Extended Waveform Tomography as Tomography

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Apology Haiku

Waveform tomography...

terrible name -

really, waveform inversion

Principles

- Extended modeling permits data fit, so avoids cycle skip
- Linear inversion based velocity estimation ("IVA") via differential semblance avoids gradient artifacts
- At heart, IVA is tomography

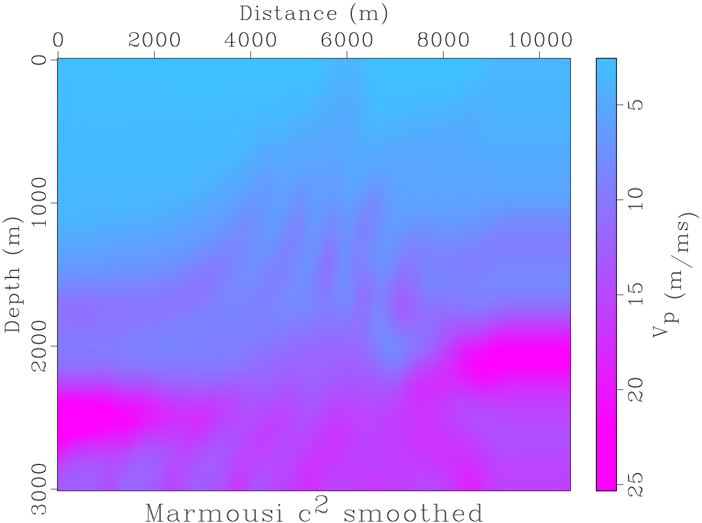
Agenda

Hug your data

Data fit ⇒ bettter gradients

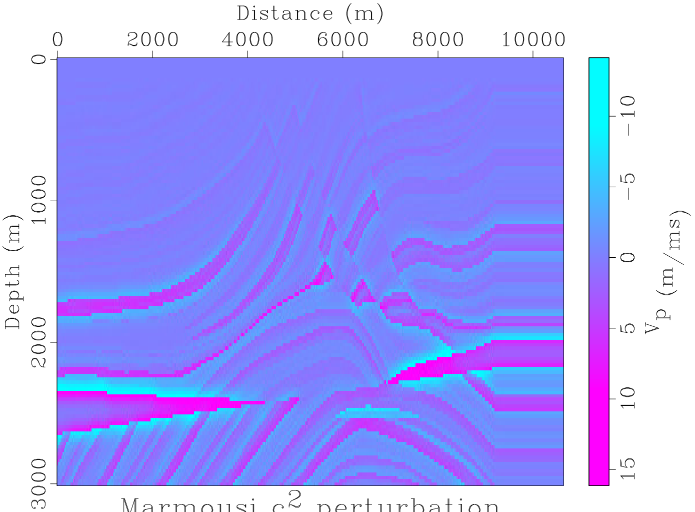
The Inner Tomographer

Data fit via extended modeling



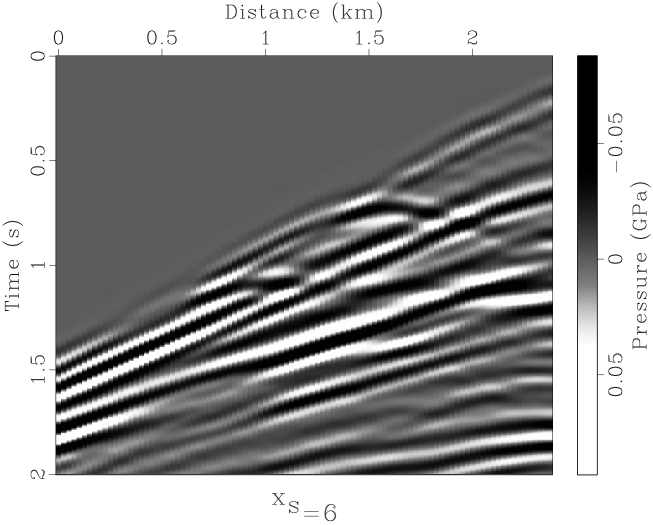
Marmousi c^2 smoothed

Data fit via extended modeling



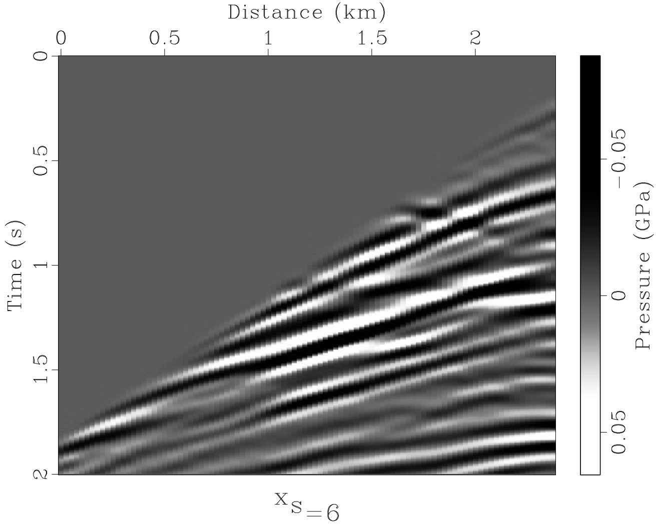
Marmousi c^2 perturbation

Data fit via extended modeling



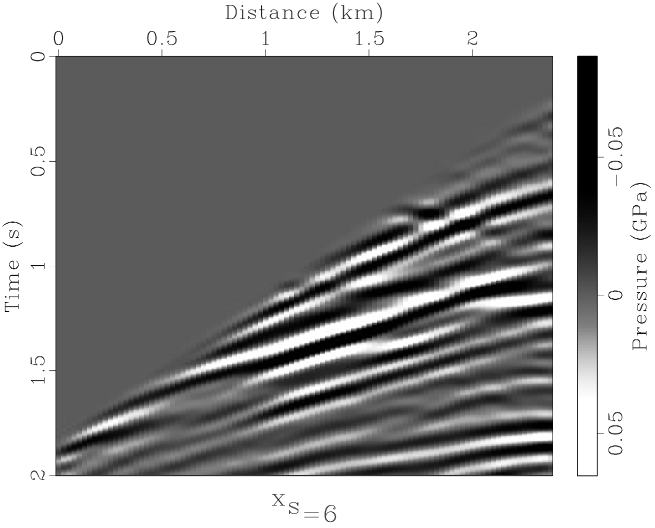
Born data, $x_s = 6$ km

Data fit via extended modeling



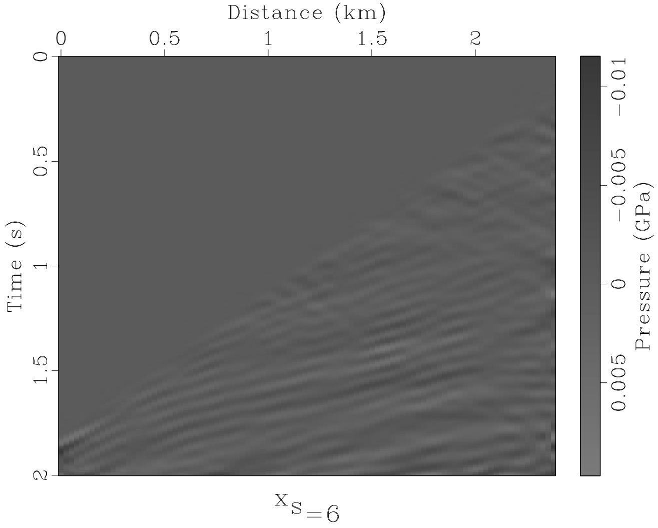
Muted Born data, $x_s = 6$ km

Data fit via extended modeling



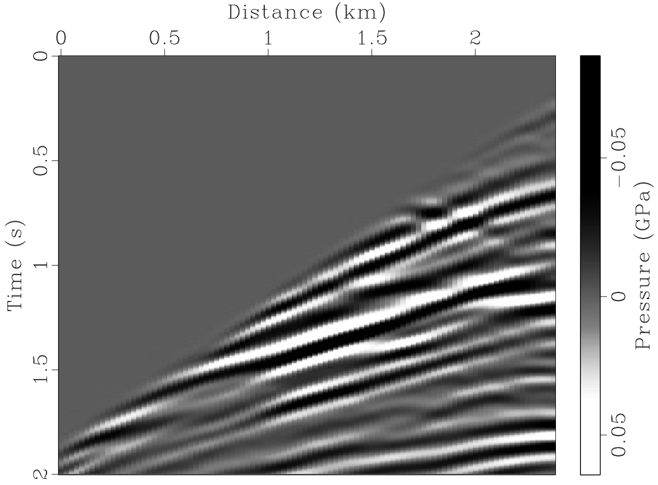
Resim from inversion, Marmousi smoothed v

Data fit via extended modeling



Residual, Marmousi smoothed v: RMS $\simeq$ .08

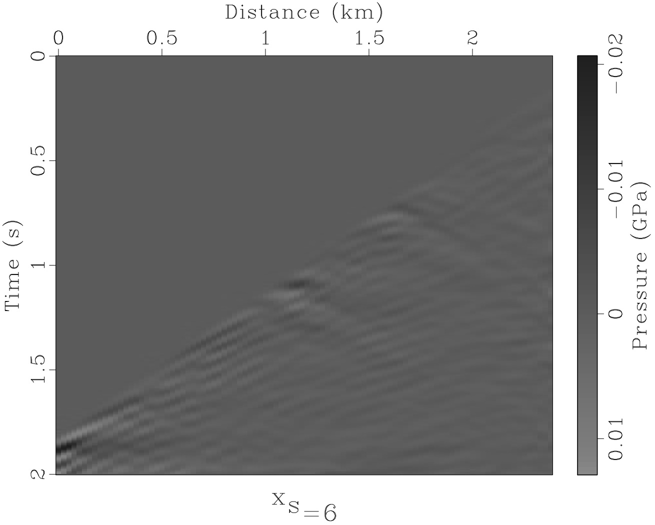
Data fit via extended modeling



$x_{S=6}$

Resim from inversion, H_2O v

Data fit via extended modeling



Residual, H_2O v: RMS $\simeq .06$

Data fit via extended modeling

Moral: no cycle skip if you fit the data!

Agenda

Hug your data

Data fit $\Rightarrow$ bettter gradients

The Inner Tomographer

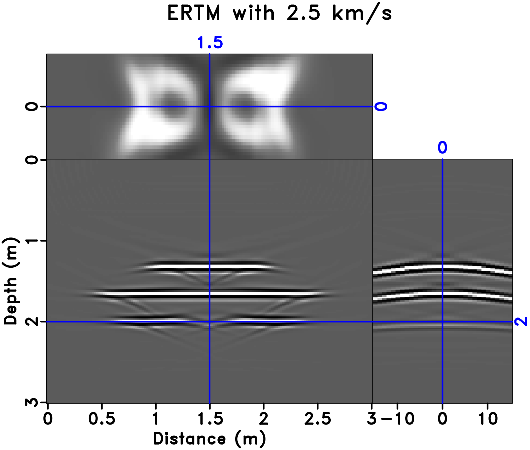
“Traditional” differential semblance

$$\min_c(\tilde{J}_0[c] = \sum_{x,z,h} |hI(x,z,h)|^2)$$

$I(x,z,h)$ =space-shift image volume

“Traditional” differential semblance

$I(x, z, h)$ at $c = 2.5$km/s (true $c = 3$km/s)



thanks: Y. Liu

“Traditional” differential semblance

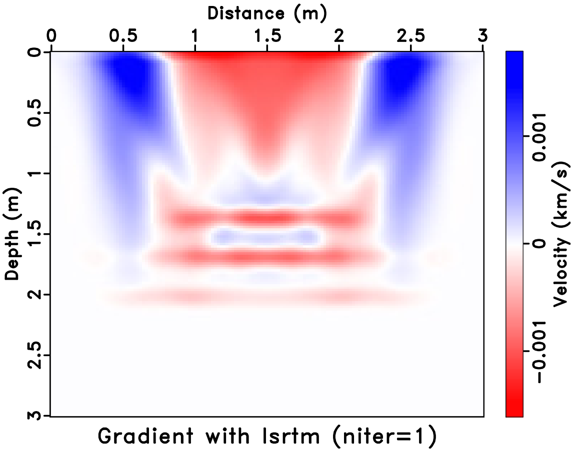
Gradient:

$$\nabla \tilde{J}_0[c] \simeq \frac{\delta I}{\delta c}^{*} (h^2 I)$$

$$\frac{\delta I}{\delta c}^{*} = \text{“tomographic operator”}$$

"Traditional" differential semblance

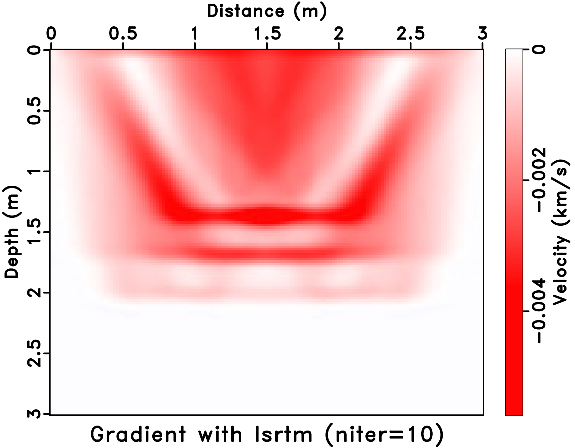
RTM-based DS gradient, (wrong) $c = 2.5\text{km/s}$



thanks: Y. Liu

Inversion VA, aka EFWI

EFWI gradient, (wrong) $c = 2.5$km/s



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The Inner Tomographer

Extended modeling:

- m depends on non-physical space-time degrees of freedom
- physical models are extended: m physical $\Leftrightarrow Am = 0$
- $\mathcal{F}$ is ordinary modeling on physical models

Extended FWI = FWI based on extended modeling, with penalty for non-physicality:

$$J[m] = \frac{1}{2}\|\mathcal{F}[m] - d\|^2 + \frac{\alpha^2}{2}\|Am\|^2$$

Example: horiz. space-shift (or subsurface offset) extended modeling, constant density acoustics in Born approximation

$m =$ (background model $c(x, z)$, extended reflectivity $r(x, z, h)$)

$\mathcal{F}[m] = F[c]r =$ pressure perturbation sampled at src/rcvrs - space-shift *demigration* physical models $= (c(x,z), r(x,z)\delta(h))$

rel'n to Born: $r = 2\delta c/c$

reg op $A =$ *annihilator* of physical models

Expl: multiplication by h (many other possibilities, eg. Albertin 09, Sava & Yang 12)

Born-based EFWI for acoustics:

$$J[c,r] = \frac{1}{2}\|F[c]r - d\|^2 + \frac{\alpha^2}{2}\|Ar\|^2$$

Main fact: $J[c,r]$ is as oscillatory as data

Reduced objective:

$$\tilde{J}[c] = \min_r J[c, r] = J[c, r[c]]$$

where

$$r[c] = N[c]^{-1} F[c]^* d, \;\; N[c] = F[c]^* F[c] + \alpha^2 A^* A$$

NB: this is *variable projection*!

$$\tilde{J}[c] = \frac{1}{2}\|(F[c]N[c]^{-1}F[c]^* - I)d\|^2 +$$
$$\frac{\alpha^2}{2}\|AN[c]^{-1}F[c]^*d\|^2$$

Both terms have form $\langle d, Pd \rangle$, $P =$ *pseudodiffl op* with symbol smooth in c

$\Rightarrow$ smooth independent of data spectrum [see Stolk & S. 03 for choice of A]

For this problem, VPM completely changes character of objective

Hessian at consistent data

$$\nabla \tilde{J}[c] = DF[c]^*(r[c], F[c]r[c] - d)$$

$$DF[c]^* = D^2\mathcal{F}[c]^* = \text{ “tomographic operator”}$$

Hessian at consistent data

Key to understanding Hessian:

$$DF[c]\delta c = F[c](Q[c]\delta c)$$

where $Q[c]\delta c$ is pseudo of order 1:

$$(Q[c]\delta c)r(z,x,h) = \int\int\int dk_z dk_x dk_h \times$$

$$(\delta\tau(x_s, z, x-h) + \delta\tau(x_r, z, x+h))(ik_z)$$

$$\hat{r}(k_z, k_z, k_h)e^{i(k_z z + k_x x + k_h h)}$$

Hessian at consistent data

$\delta\tau = D\tau[c]\delta c =$ traveltime perturbation

x_s, x_r determined by $z, x, h, k_x/k_z, k_h/k_z$

Relation generally complex, *but* at $h = 0$,

$$\begin{aligned} k_x/k_z &= \tan\psi \\ k_h/k_z &= \tan\theta \end{aligned}$$

$\psi =$ dip angle, $\theta =$ scattering angle

Hessian at consistent data

How to see this: express $DF[c]\delta c$ as GRT, apply pseudo inverse on left (cf. Jie's talk), use stationary phase

Hessian at consistent data

Compute Hessian at consistent point:

$$\begin{aligned} F[c]r &= d \\ Ar &= 0 \end{aligned}$$

Use DF[c]=F[c]Q[c], $h = 0$; lots of cancellations occur:

$$D^2\tilde{J}[c](\delta c_1, \delta c_2) =$$

$$\alpha^2 \langle A(Q[c]\delta c_1)r, (I - AN[c]^{-1}A^*)A(Q[c]\delta c_2)r \rangle$$

Hessian at consistent data

Since $Ar = 0$, $AQr = [A, Q]r$. Calc. of pseudos: symbol of commutator = Poisson braced of symbols

Symbol of $A = h$, symbol of $Q = (\delta\tau_s + \delta\tau_r)(\ldots, \tan\psi, \tan\theta) ik_z$

$\Rightarrow$ symbol of $[A, Q] = \delta p_s \frac{\partial x_s}{\partial \tan\theta} + \delta p_r \frac{\partial x_r}{\partial \tan\theta}$

$p_r = \frac{\partial \tau_r}{\partial x_r}$ etc

Hessian at consistent data

Upshot: $D^2\tilde{J}[c](\delta c_1, \delta c_2)$ is weighted integral of perturbations in slownesses, weighted by

- energy in reflectivity
- geometric factor (rate of change of scattering angle wrt src, rec x)

$\Rightarrow$ near consistent data, DSO $\simeq$ a form of slope tomography.

Where from here

- better understanding of tomography problem at heart of space-shift DSO: linear combinations of source, receiver slope.
- better formalize role of reflectivity - denser reflectors $\Rightarrow$ better resolution?
- 3D

Acknowledgements

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