Extended Waveform Tomography as Tomography

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Waveform tomography...
terrible name -
really, waveform inversion
Principles

▸ Extended modeling permits data fit, so avoids cycle skip
▸ Linear inversion based velocity estimation ("IVA") via differential semblance avoids gradient artifacts
▸ At heart, IVA is tomography
Agenda

Hug your data

Data fit ⇒ better gradients

The Inner Tomographer
Data fit via extended modeling
Data fit via extended modeling
Data fit via extended modeling

Born data, $x_s = 6 \text{ km}$
Data fit via extended modeling

Muted Born data, $x_s = 6$ km
Data fit via extended modeling

Resim from inversion, Marmousi smoothed v
Data fit via extended modeling

Residual, Marmousi smoothed v: RMS ≃ 0.08
Data fit via extended modeling

Resim from inversion, $H_2O\nu$
Data fit via extended modeling

Residual, $H_2O$ v: RMS $\simeq 0.06$
Data fit via extended modeling

Moral: no cycle skip if you fit the data!
Agenda

Hug your data

Data fit $\Rightarrow$ better gradients

The Inner Tomographer
“Traditional” differential semblance

\[ \min_c(\tilde{J}_0[c] = \sum_{x,z,h} |hl(x, z, h)|^2) \]

\[ l(x, z, h) = \text{space-shift image volume} \]
“Traditional” differential semblance

\[ l(x, z, h) \] at \( c = 2.5\text{km/s} \) (true \( c = 3\text{km/s} \))

thanks: Y. Liu
“Traditional” differential semblance

Gradient:

$$\nabla \tilde{J}_0[c] \simeq \frac{\delta l^*}{\delta c} (h^2 l)$$

$$\frac{\delta l^*}{\delta c} = \text{“tomographic operator”}$$
“Traditional” differential semblance

RTM-based DS gradient, (wrong) $c = 2.5\text{km/s}$

thanks: Y. Liu
Inversion VA, aka EFWI

EFWI gradient, (wrong) $c = 2.5 \text{km/s}$

thanks: Y. Liu
Agenda

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Data fit ⇒ better gradients

The Inner Tomographer
Extended modeling:

- $m$ depends on non-physical space-time degrees of freedom
- physical models are extended: $m$ physical $\iff Am = 0$
- $\mathcal{F}$ is ordinary modeling on physical models
Extended FWI = FWI based on extended modeling, with penalty for non-physicality:

\[ J[m] = \frac{1}{2} \| \mathcal{F}[m] - d \|^2 + \frac{\alpha^2}{2} \| Am \|^2 \]
Example: horiz. space-shift (or subsurface offset) extended modeling, constant density acoustics in Born approximation

\[ m = (\text{background model } c(x, z), \text{ extended reflectivity } r(x, z, h) ) \]

\[ \mathcal{F}[m] = F[c]r = \text{pressure perturbation sampled at src/rcvrs - space-shift demigration} \]
physical models = \((c(x, z), r(x, z)\delta(h))\)

rel’n to Born: \(r = 2\delta c/c\)

reg op \(A = \text{annihilator}\) of physical models

Expl: multiplication by \(h\) (many other possibilities, eg. Albertin 09, Sava & Yang 12)
Born-based EFWI for acoustics:

\[ J[c, r] = \frac{1}{2} \| F[c]r - d \|^2 + \frac{\alpha^2}{2} \| Ar \|^2 \]

Main fact: \( J[c, r] \) is as oscillatory as data
Reduced objective:

\[ \tilde{J}[c] = \min_r J[c, r] = J[c, r[c]] \]

where


NB: this is variable projection!
\[ \tilde{J}[c] = \frac{1}{2} \|(F[c]N[c]^{-1}F[c]^* - I)d\|^2 + \]

\[ \frac{\alpha^2}{2} \|AN[c]^{-1}F[c]^*d\|^2 \]

Both terms have form \( \langle d, Pd \rangle \), \( P = \text{pseudodiff op} \) with symbol smooth in \( c \)

\( \Rightarrow \) smooth independent of data spectrum [see Stolk & S. 03 for choice of A]
For this problem, VPM completely changes character of objective
Hessian at consistent data

\[ \nabla \tilde{J}[c] = DF[c]^* (r[c], F[c]r[c] - d) \]

\[ DF[c]^* = D^2 \mathcal{F}[c]^* = \text{“tomographic operator”} \]
Hessian at consistent data

Key to understanding Hessian:

\[ DF[c] \delta c = F[c](Q[c] \delta c) \]

where \( Q[c] \delta c \) is pseudo of order 1:

\[
(Q[c] \delta c)r(z, x, h) = \int \int \int dk_z dk_x dk_h \times \\
(\delta \tau(x_s, z, x - h) + \delta \tau(x_r, z, x + h))(ik_z) \\
\hat{r}(k_z, k_z, k_h) e^{i(k_z z + k_x x + k_h h)}
\]
Hessian at consistent data

\[ \delta \tau = D \tau [c] \delta c = \text{traveltime perturbation} \]

\( x_s, x_r \) determined by \( z, x, h, k_x/k_z, k_h/k_z \)

Relation generally complex, \textit{but} at \( h = 0 \),

\[ \frac{k_x}{k_z} = \tan \psi \]
\[ \frac{k_h}{k_z} = \tan \theta \]

\( \psi \) = dip angle, \( \theta \) = scattering angle
Hessian at consistent data

How to see this: express $DF[c] \delta c$ as GRT, apply pseudo inverse on left (cf. Jie’s talk), use stationary phase
Hessian at consistent data

Compute Hessian at consistent point:

\[ F[c]r = d \]
\[ Ar = 0 \]

Use \( DF[c] = F[c]Q[c] \), \( h = 0 \); lots of cancellations occur:

\[
D^2 \tilde{J}[c](\delta c_1, \delta c_2) = \\
\alpha^2 \langle A(Q[c]\delta c_1)r, (I - AN[c]^{-1}A^*)A(Q[c]\delta c_2)r \rangle
\]
Hessian at consistent data

Since $Ar = 0$, $AQr = [A, Q]r$. Calc. of pseudos:

symbol of commutator = Poisson braced of symbols

Symbol of $A = h$, symbol of $Q = (\delta\tau_s + \delta\tau_r)(..., \tan \psi, \tan \theta)ik_z$

⇒ symbol of $[A, Q] = \delta p_s \frac{\partial x_s}{\partial \tan \theta} + \delta p_r \frac{\partial x_r}{\partial \tan \theta}$

$p_r = \frac{\partial \tau_r}{\partial x_r}$ etc
Hessian at consistent data

Upshot: $D^2 \tilde{\mathcal{J}}[c](\delta c_1, \delta c_2)$ is weighted integral of perturbations in slownesses, weighted by

- energy in reflectivity
- geometric factor (rate of change of scattering angle wrt src, rec $x$)

⇒ near consistent data, DSO $\simeq$ a form of slope tomography.
Where from here

- better understanding of tomography problem at heart of space-shift DSO: linear combinations of source, receiver slope.
- better formalize role of reflectivity - denser reflectors $\Rightarrow$ better resolution?
- 3D
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