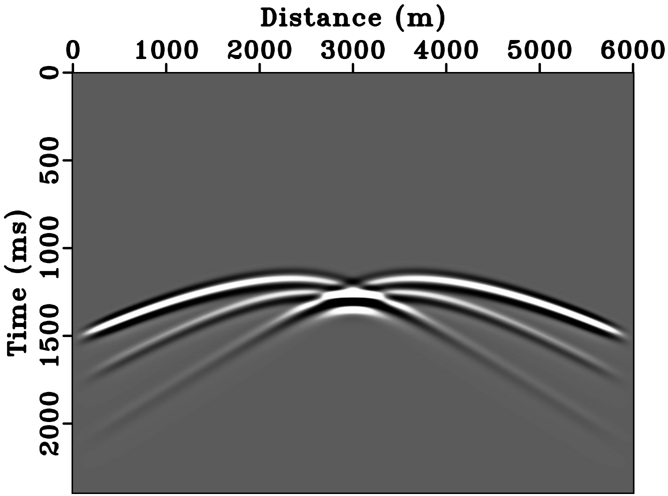


Figure : Data of shot 51 at the center

Inverted $\bar{r}$ with v_{init}

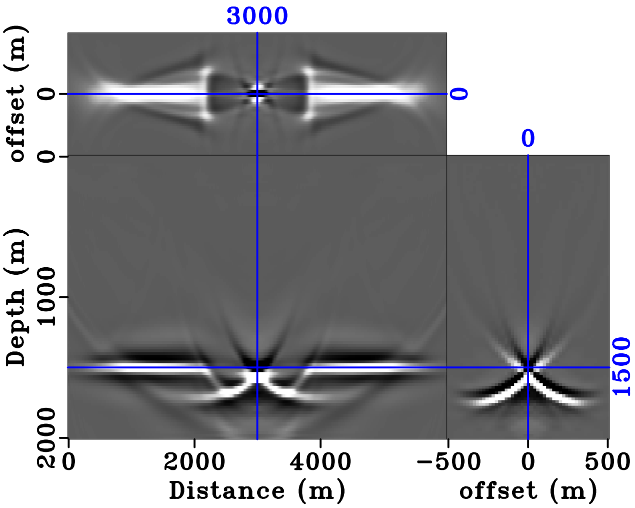
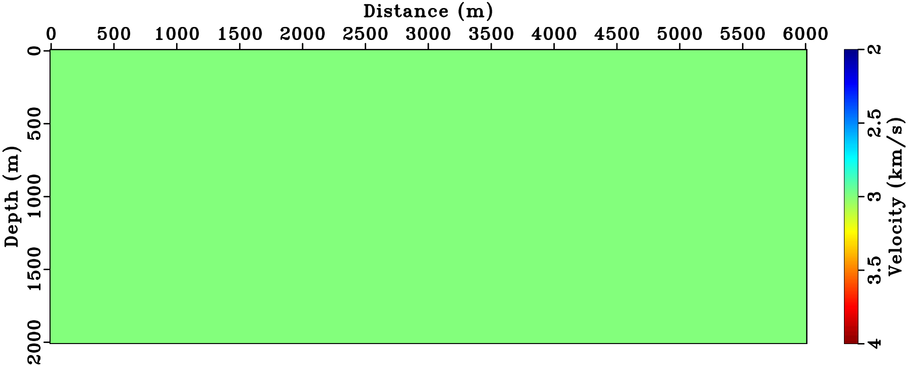


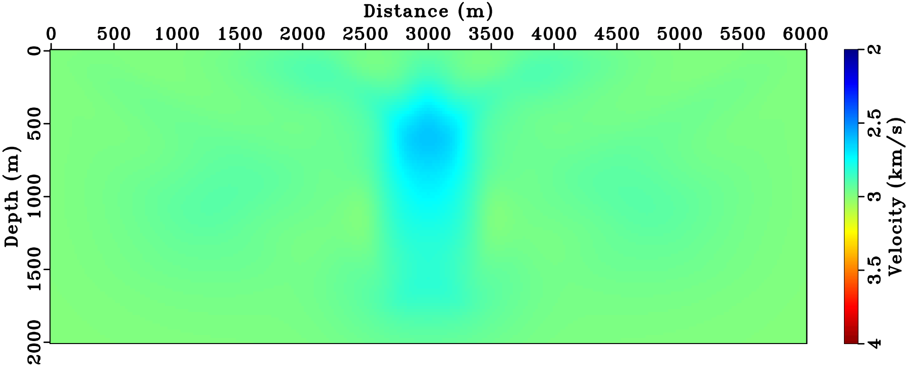
Figure : 30 CG iterations

v_{init}

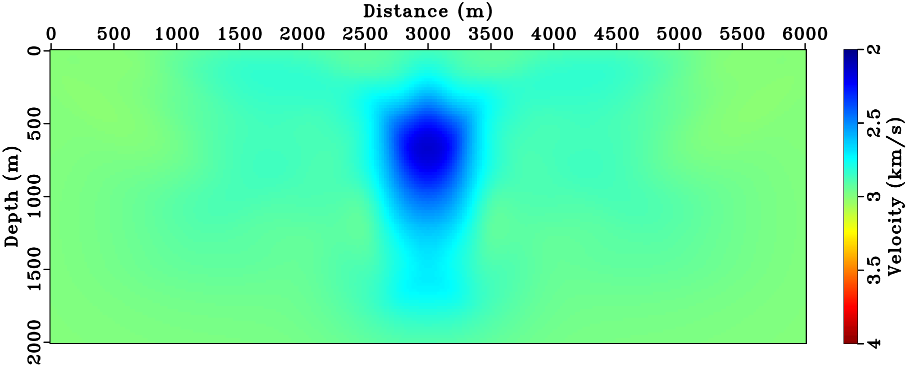


Distance (m)
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
5500
6000
Depth (m)
0
500
1000
1500
2000
Velocity (km/s)
2
2.5
3
3.5
4

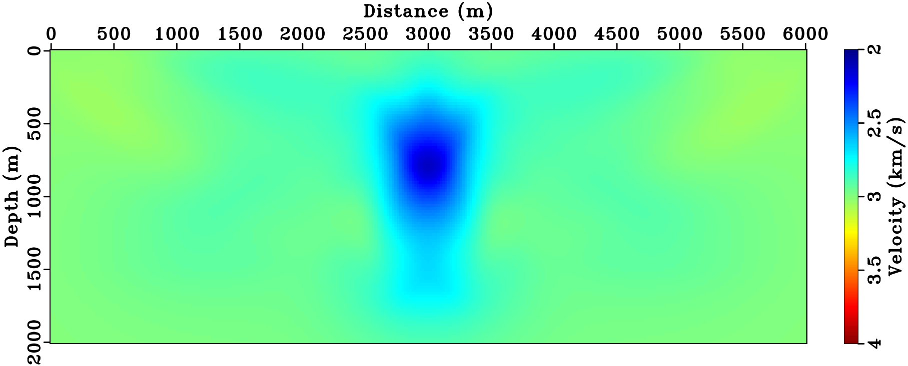
Iteration 1, v_1



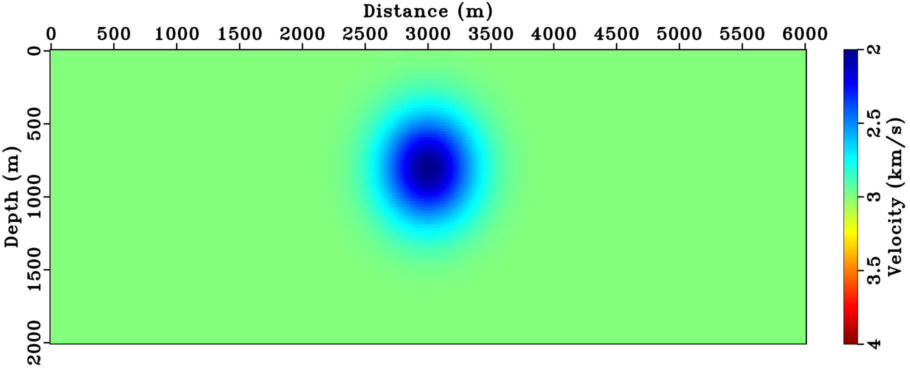
Iteration 2, v_2



Iteration 3, v_3



v_{true}



Distance (m)
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000
5500
6000
Depth (m)
0
500
1000
1500
2000
Velocity (km/s)
2
2.5
3
3.5
4

Inverted $\bar{r}$ with v_2

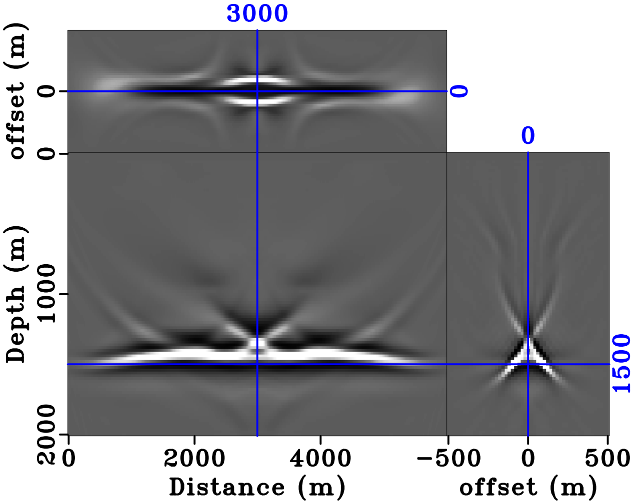


Figure : 30 CG iterations

Acknowledgement

Thanks to

TRIP sponsors and members

- Golub, G., and V. Pereyra, 1973, The differentiation of pseudoinverses and nonlinear least squares problems whose variables separate: SIAM Journal on Numerical Analysis, **10**, 413–432.
- Symes, W., and M. Kern, 1994, Inversion of reflection seismograms by differential semblance analysis: Algorithm structure and synthetic examples: Geophysical Prospecting, **in press**.
- Symes, W. W., 2008, Migration velocity analysis and waveform inversion: Geophysical Prospecting, **56**, 765–790.