Linearized Extended Waveform Inversion

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TRIP review meeting
May 6, 2014

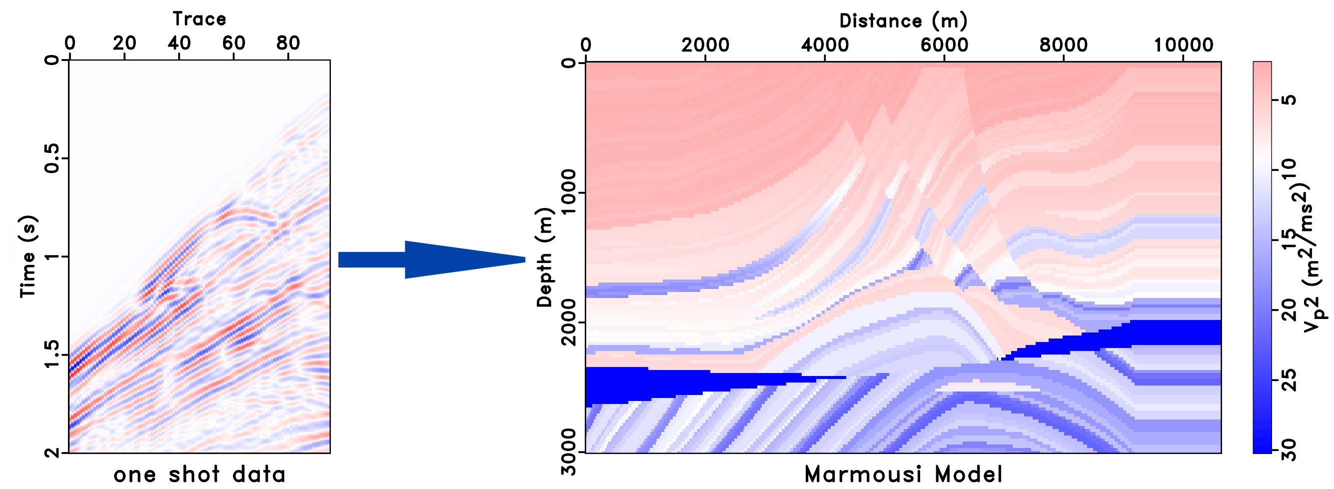
Outline

- Introduction
- Linearized extended waveform inversion and reduced objective function
- Image stability of normal operator
- Smoothness and unimodality of reduced objective function (ROF)
- Conclusion and current work

Introduction

Goal:

From a lot of reflection data, get the model of the earth.



Methods:

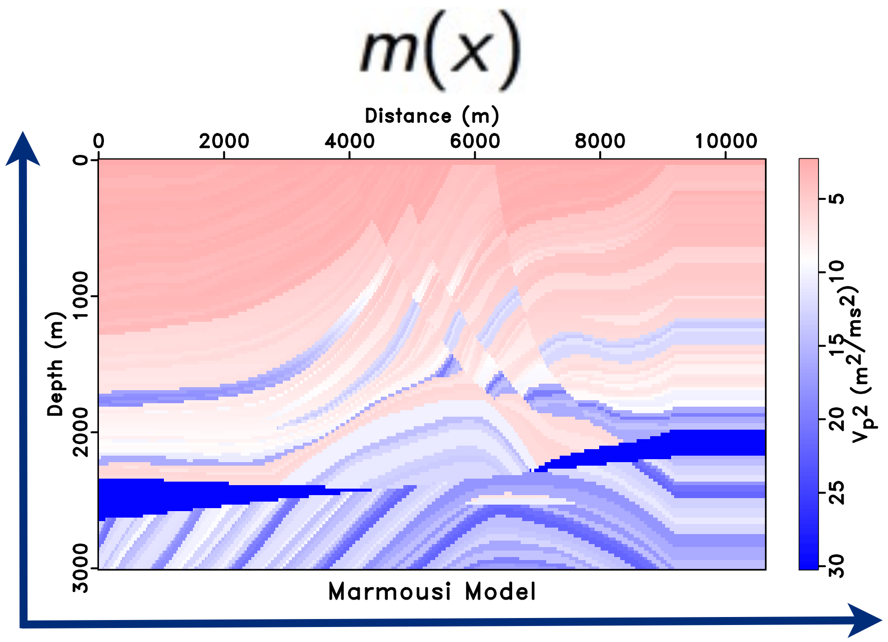
Seismic inversion (Gauthier et al., 1986; Santosa & Symes, 1989; Virieux & Operto, 2009, etc.)

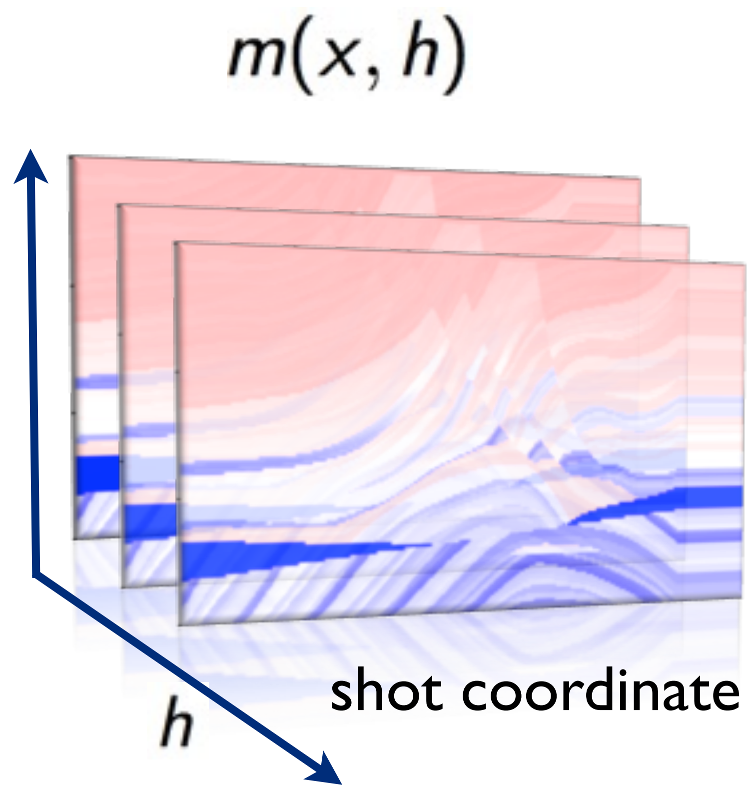
- Over-determined: highly redundant in the observed data;
- Local minima: cycle skipping.

Migration velocity analysis (Yilmaz, Seismic Data Analysis, Chapter 10; Biondi, 3D Seismic Imaging, Chapter 11 & 12)

- Part of conventional workflow
- Give correct position of reflectors

Model Extension





Model extension (Symes & Kern 1994; Symes 2008; Sun 2012; Biondi & Almomin 2012; Zhang & Biondi 2012)

- $M = \{m(x)\}$ Physical model space, velocity, density, bulk modulus, ...
- $\bar{M} = \{m(x, h)\}$ Extended model space, $M \subset \bar{M}$.
- h extended coordinate: shot position, offset, subsurface offset, or scattering angle, ...

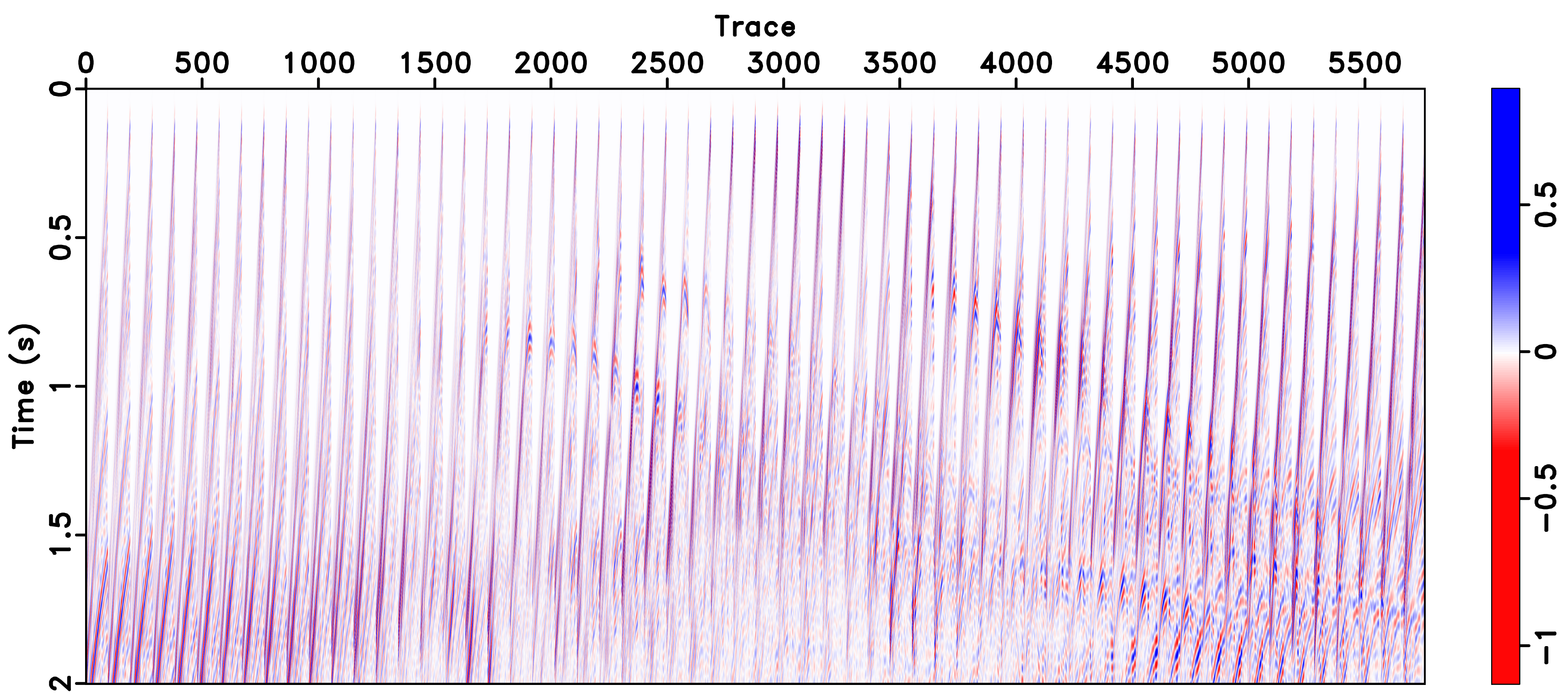
Extended Forward Modeling

$\bar{F} : \bar{M} \mapsto D$ extended forward map

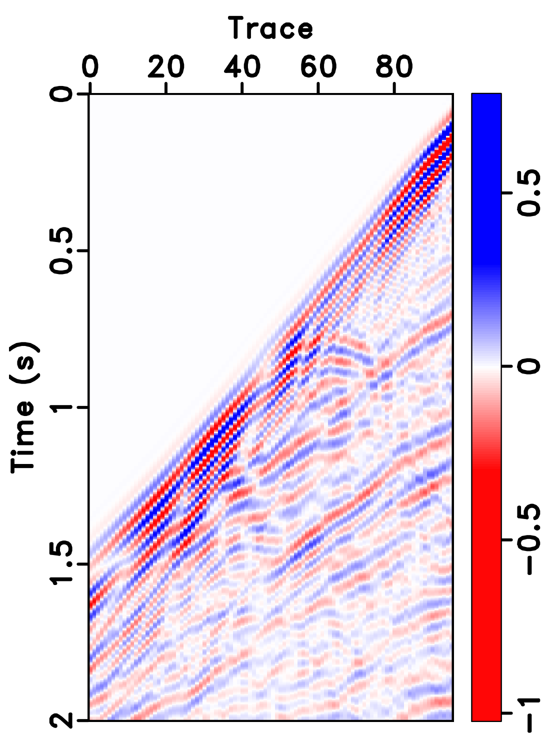
$$\bar{F}[\bar{m}] = d.$$

Ex: acoustic constant density, shot coordinate extension with $d = u(x_r, t, x_s)$ and $\bar{m} = c(x, x_s)^2$

$$\frac{\partial^2 u(x,t,x_s)}{\partial t^2} - c(x,x_s)^2 \Delta u(x,t,x_s) = f(x,t,x_s).$$



60 shots simulation data



Simulation data

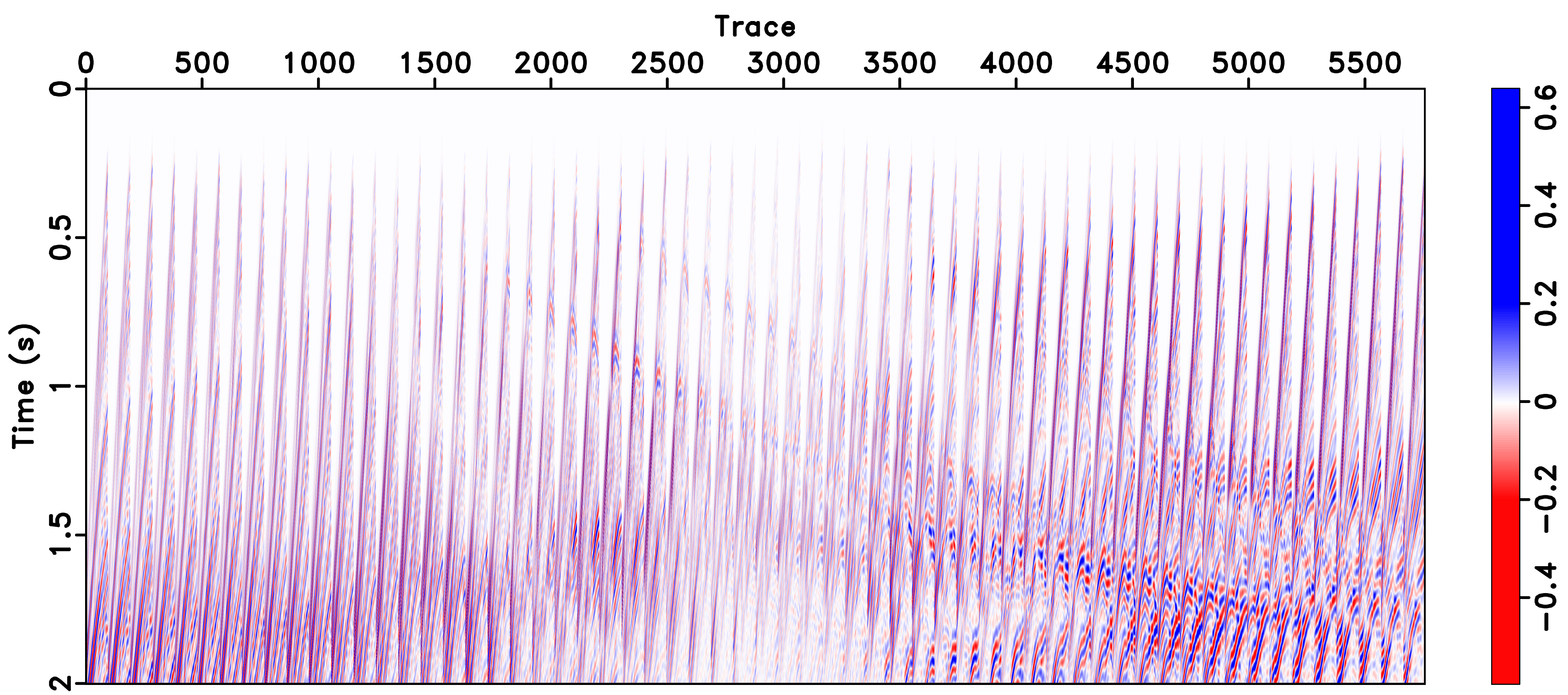
Extended Born Modeling

$D\bar{F} : M \times \bar{M} \mapsto D$ extended Born map

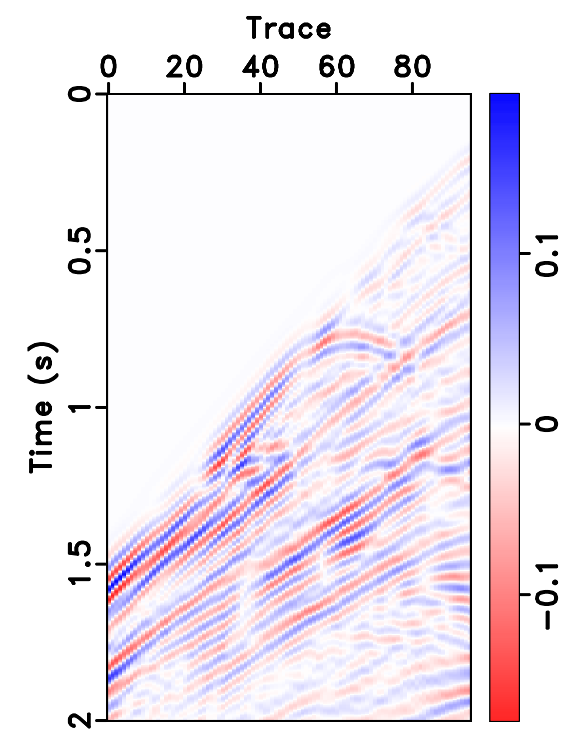
$$D\bar{F}[m_l]\delta\bar{m} = \delta d.$$

Ex: acoustic constant density, shot coordinate extension with $\delta d = \delta u(x_r, t, x_s)$, $m_l = c(x)^2$ and $\delta\bar{m} = \delta c(x, x_s)^2$

$$\frac{\partial^2 \delta u(x,t,x_s)}{\partial t^2} - c(x)^2 \Delta \delta u(x,t,x_s) = \delta c(x,x_s)^2 \Delta u(x,t,x_s).$$



60 shots Born data

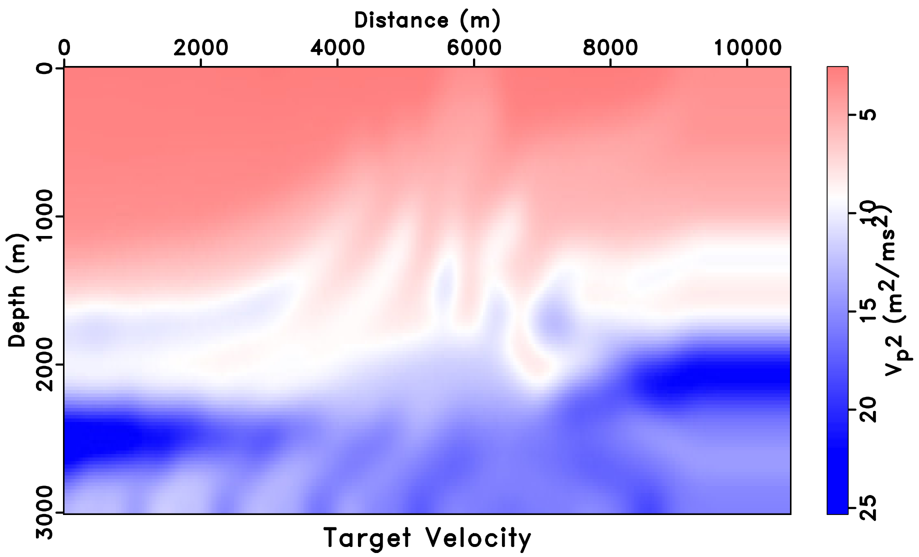


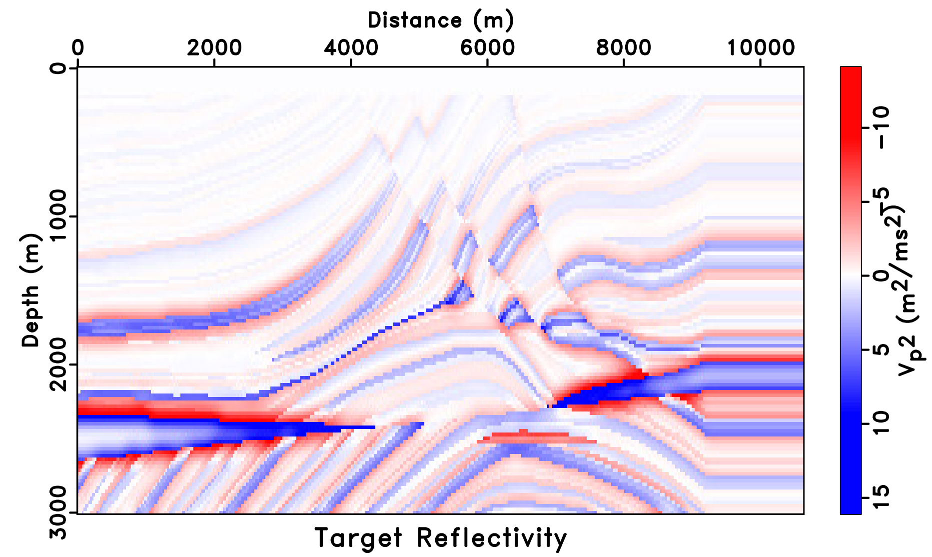
Born data

Linearized Extended Waveform Inversion

Model separation: $\bar{m} \simeq m_l + \delta\bar{m}$.

- m_l smooth background model, physical.
- $\delta\bar{m}$ reflectivity, extended.





Linearized extended waveform inversion: given reflection data $\delta d \in D$, find m_l, $\delta\bar{m}$ so that

$$D\bar{F}[m_l]\delta\bar{m} \simeq \delta d.$$

(Symes & Carazzone, 1991; Kern & Symes, 1994; Chauris & Noble, 2001; Mulder & ten Kroode, 2002; Shen & Symes, 2008; Symes 2008.)

Reduced Objective Function (ROF)

Define

$$J[m_l, \delta\bar{m}] = \frac{1}{2}\|D\bar{F}[m_l]\delta\bar{m} - \delta d\|^2 + \frac{\alpha^2}{2}\|A\delta\bar{m}\|^2$$

- A annihilator, $A\delta m = 0$ for all $\delta m \in M$.
- $A = \dfrac{\partial}{\partial x_s}$ for shot coordinate model extension.

Reduced objective function (Symes & Kern 1994)

$$\tilde{J}[m_l] = \min_{\delta\bar{m}} J[m_l, \delta\bar{m}].$$

with

$$\delta\bar{m}[m_l] = (D\bar{F}[m_l]^T D\bar{F}[m_l] + \alpha^2 A^T A)^{-1} D\bar{F}[m_l]^T \delta d.$$

An example of variable projection method (van Leeuwen & Mulder 2009)

Image Stability of Normal Operator

Normal operator

$$N[m_l] = DF[m_l]^T DF[m_l].$$

- $N[m_l]$ essentially pseudodifferential operator.
- $N[m_l]$ is smooth function of m_l.

Show $N[m_l]\delta m$ with different m_l and fixed δm

$$N[m_l]\delta m \approx L^{\frac{n-1}{2}} P\delta m$$

with L the Laplacian operator and P acts as multiplication.

Symes, 2008 Approximate linearized inversion by optimal scaling of prestack depth migration

Image Stability of Normal Operator

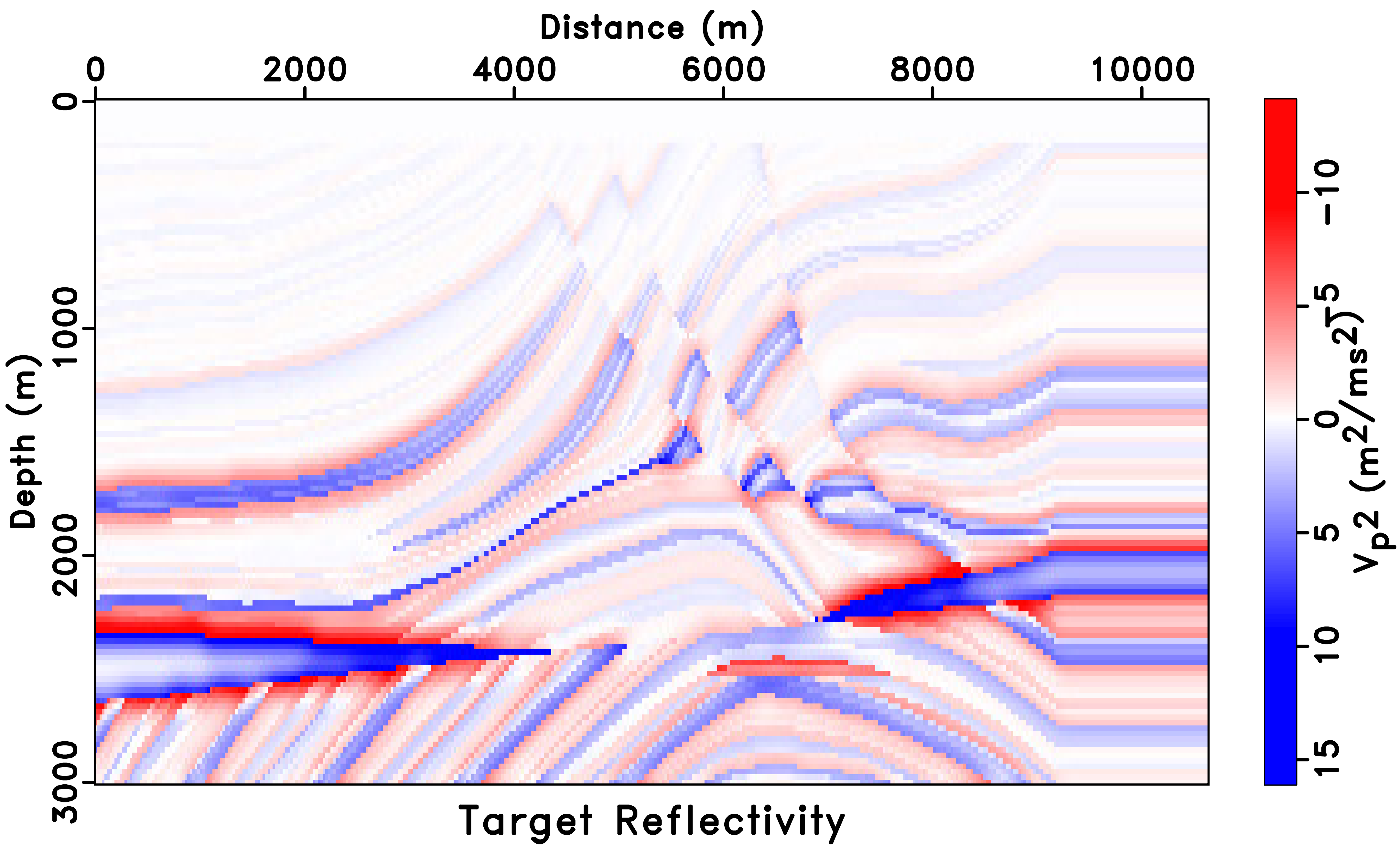
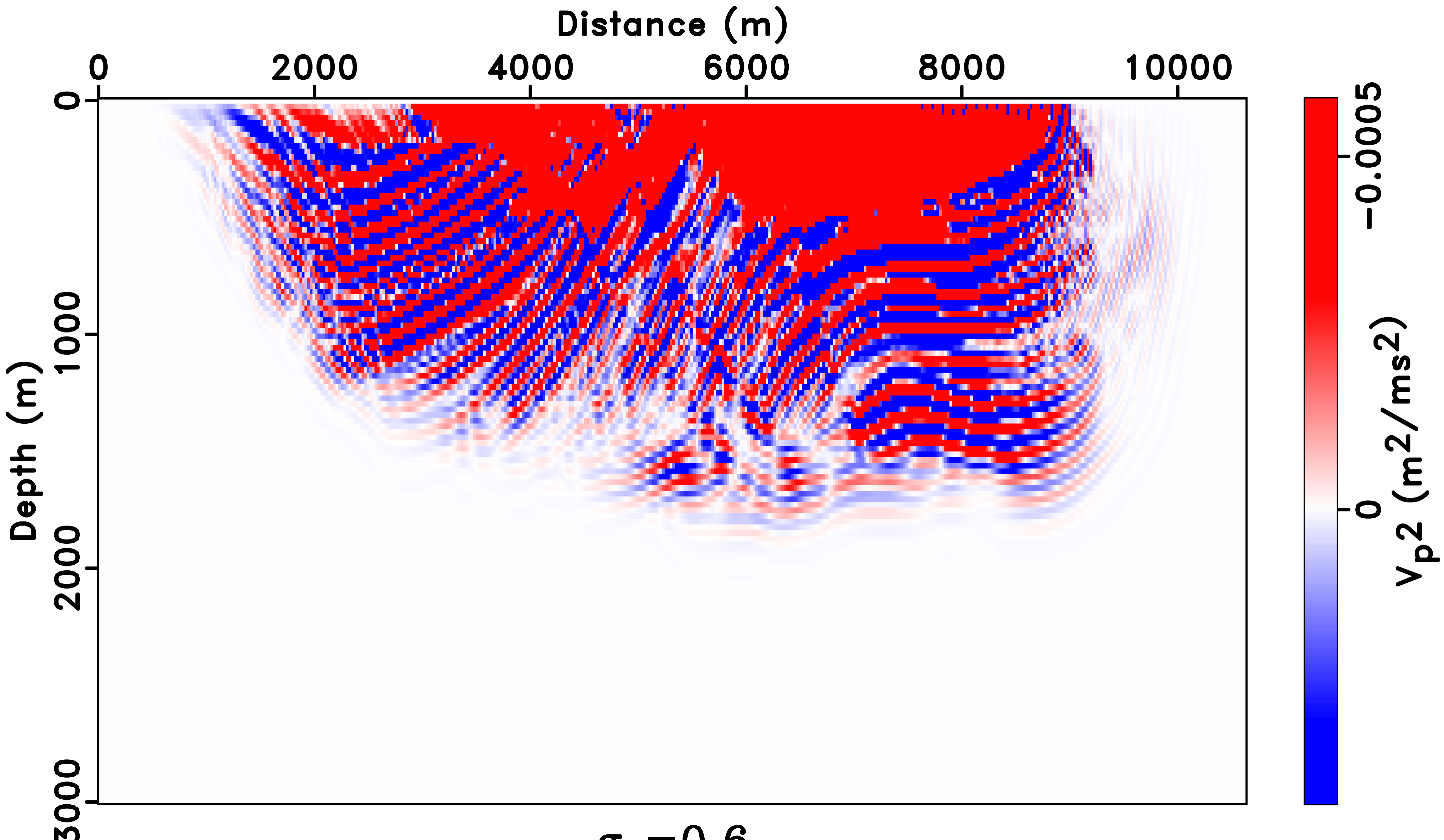


Image Stability of Normal Operator

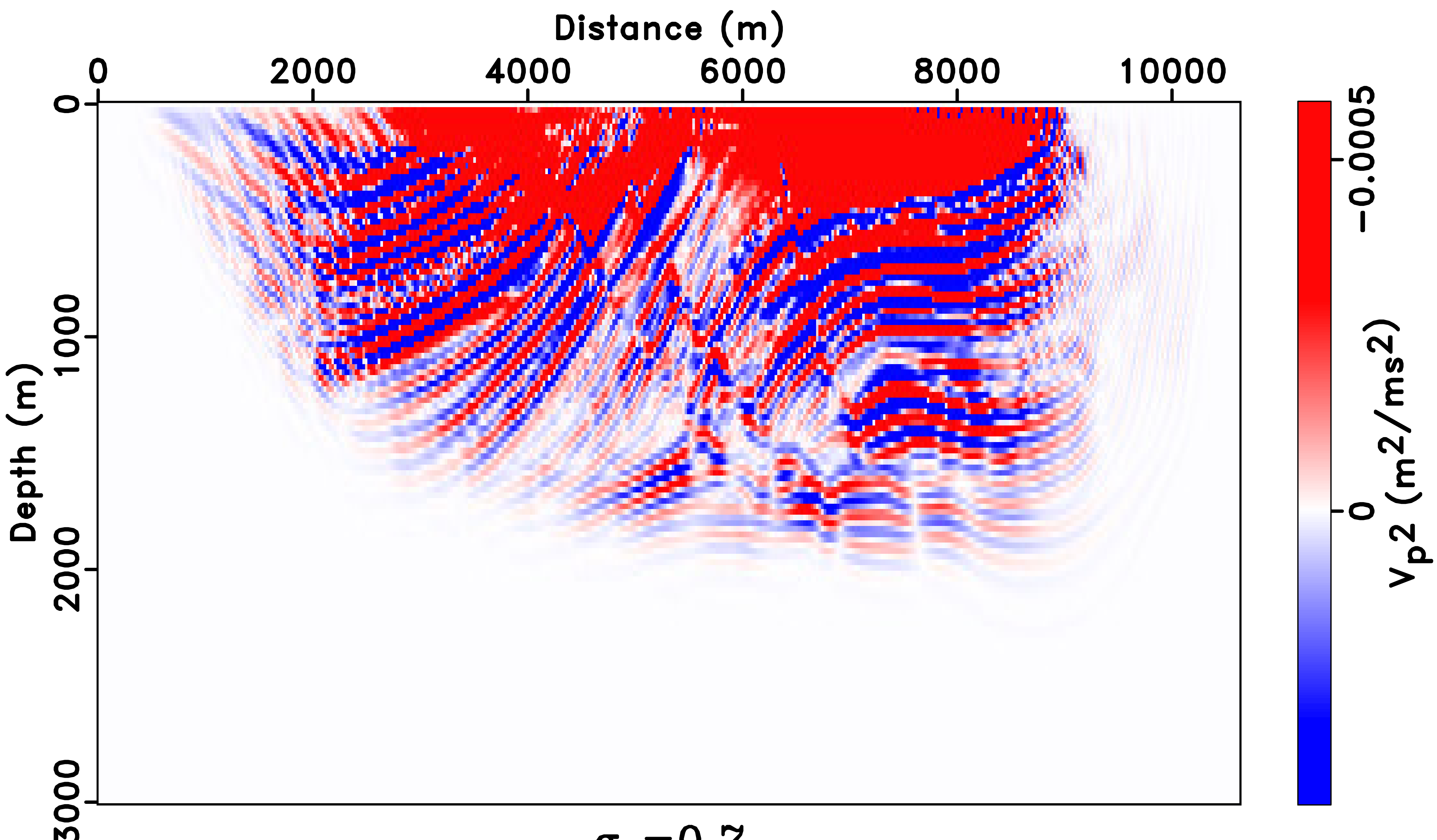
$$N[m_l]\delta m \approx L^{\frac{n-1}{2}} P\delta m \text{ with } m_l = \sigma m_{\text{sm}}.$$



$\sigma = 0.6$

Image Stability of Normal Operator

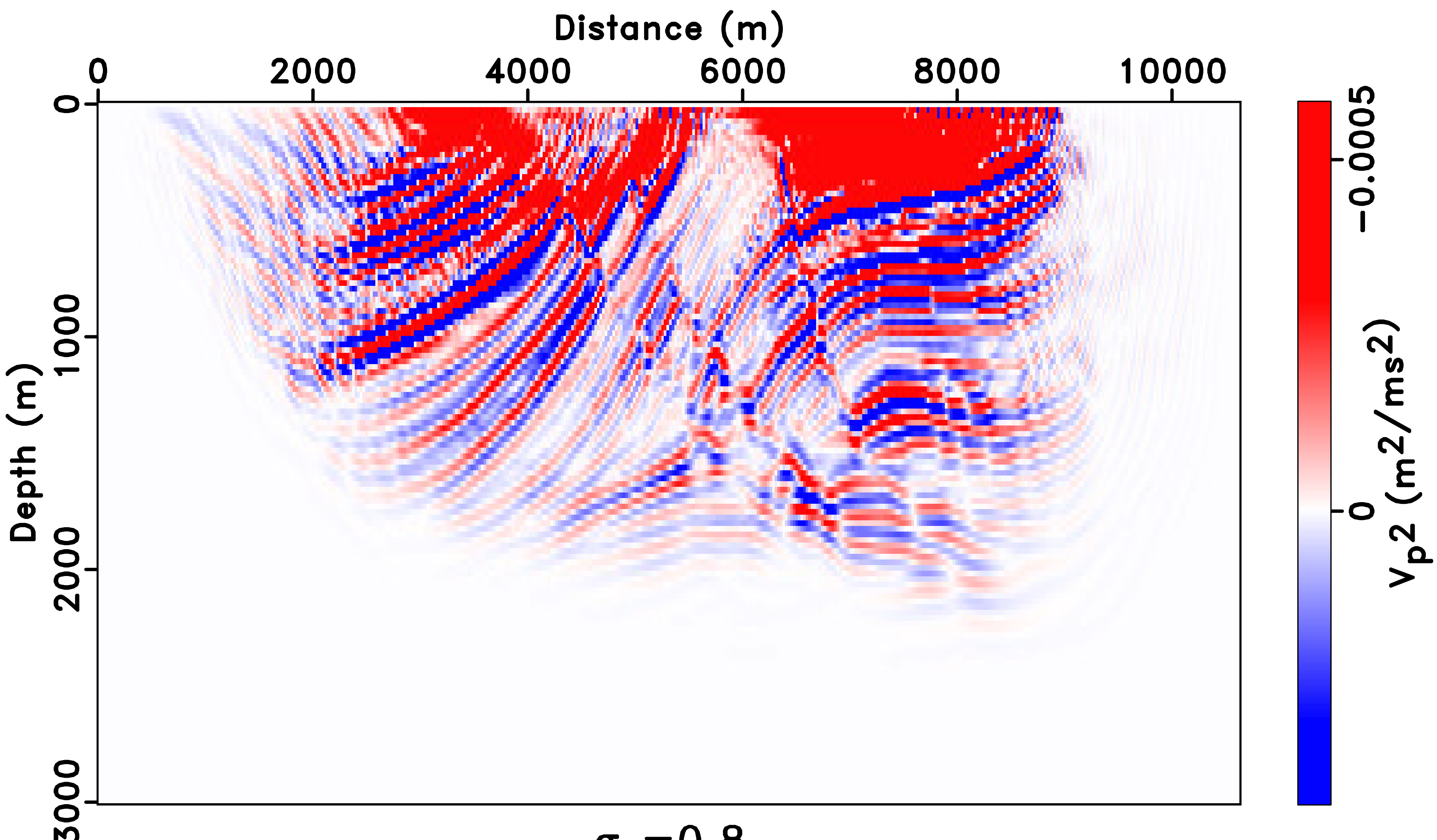
$$N[m_l]\delta m \approx L^{\frac{n-1}{2}} P\delta m \text{ with } m_l = \sigma m_{\text{sm}}.$$



$\sigma = 0.7$

Image Stability of Normal Operator

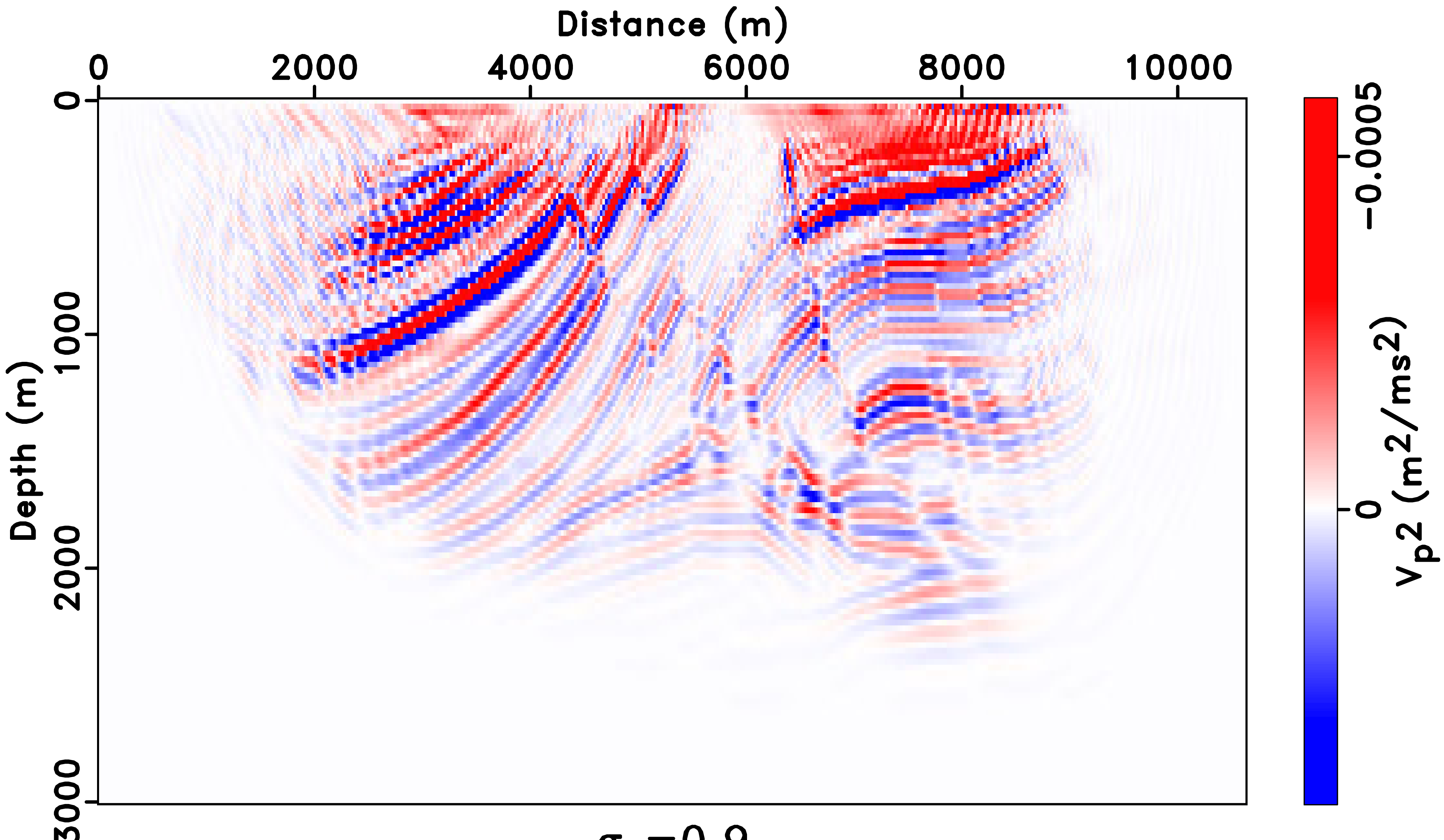
$$N[m_l]\delta m \approx L^{\frac{n-1}{2}} P\delta m \text{ with } m_l = \sigma m_{\mathrm{sm}}.$$



$\sigma = 0.8$

Image Stability of Normal Operator

$$N[m_l]\delta m \approx L^{\frac{n-1}{2}} P \delta m \text{ with } m_l = \sigma m_{\text{sm}}.$$



σ =0.9

Image Stability of Normal Operator

$$N[m_l]\delta m \approx L^{\frac{n-1}{2}} P\delta m \text{ with } m_l = \sigma m_{\text{sm}}.$$

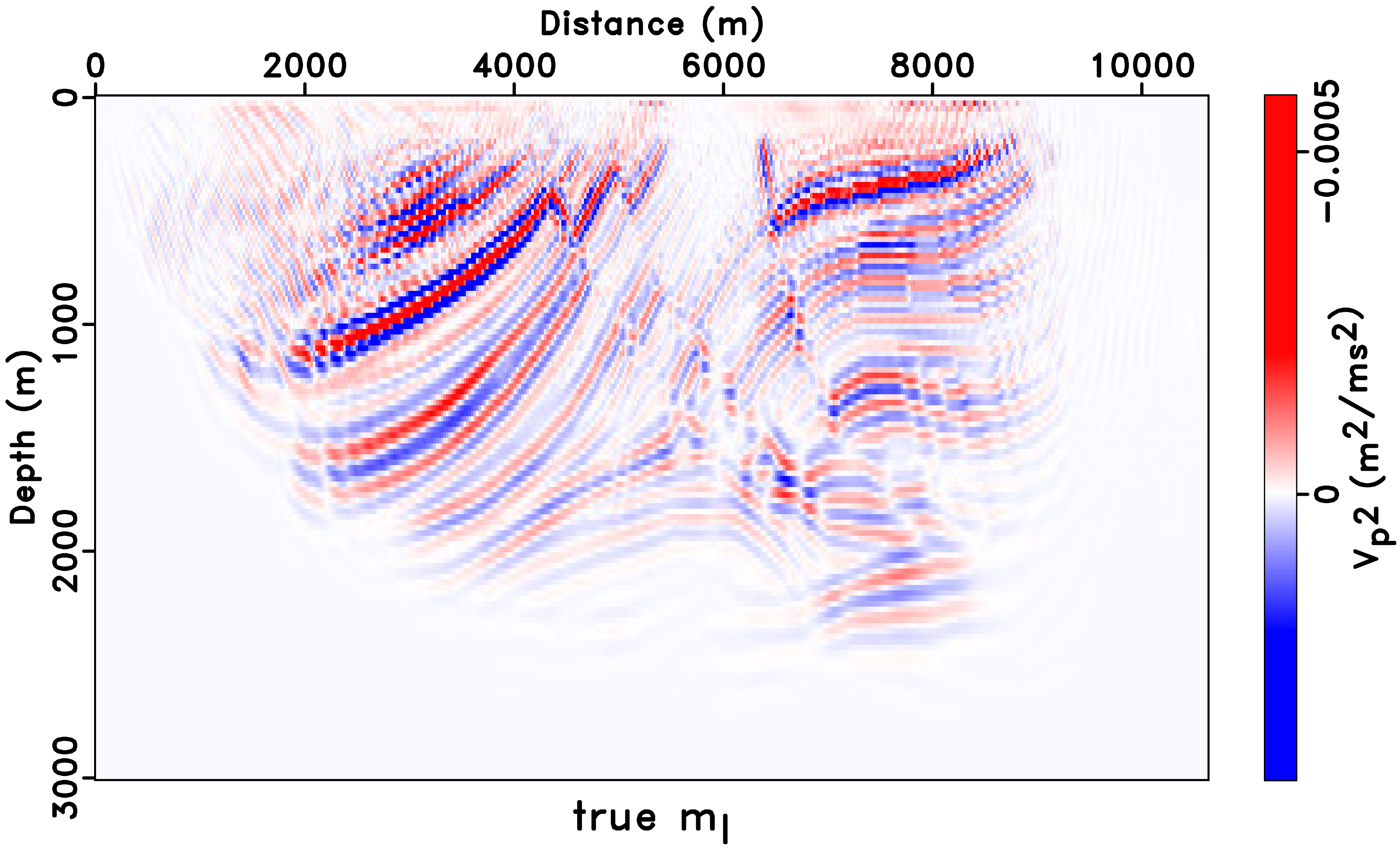
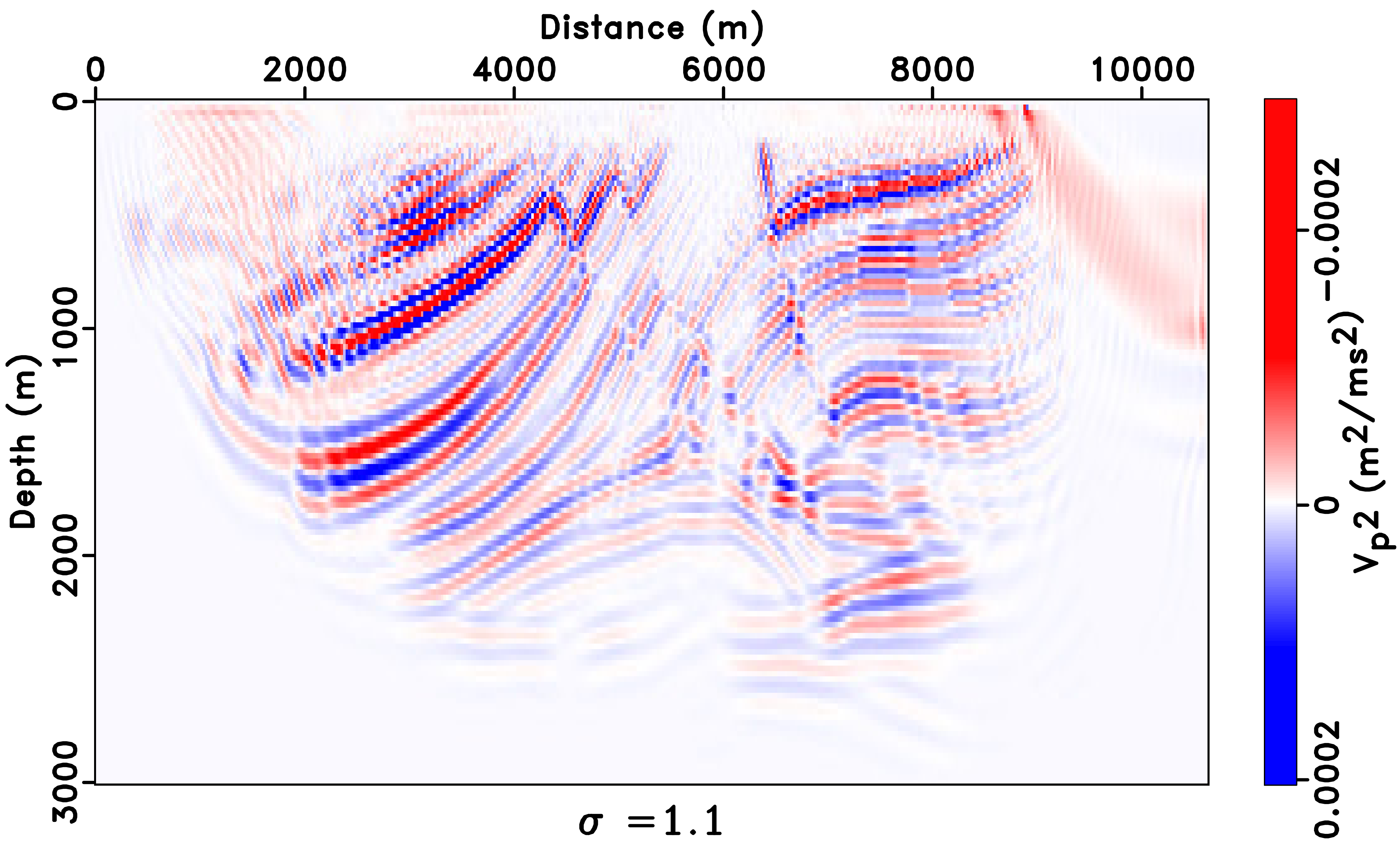


Image Stability of Normal Operator

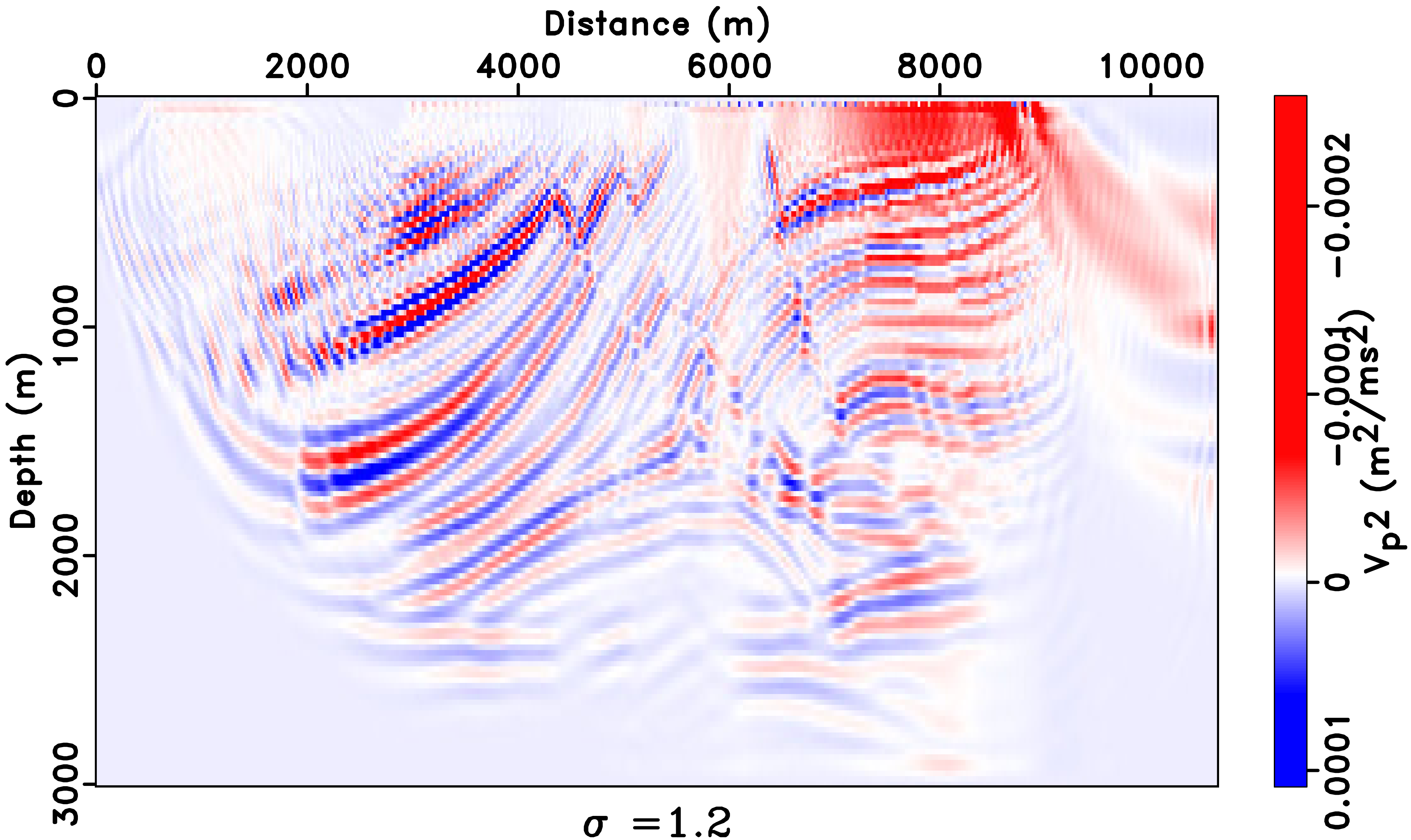
$$N[m_l]\delta m \approx L^{\frac{n-1}{2}} P\delta m \text{ with } m_l = \sigma m_{\text{sm}}.$$



$\sigma = 1.1$

Image Stability of Normal Operator

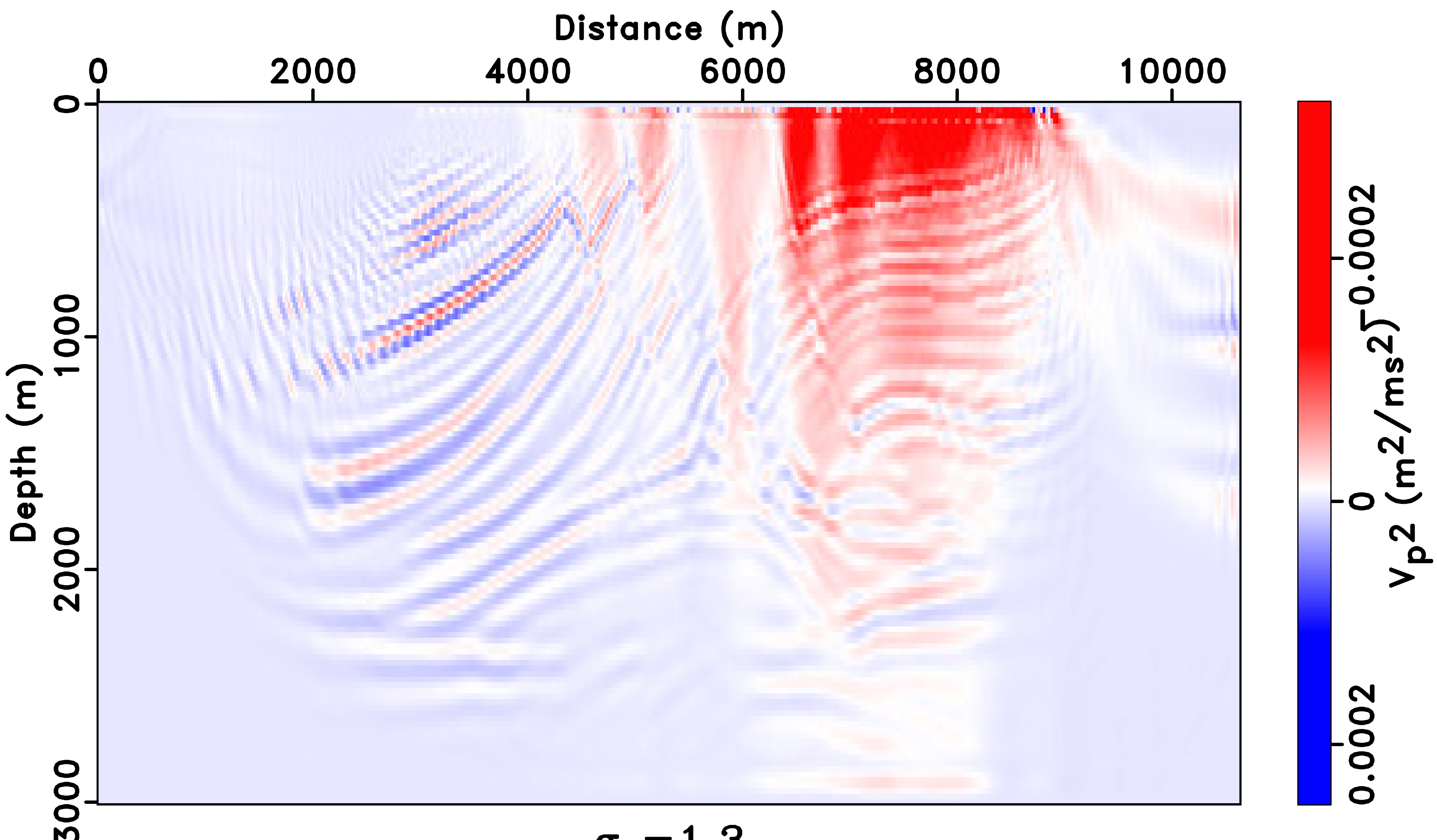
$$N[m_l]\delta m \approx L^{\frac{n-1}{2}} P\delta m \text{ with } m_l = \sigma m_{\text{sm}}.$$



$\sigma = 1.2$

Image Stability of Normal Operator

$$N[m_l]\delta m \approx L^{\frac{n-1}{2}} P\delta m \text{ with } m_l = \sigma m_{\text{sm}}.$$



$\sigma = 1.3$

Scan Tests of Reduced Objective Function

Reduced objective function

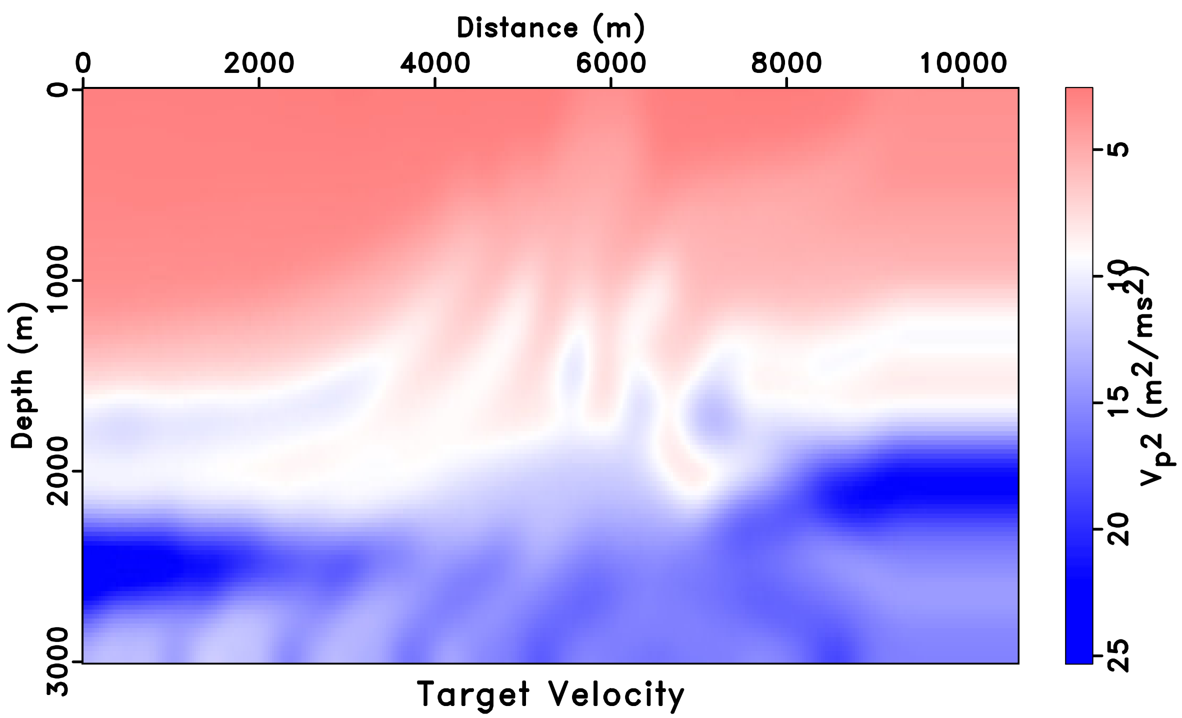
$$\tilde{J}[m_l] = \min_{\delta\bar{m}} J[m_l, \delta\bar{m}] = \frac{1}{2}\|D\bar{F}[m_l]\delta\bar{m} - \delta d\|^2 + \frac{\alpha^2}{2}\|A\delta\bar{m}\|^2.$$

with $\delta\bar{m}[m_l]$ the solution of "extended least squares migration (LSM)" (called PICLI method in Ehinger and Lailly, 1993)

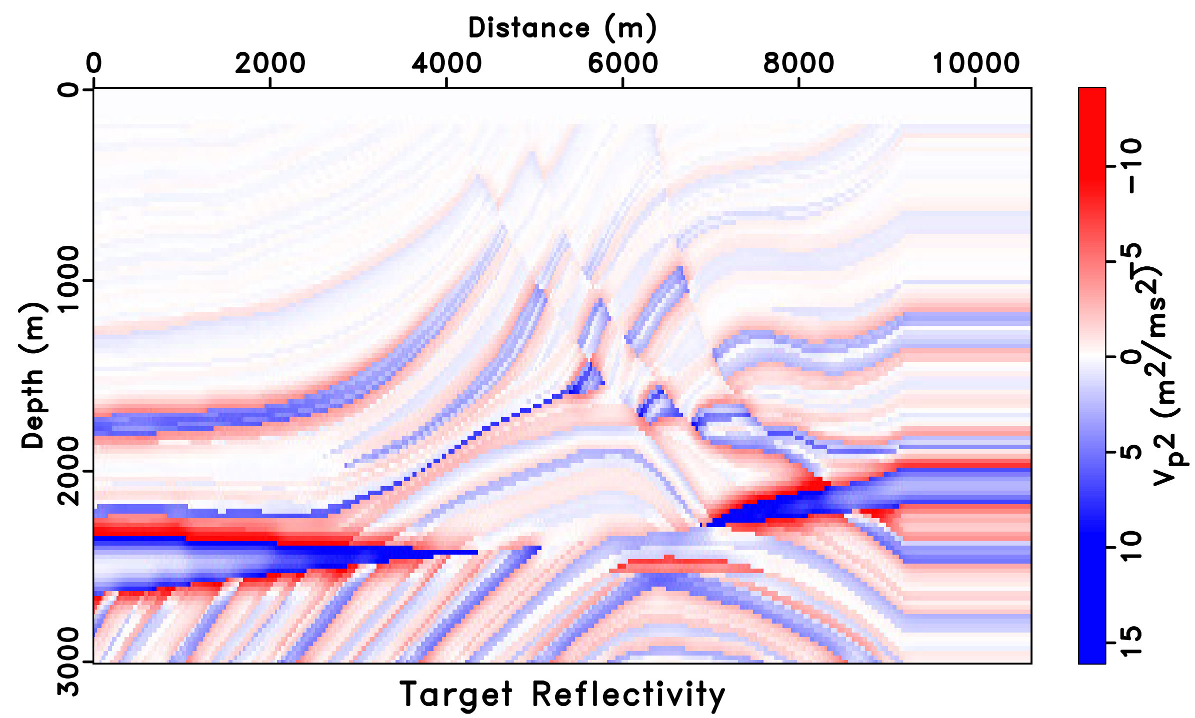
$$(D\bar{F}[m_l]^T D\bar{F}[m_l] + \alpha^2 A^T A)\delta\bar{m}[m_l] = D\bar{F}[m_l]^T \delta d.$$

Ehinger and Lailly, 1993, Prestack imaging by coupled linearized inversion: SPIE Proceedings

Marmousi Model



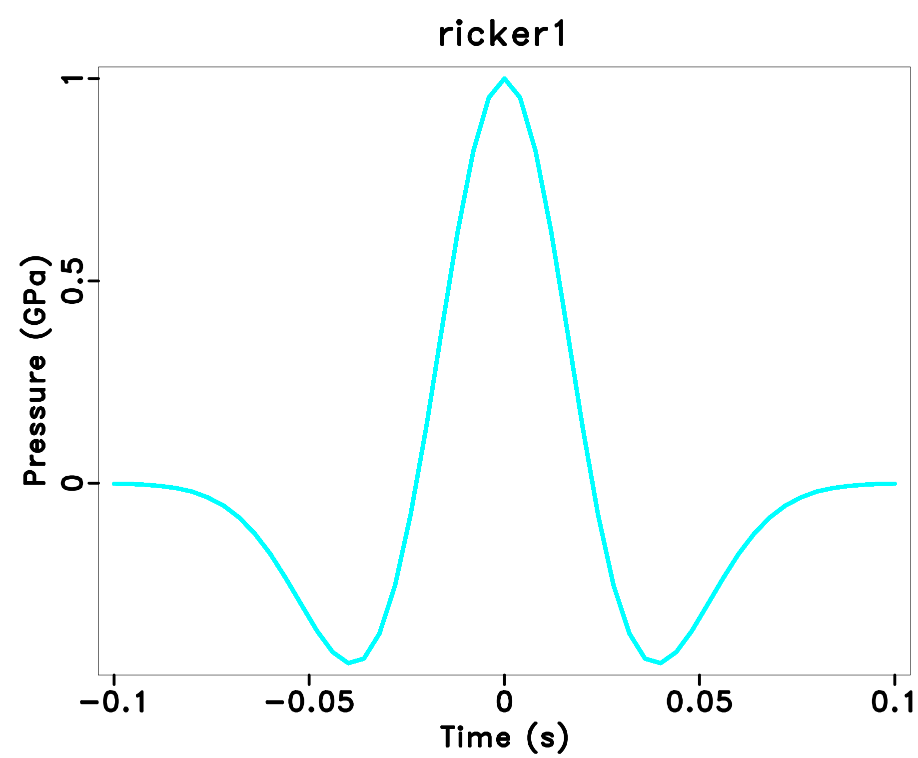
m_{sm}



δm

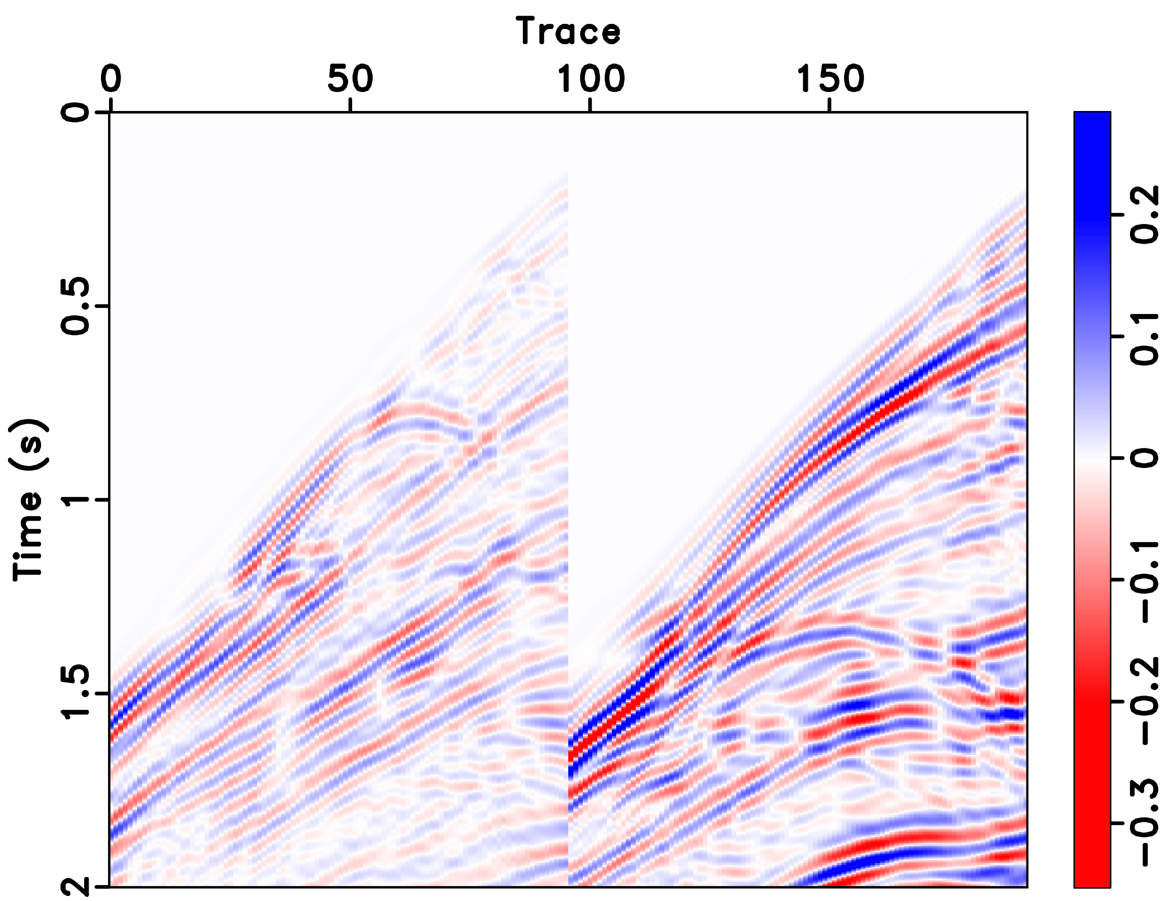
Acquisition geometry:

- 60 shots starting at 3km, with 100 meters spacing
- 96 receivers are placed behind each shot, with 25 meters spacing
- 200 meters between the first receiver and a shot
- shos are 12 meters below the sea surface
- receivers are 8 meters below the sea surface

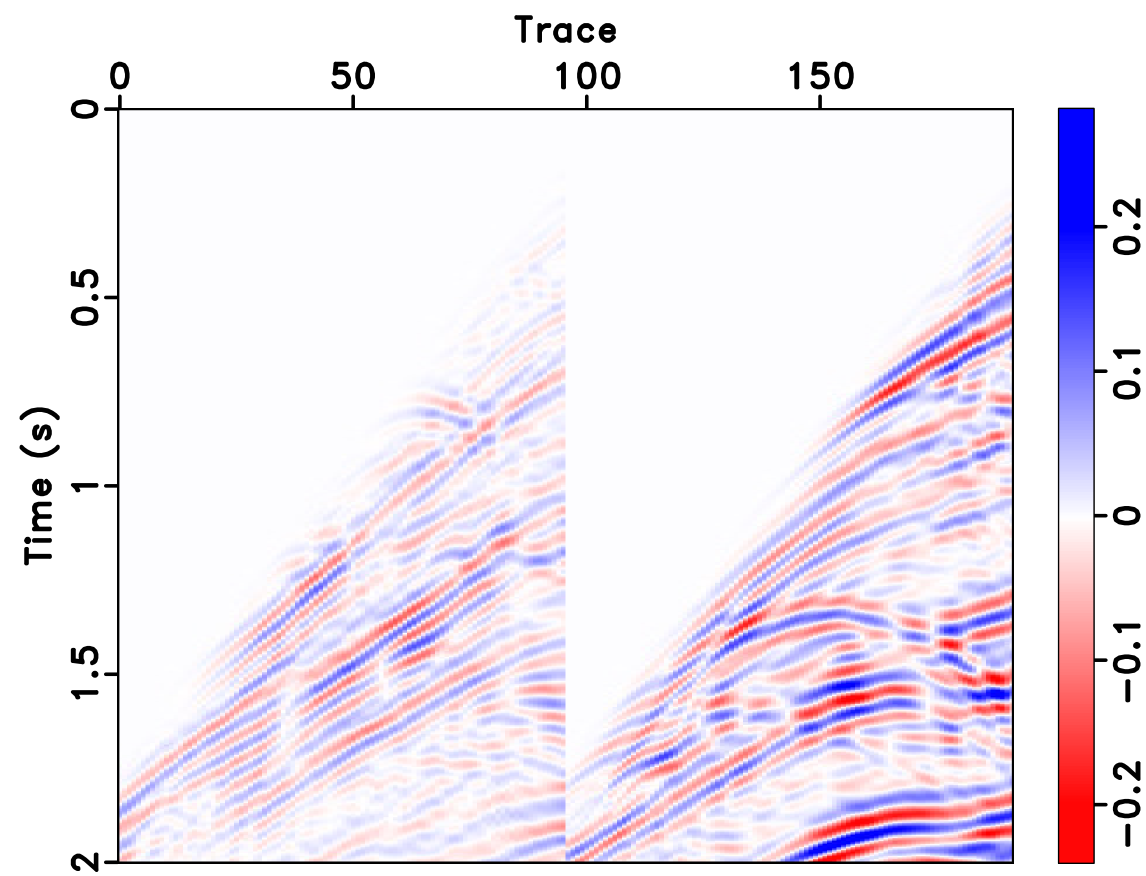


Born data

$$\delta d = DF[m_l]\delta m$$



computed data



muted data

Image Gathers of RTM

$$D\bar{F}[m_l]^T \delta d$$

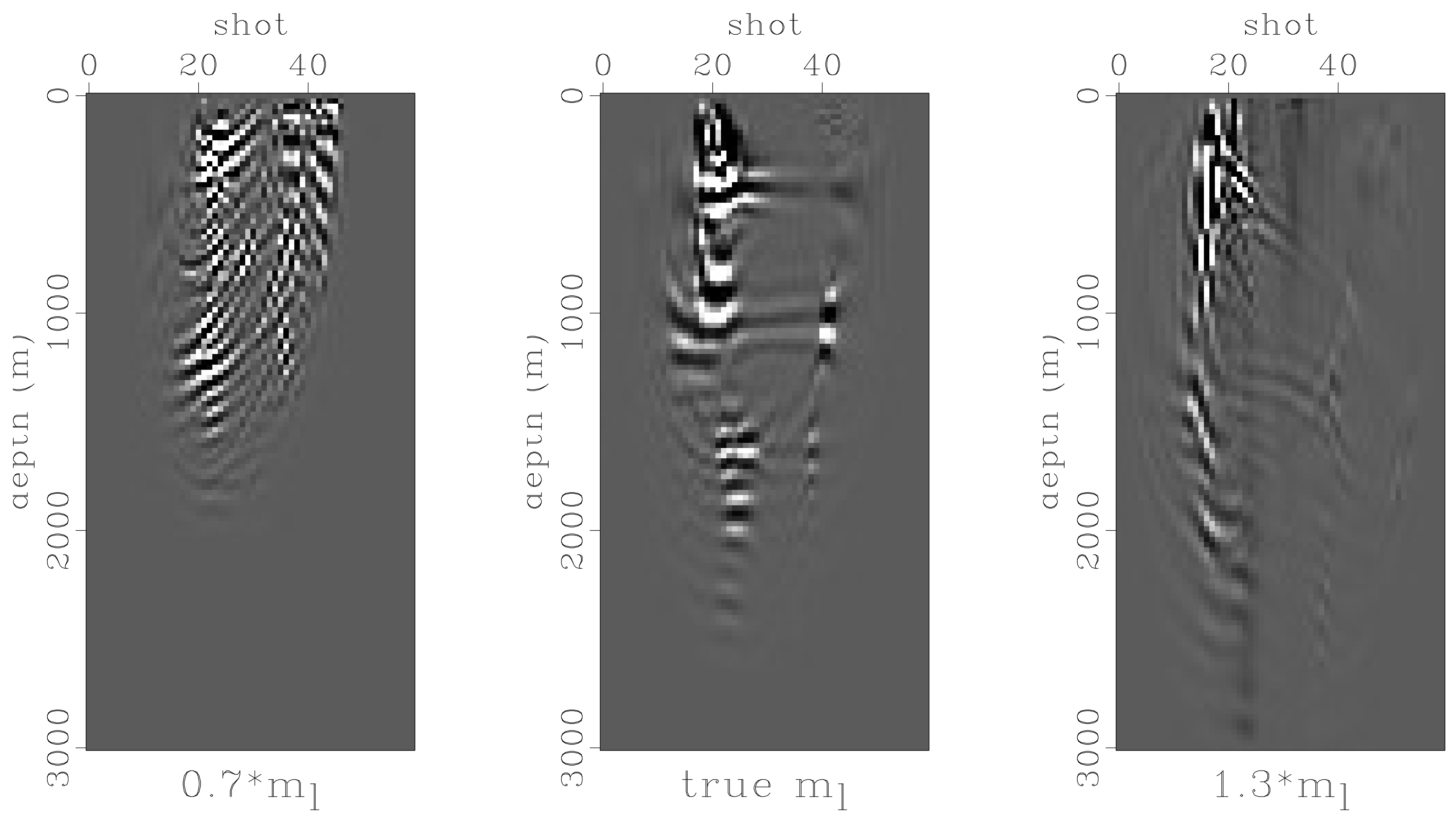


Image Gathers of Extended LSM

$$(D\bar{F}[m_l]^T D\bar{F}[m_l] + \alpha^2 A^T A)\delta\bar{m} = D\bar{F}[m_l]^T \delta d.$$

$$\alpha = 0.01$$

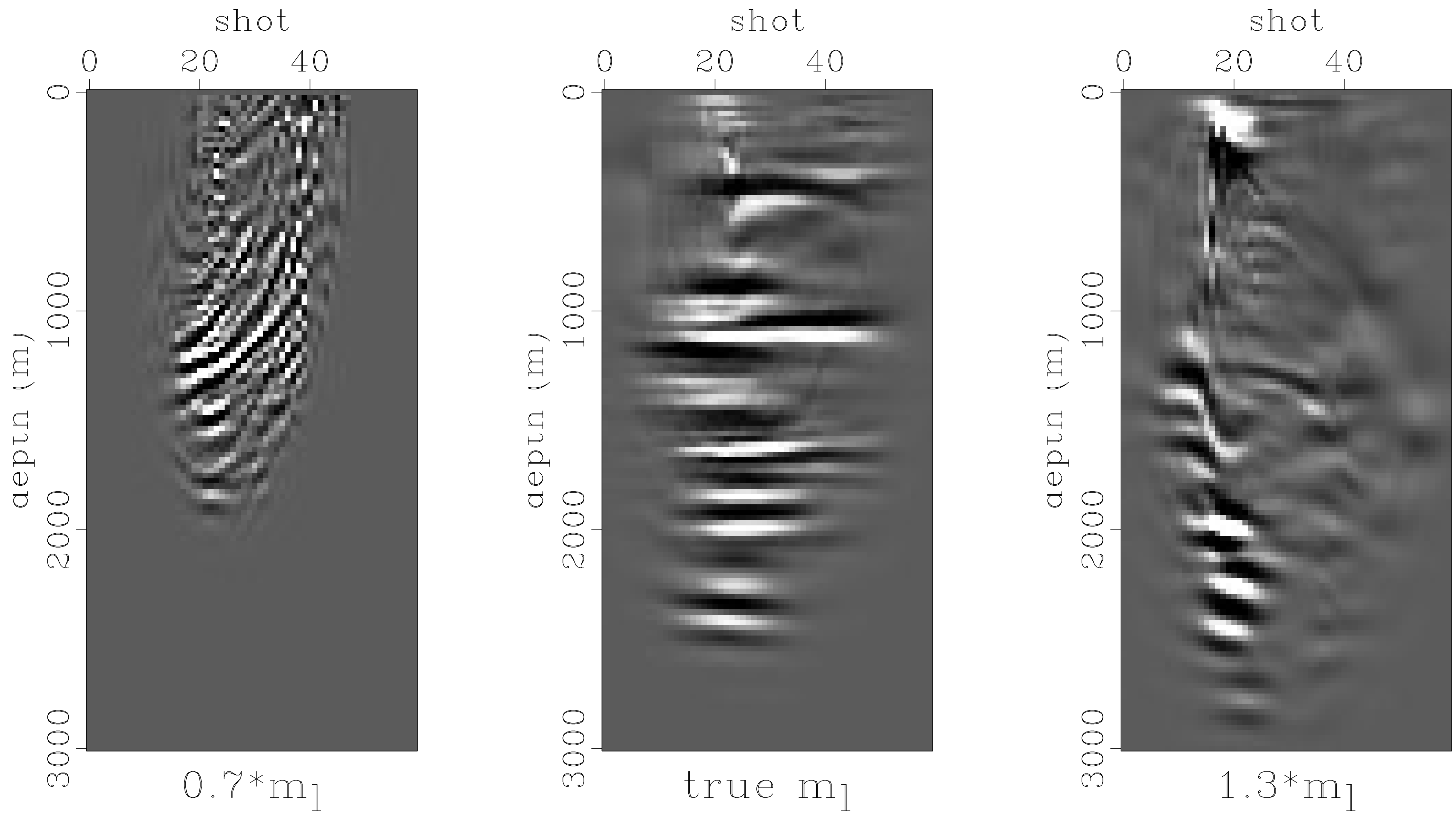
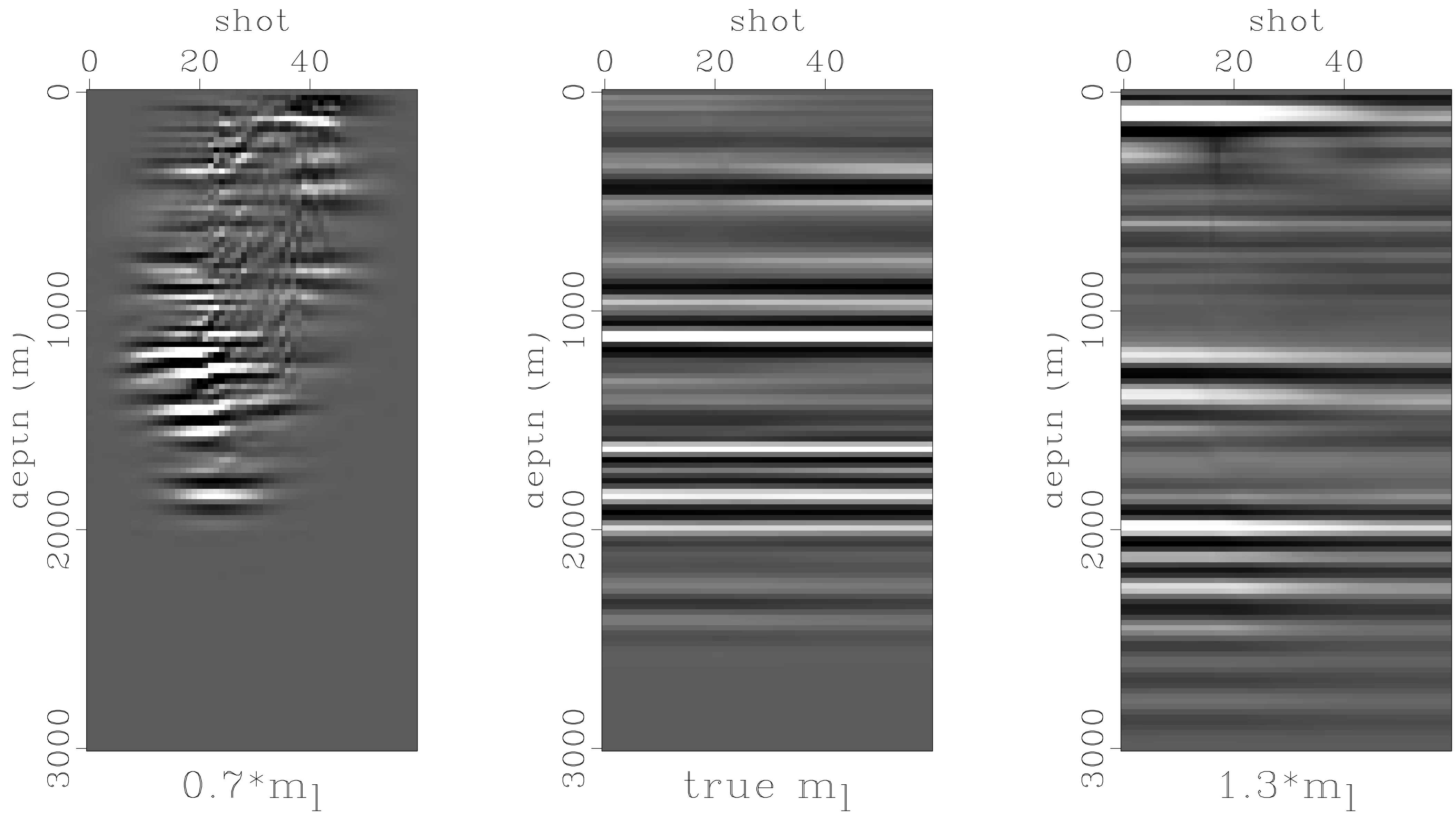


Image Gathers of Extended LSM

$$(D\bar{F}[m_l]^T D\bar{F}[m_l] + \alpha^2 A^T A)\delta\bar{m} = D\bar{F}[m_l]^T \delta d.$$

$$\alpha = 0.1$$

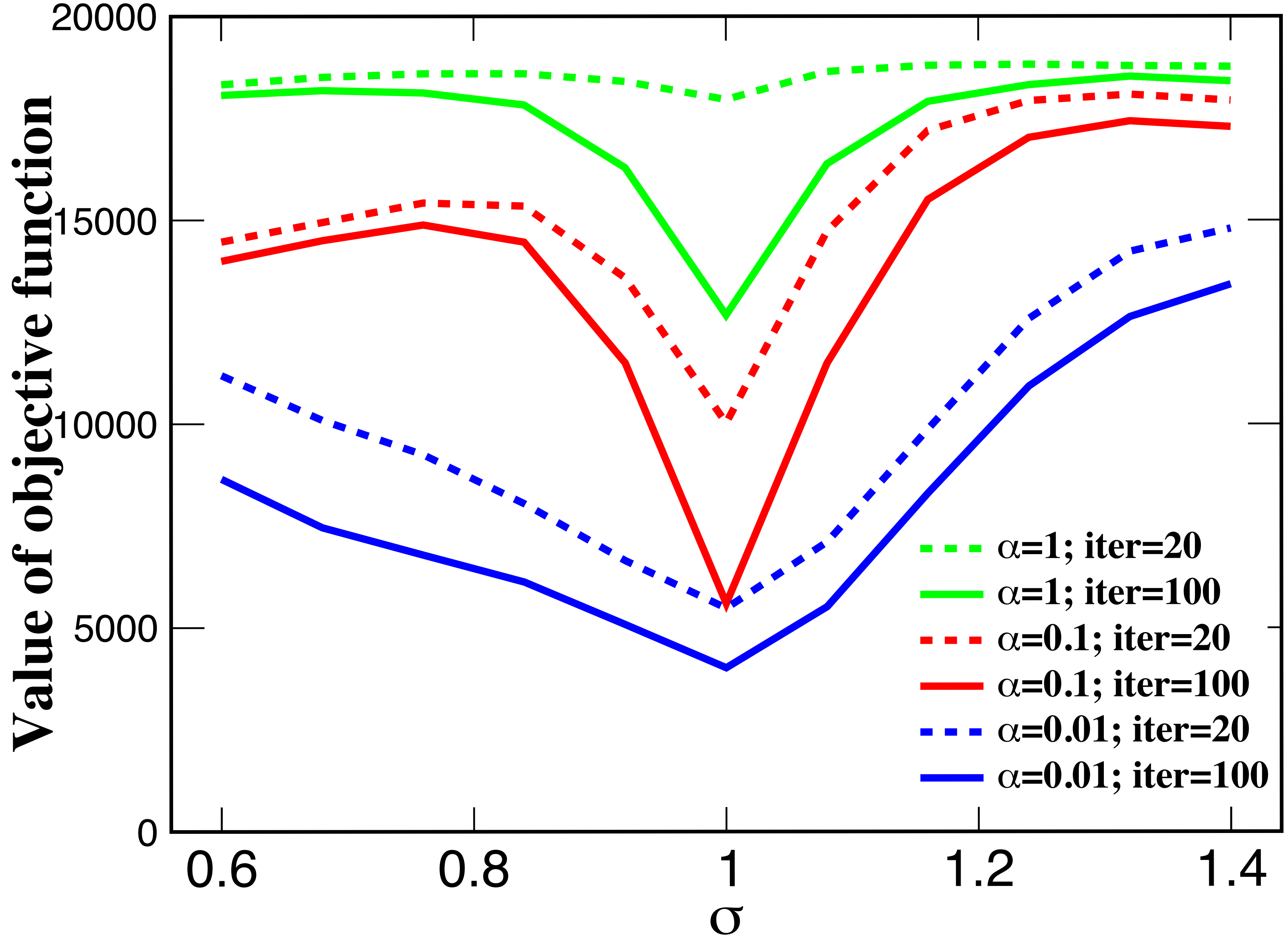


Smoothness and Unimodality of ROF

Scan test of $\tilde{J}[m_l]$ along line segment

$$m_l = \sigma m_{\mathsf{sm}}$$

with $\alpha = 0.01, 0.1, 1.0$ and $\sigma \in [0.6, 1.4]$.

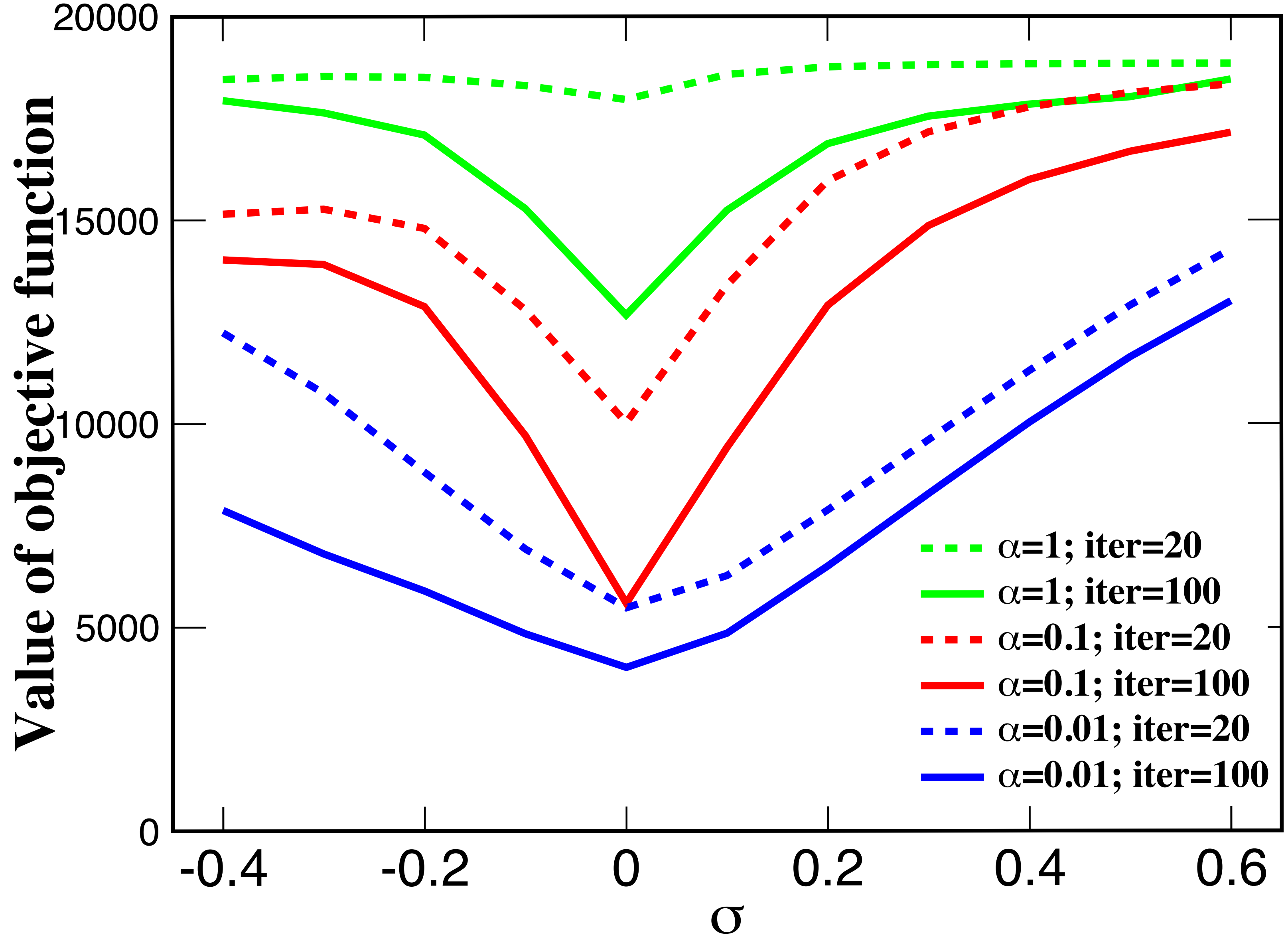


Smoothness and Unimodality of ROF

Scan test of $\tilde{J}[m_l]$ along line segment

$$m_l = (1-\sigma)m_{\text{sm}} + \sigma m_0$$

with $\alpha = 0.01, 0.1, 1.0$, $\sigma \in [-0.4, 0.6]$ and $m_0(x) = 1500\text{m/s}$.



Conclusion

- Numerical results suggest image stability of the normal operator;
- Extended reflectivity from least squares migration along shot coordinate resemble each other closely for large α;
- Scan tests along line segments show that the reduced objective function has large basin of attraction to the global minimum and is smooth for certain α;

Work in progress I

Gradient computation of the reduced objective function

Reduced objective function

$$\tilde{J}[m_l] = \min_{\delta\bar{m}} J[m_l, \delta\bar{m}] = \frac{1}{2}\|D\bar{F}[m_l]\delta\bar{m} - \delta d\|^2 + \frac{\alpha^2}{2}\|A\delta\bar{m}\|^2.$$

with $\delta\bar{m}[m_l]$ the solution of "extended least squares migration (LSM)"

$$(D\bar{F}[m_l]^T D\bar{F}[m_l] + \alpha^2 A^T A)\delta\bar{m}[m_l] = D\bar{F}[m_l]^T \delta d.$$

Gradient of the reduced objective function (Symes & Kern 1994):

$$\nabla\tilde{J}[m_l] = D^2\bar{F}[m_l]^T[\delta\bar{m}[m_l], D\bar{F}[m_l]\delta\bar{m} - \delta d]$$

with $D^2\bar{F}[m_l]^T$ the tomographic operator (Biondi and Almomin, 2013).

Work in progress II

Use Lagrange multiplier method to compute optimal α automatically

Constrained optimization problem:

$$\begin{aligned} \min_{\delta\bar{m}} \quad & \frac{1}{2}\|A\delta\bar{m}\|^2 \\ \text{subject to} \quad & \|D\bar{F}[m_l]\delta\bar{m} - \delta d\|^2 \leq \epsilon^2 \end{aligned}$$

with ϵ the noise level of data.

Equivalent to our "extended least squares migration"

$$\min_{\delta\bar{m}} \frac{1}{2}\|D\bar{F}[m_l]\delta\bar{m} - \delta d\|^2 + \frac{\alpha^2}{2}\|A\delta\bar{m}\|^2$$

Work in progress III

Apply preconditioner to accelerate the convergence rate of extended least squares migration

Optimal scaling preconditioner

From

$$N[m_l] \approx L^{\frac{n-1}{2}} P$$

get

$$(N[m_l])^{-1} \approx P^{\dagger} L^{-\frac{n-1}{2}}$$

with $P^{\dagger} = (P + \epsilon I)^{-1}$ for a possitive ϵ.

Symes, 2008 Approximate linearized inversion by optimal scaling of prestack depth migration

Acknowledgements

Great thanks to

- Wonderful audience;
- Current and former TRIP team members;
- Sponsors of The Rice Inversion Project.