Computational and Applied Mathematics

Viscoacoustic full waveform inversion: what can be resolved?

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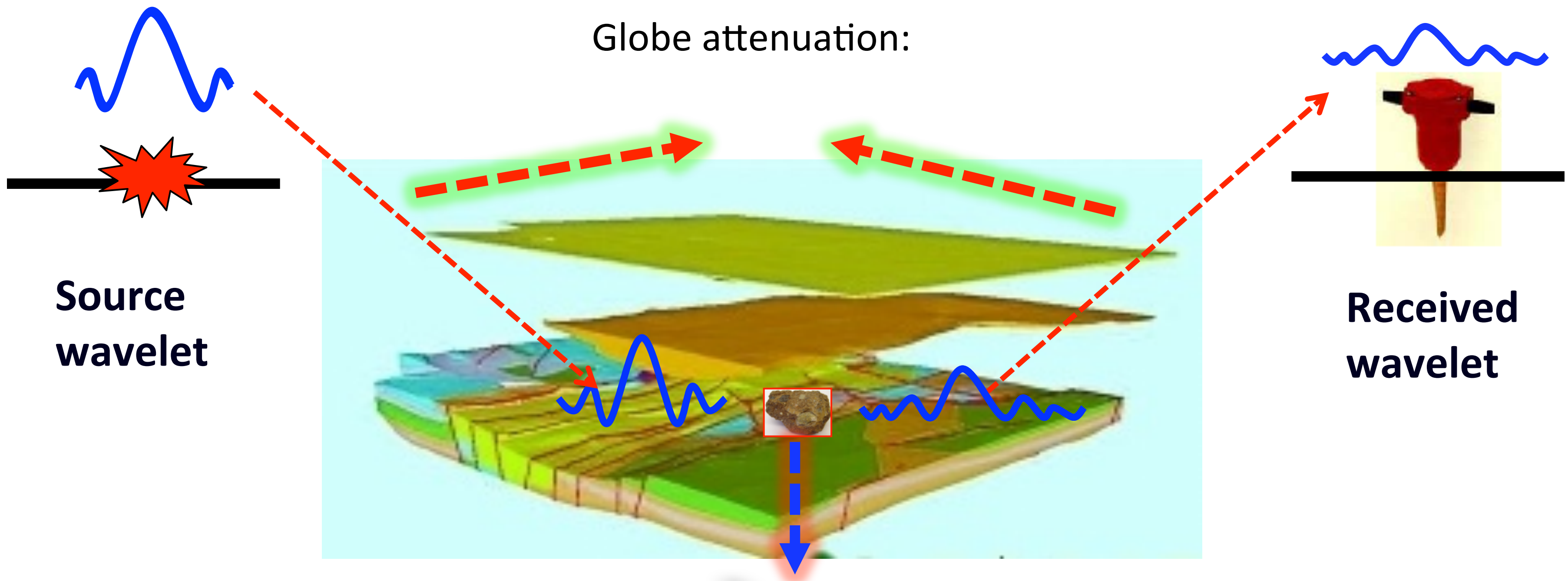
Outline

- Introduction
- Gradient calculation by adjoint state
- Strategies for viscoacoustic FWI
- Conclusion

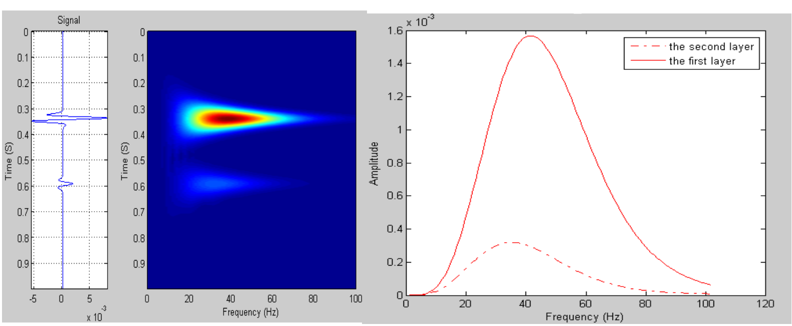
Introduction

Globe attenuation:

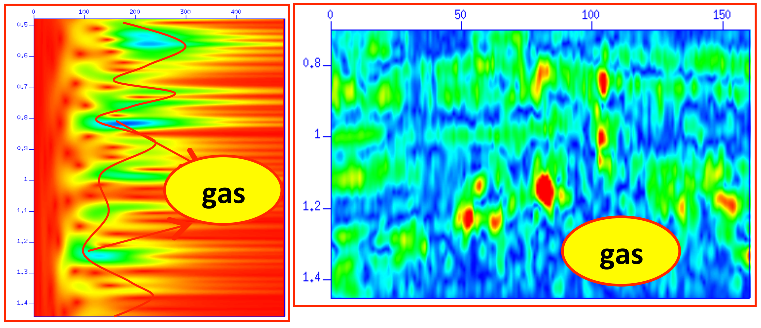


local attenuation:

Globle:



Local:



Introduction

Viscoacoustic FWI:

- ◆Viscoacoustic operator: fit the data and improve the accuracy of velocity
- ◆Multiparameter inversion: velocity and attenuation parameters

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Adjoint state

Objective function:

$$J[m] = h(p(K,\tau),K,\tau) = \frac{1}{2} \parallel p(K,\tau) - p_{obs} \parallel^2$$

Forward mapping:

$$F(p, v_x, v_z, r_l, K, \tau) = 0$$

$$K = \rho v_p^2$$

$$\tau = \frac{\tau_{\epsilon l}}{\tau_{\sigma l}} - 1$$ relative relaxation time difference

$\tau_{\sigma l}$ stress relaxation time

$\tau_{\epsilon l}$ strain relaxation time

Adjoint-state method

- Forward modeling equation

$$\begin{cases} \dot{p} = K(v_{x,x} + v_{z,z}) - K\sum_{l=0}^{L} r_l(1 - \tau_{\sigma l}/\tau_{\varepsilon l}) \\ \dot{v_x} = \frac{1}{\rho}\frac{\partial p}{\partial x} \\ \dot{v_z} = \frac{1}{\rho}\frac{\partial p}{\partial z} \\ \dot{r_l} + \frac{1}{\tau_{\sigma l}} r_l = \frac{1}{\tau_{\sigma l}}(v_{x,x} + v_{z,z}) \end{cases}$$

In order to get constant Q model, Generally Maxwell Body(GMB) is included in the forward equaion:

GMB: $$M(\omega) = M_R + \sum_{l=1}^{L} \frac{iM_l\omega}{\omega_l + i\omega}, \omega_l = \frac{M_l}{\eta_l} = \frac{1}{\tau_{\sigma l}}$$

Adjoint-state method

Adjoint wave equation:

$$\begin{cases} \dot{q} = K(v_{x,x} + v_{z,z}) + K\sum_{l=0}^{L} r_l(1 - \tau_{\sigma l}/\tau_{\varepsilon l}) \\ \dot{v_x} = \frac{1}{\rho}\frac{\partial q}{\partial x} \\ \dot{v_z} = \frac{1}{\rho}\frac{\partial q}{\partial z} \\ \dot{r_l} - \frac{1}{\tau_{\sigma l}} r_l = \frac{1}{\tau_{\sigma l}}(v_{x,x} + v_{z,z}) \end{cases}$$

Gradient for update:

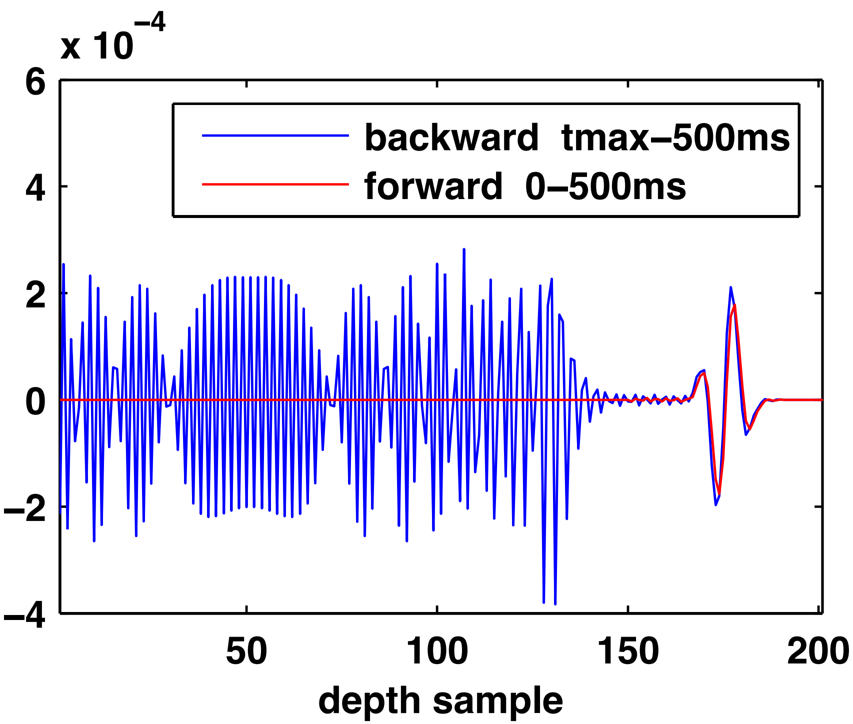
$$grad_K J[K,\tau] = -\langle q, DF_K[K,\tau]\rangle$$

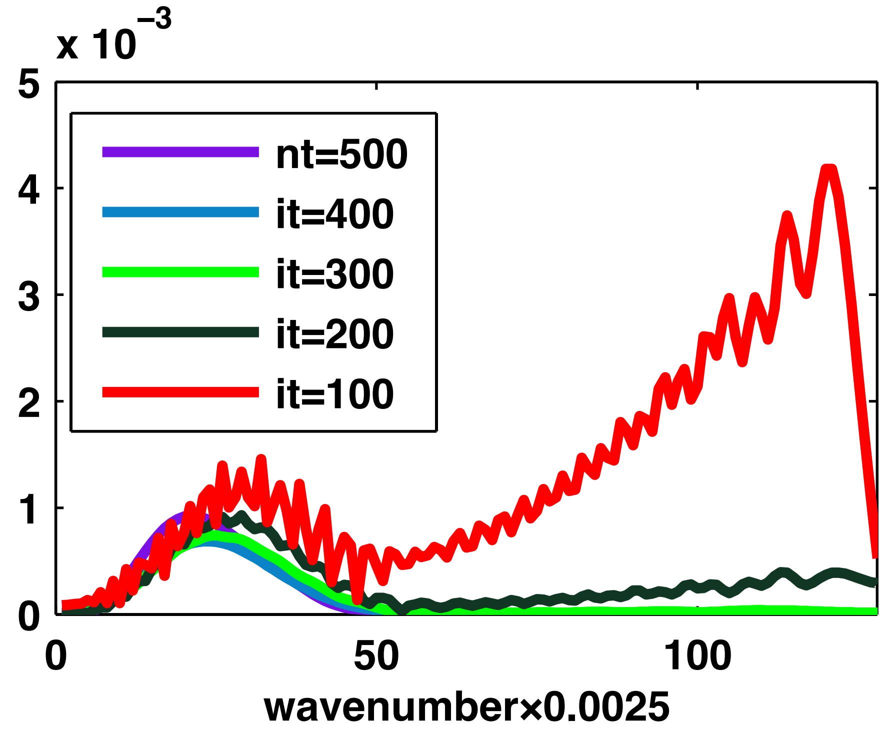
$$grad_\tau J[K,\tau] = -\langle q, DF_\tau[K,\tau]\rangle$$

similar cost to acoustic FWI

Can reverse time propagate?

- **An example of blowing up**



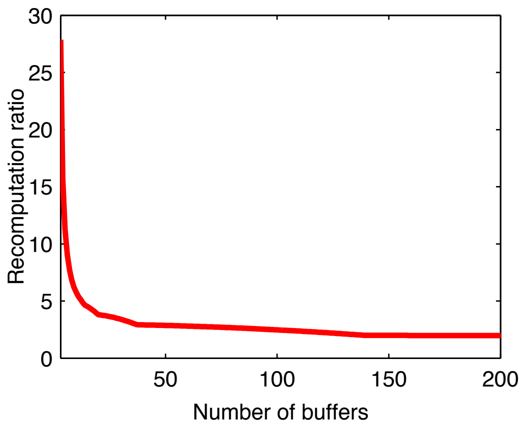


Q=50

Optimal Checkpointing

Blanch et al (1998); Griewark (1992); Symes(2007):

- **For given numbers of time steps and buffers, the recomputation ratio is minimum amongst all possible checkpointing schedules**



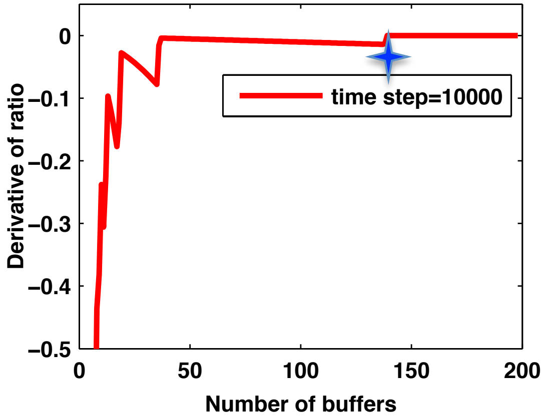
Question:

- **For given numbers of time steps, what is the best choice of the number of buffers?**

Solution:

buffers No.	ratio	d1-ratio
2	93.282	-65.409
13	4.837	-0.229
19	3.937	-0.140
37	2.934	-0. 016
140	1.999	-0.0131
150	1.985	-1.000e-04
170	1.983	-1.000e-04
190	1.981	-1.000e-04

Table 1: List of priority number of buffers (≤ 200)



Gradient calculated by adjoint state

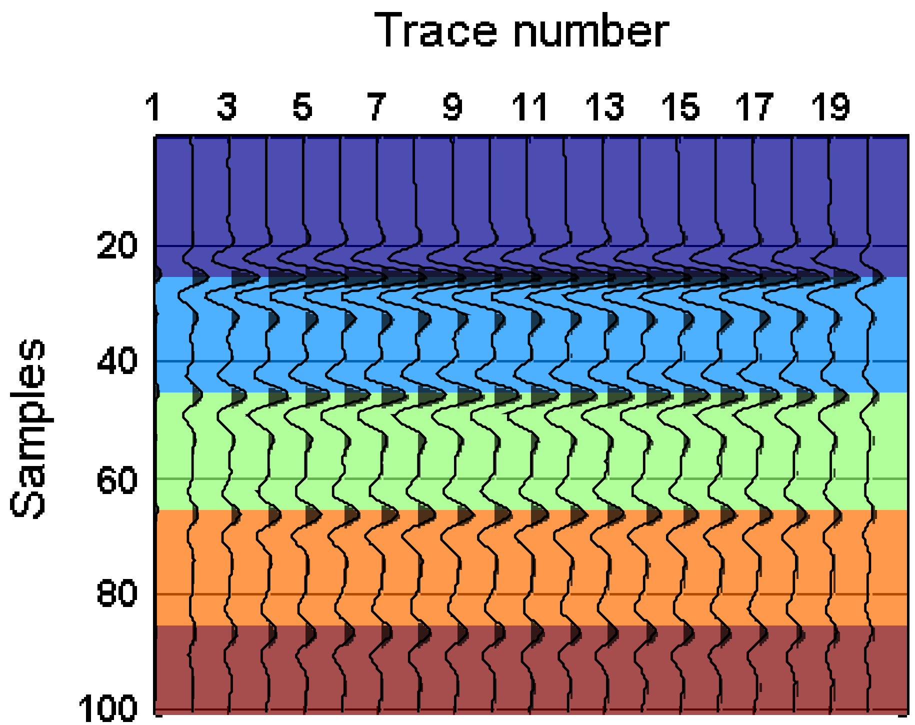
- **Compensation**

$$d(x,t) = d(x,t)e^{2\pi\omega_0 t/2Q_a}$$

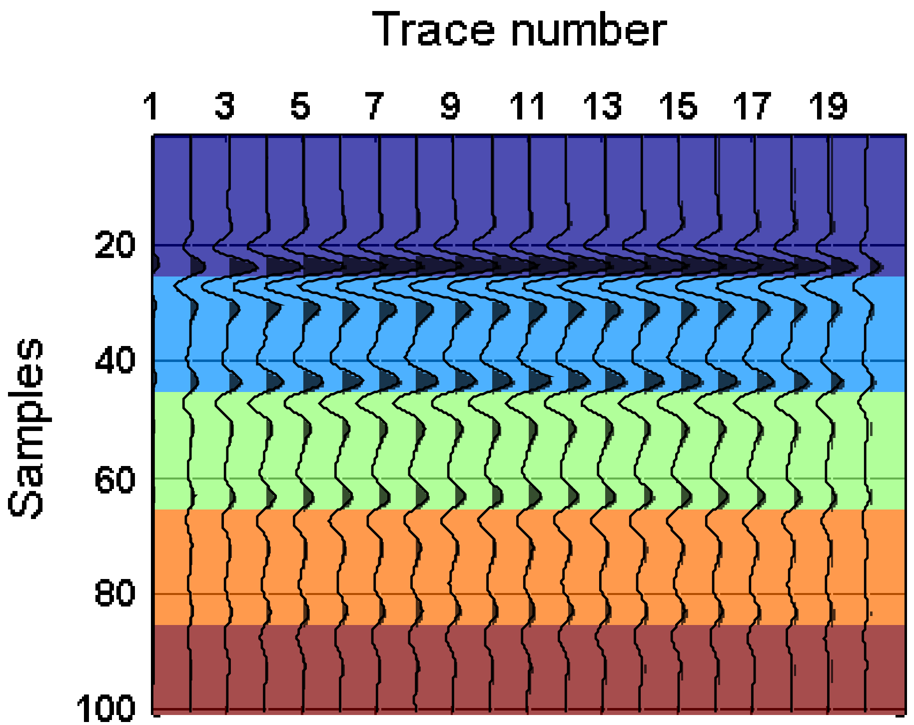
ω_0 corresponding to peak frequency

Q_a Average quality factor

Gradient

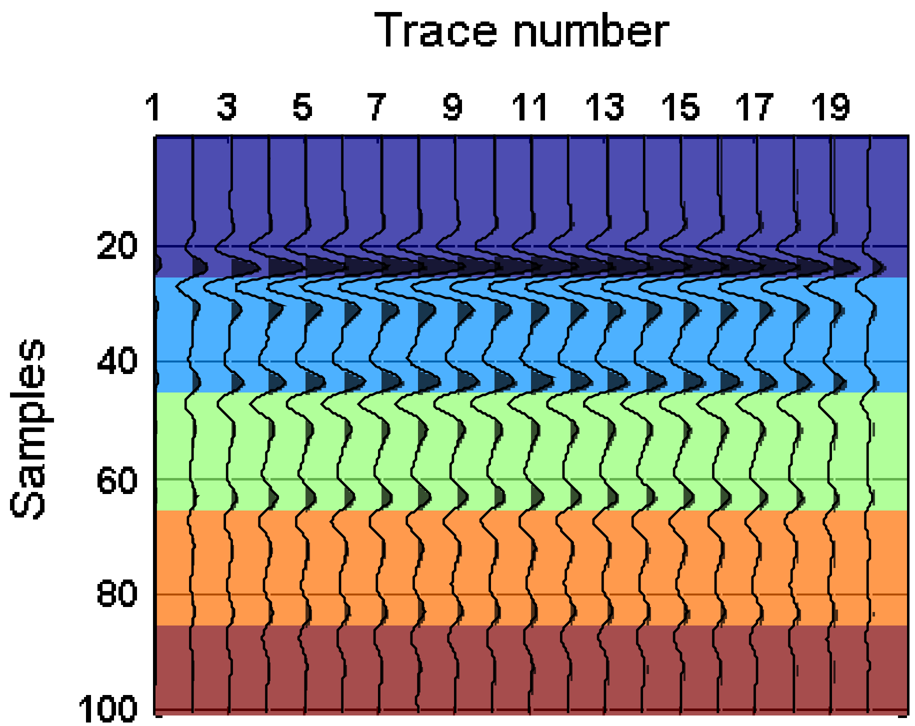


Acoustic

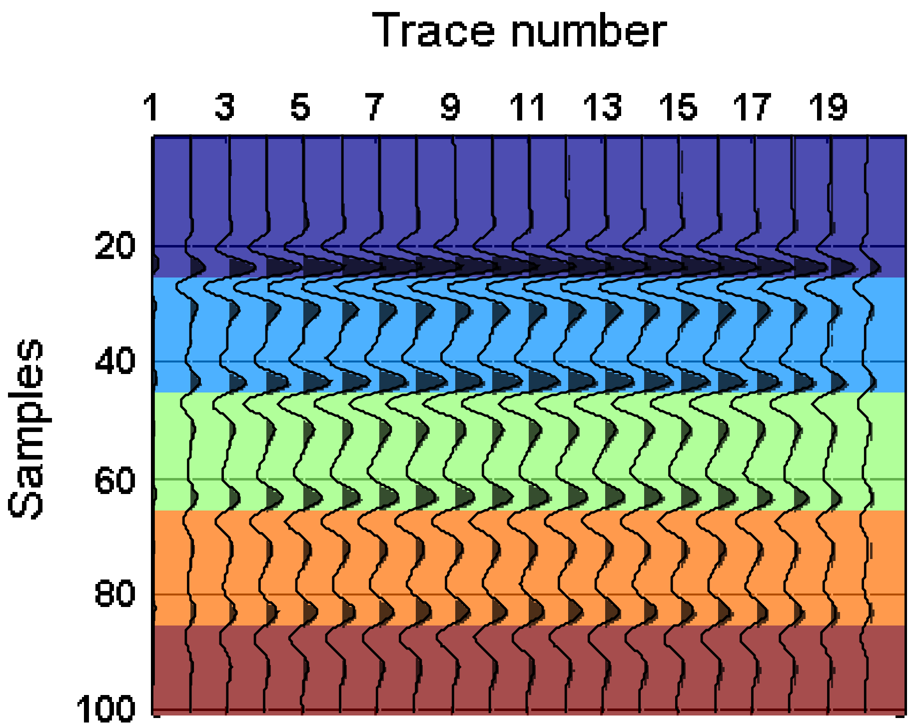


Viscoacoustic

Gradient



Viscoacoustic



Viscoacoustic compensation

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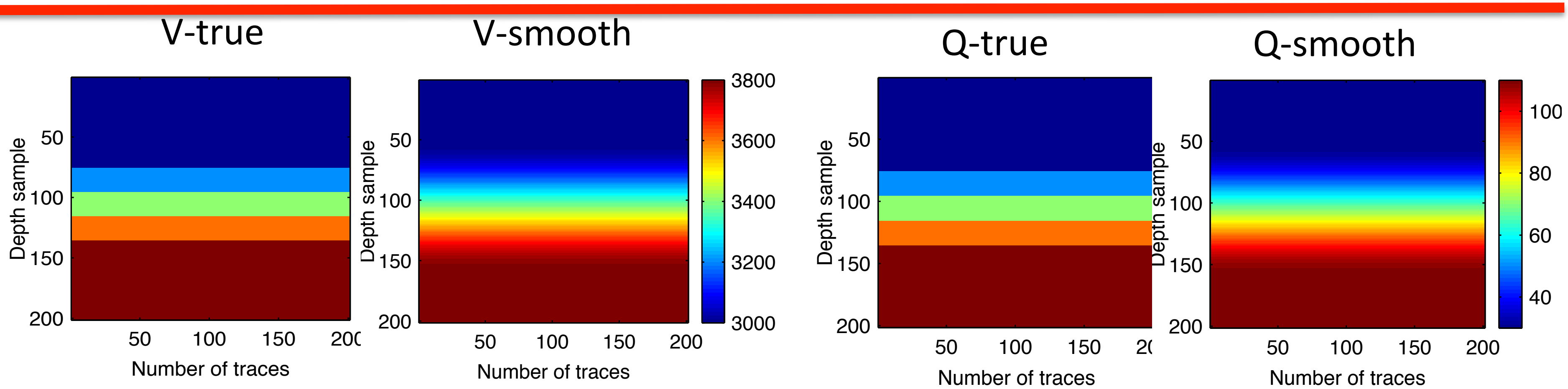
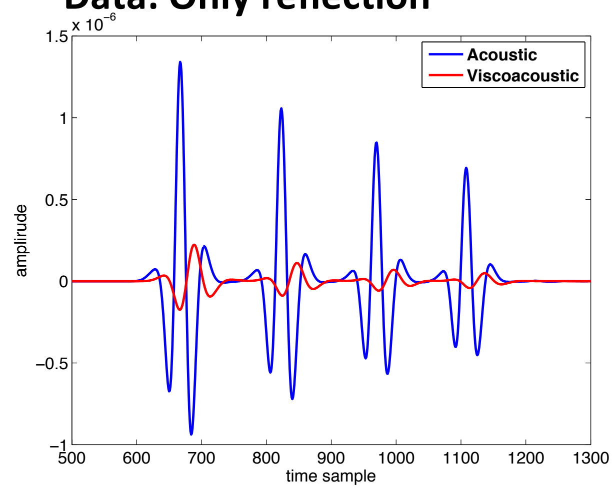
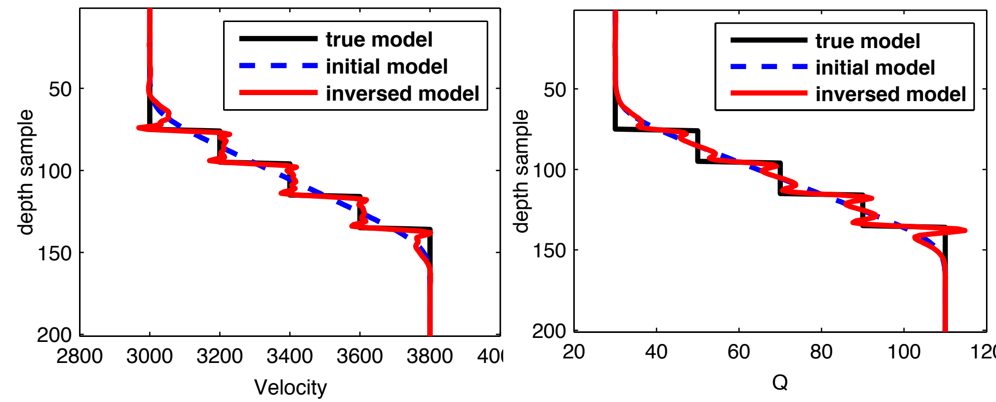
Ture models and observed data



- **Shot: 10; receriver:201, surface;**
- **Source :20HZ**
- **Offset_max: 2000m,**
- **Data: Only reflection**

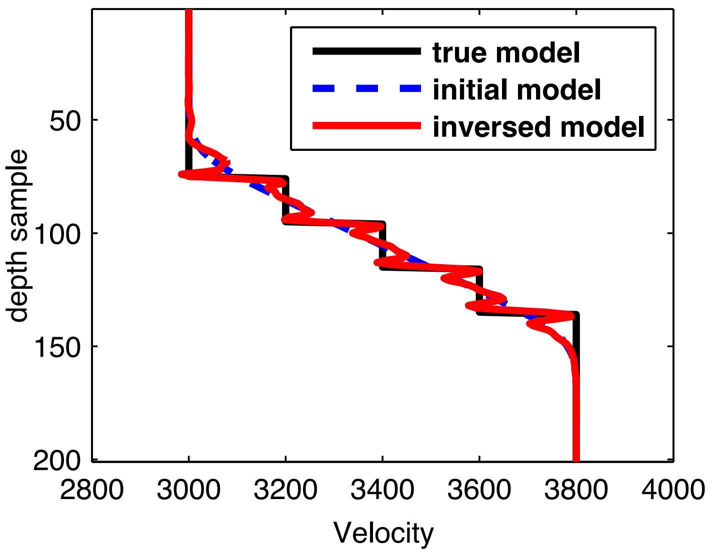


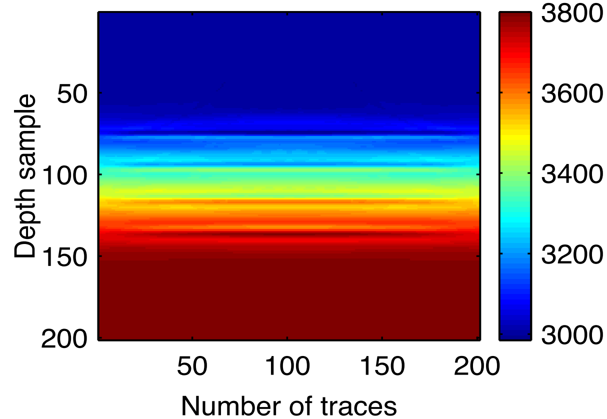


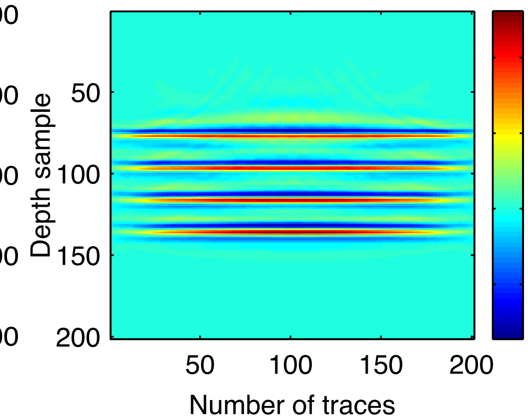
stopping criterion for iteration:

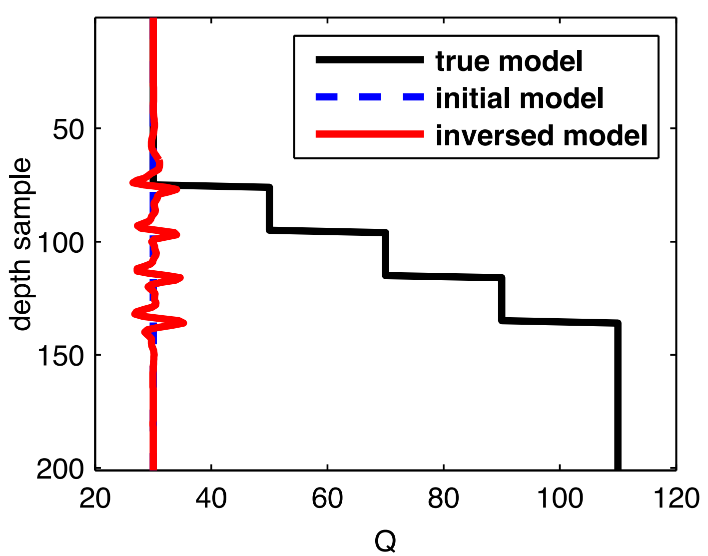
$$|gradient|_{max} <= 0.05|gradient_{initial}|_{max}$$

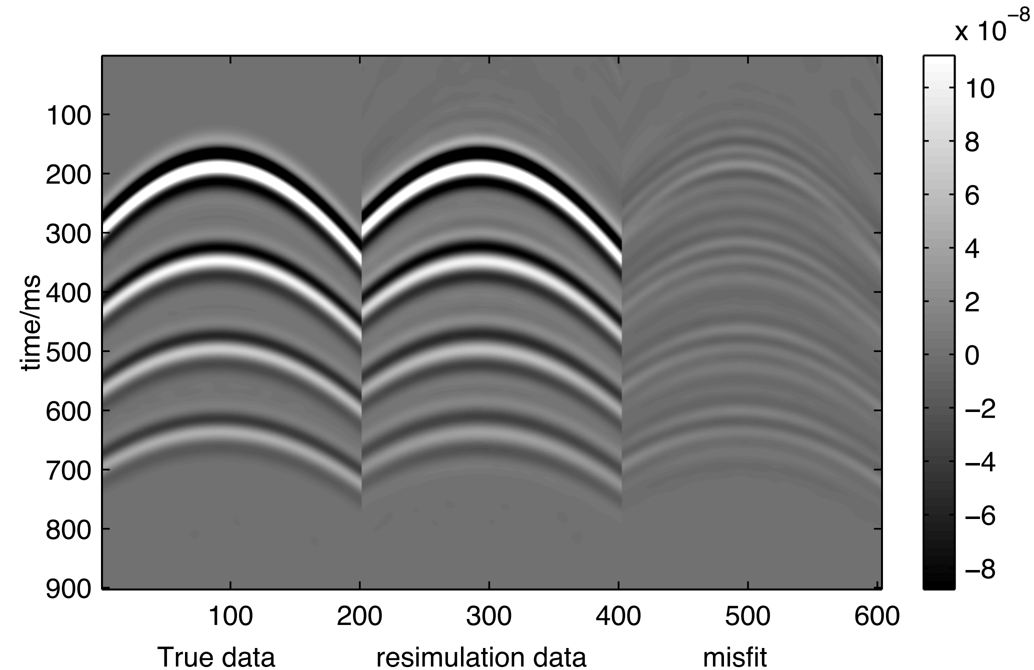
Initial V:smooth   initial Q: Q=30









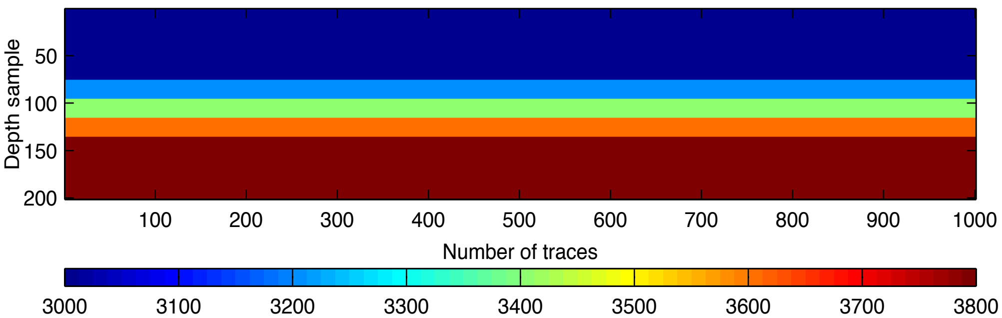


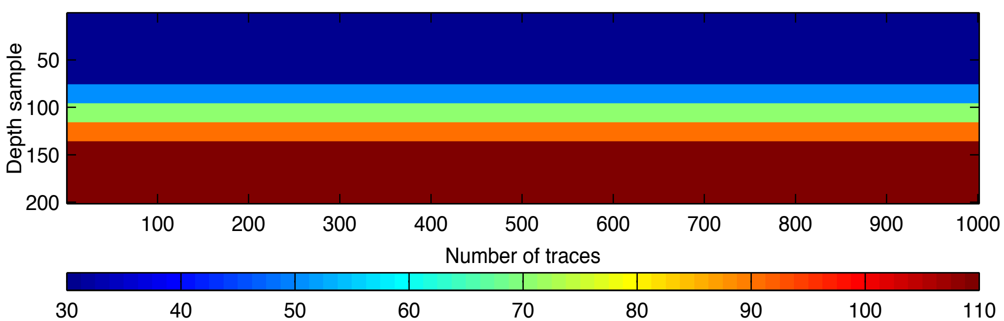
Ture models and observed data

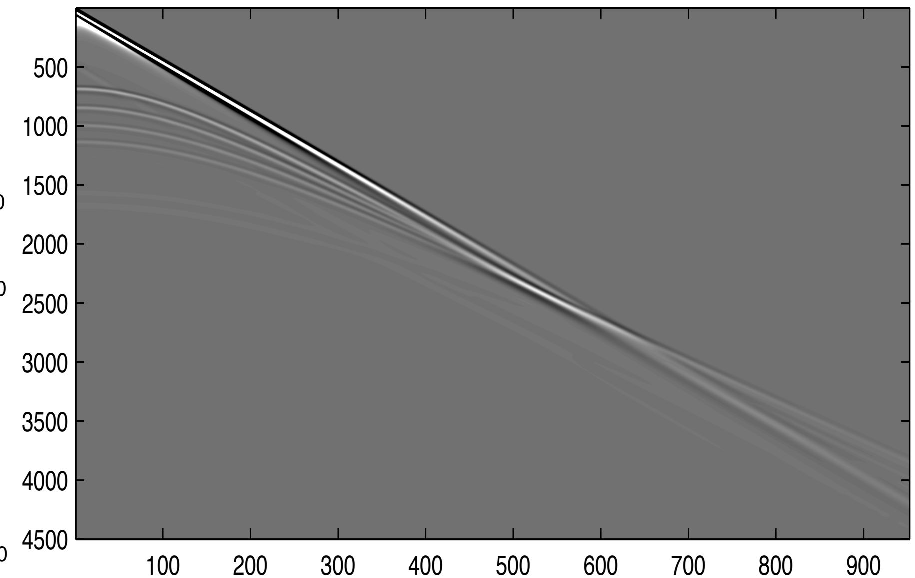
- **Shot: 10;**
- **Receriver:1001, surface;**
- **Source :20HZ**
- **Offset_max: 10000m**
- **Data:full wave, viscoacoustic**

stopping criterion for iteration

$$|gradient|_{max} <= 0.1|gradient_{initial}|_{max}$$





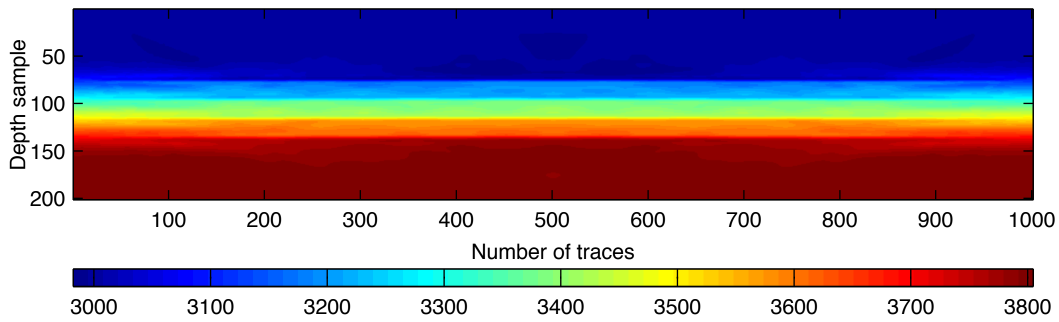


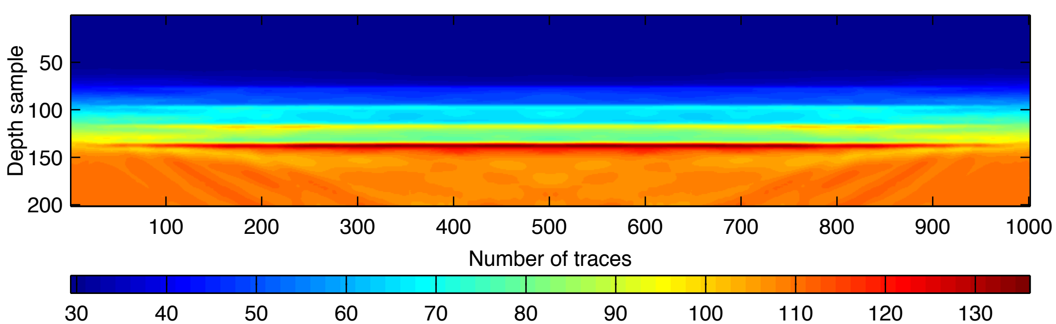
Initial V:smooth   initial Q: Q=smooth

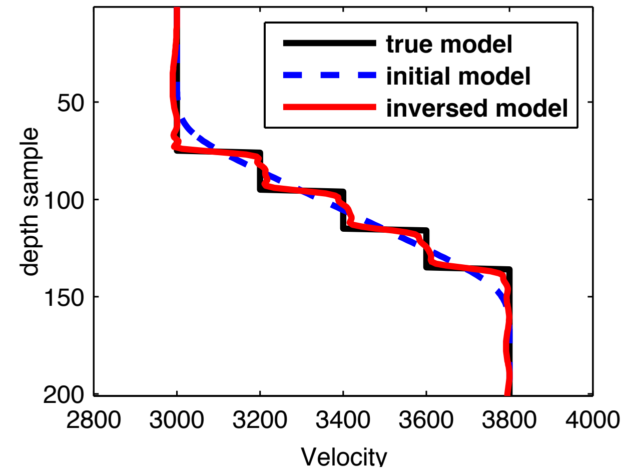
Initial V :smooth

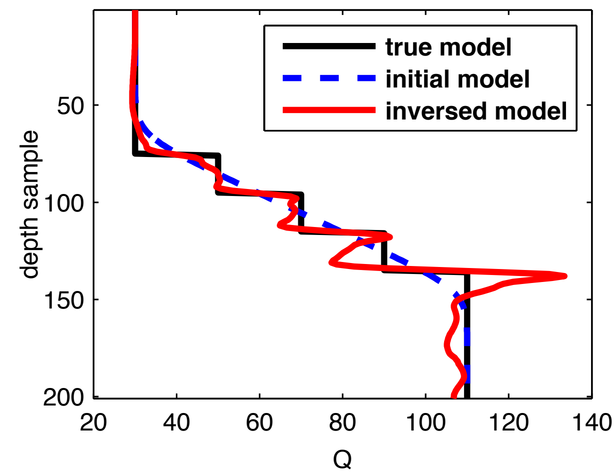
Data: all the information

Initial Q :smooth







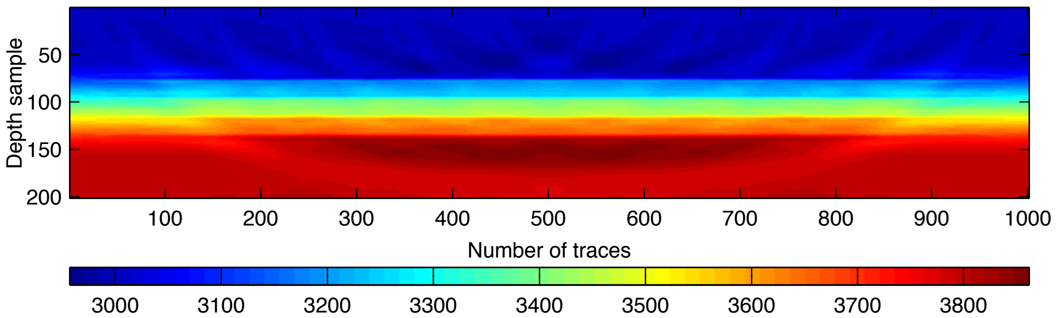


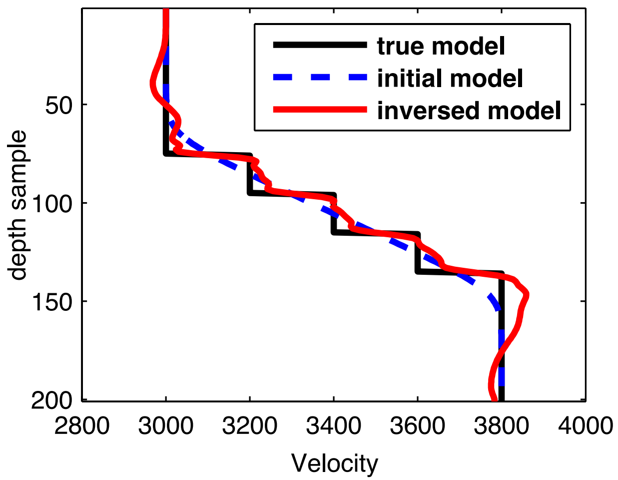
Initial V:smooth   initial Q: Q=30

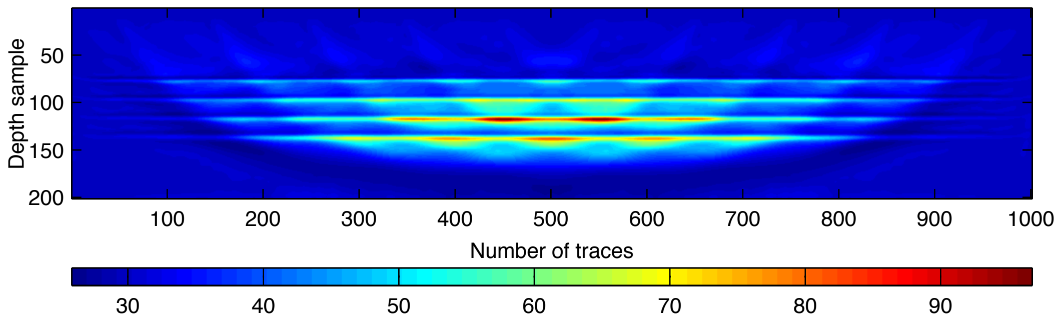
Initial V :smooth

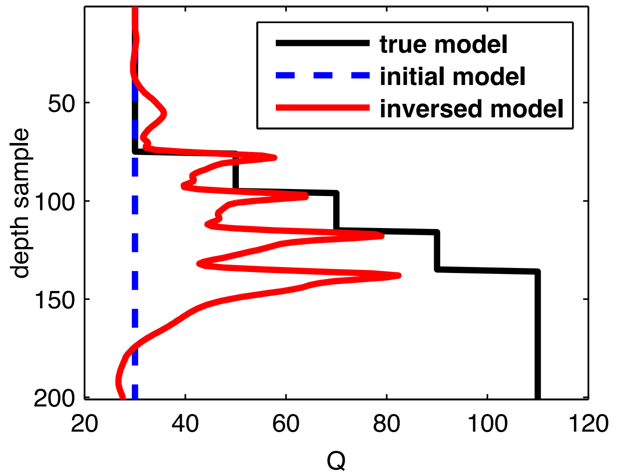
Data: all the information

Initial Q : Q=30







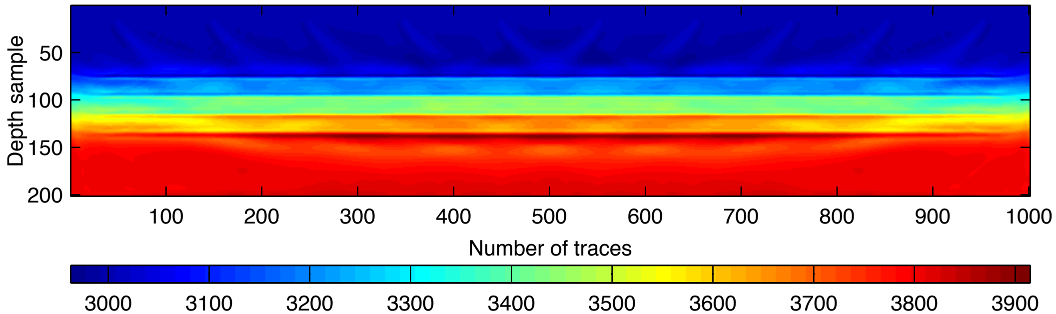


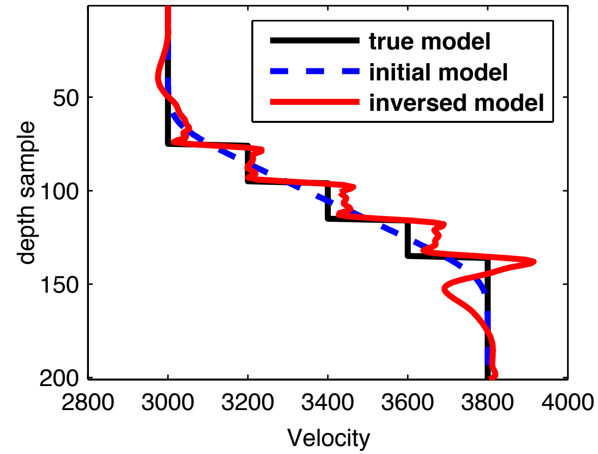
Initial V:smooth initial Q: Q=30

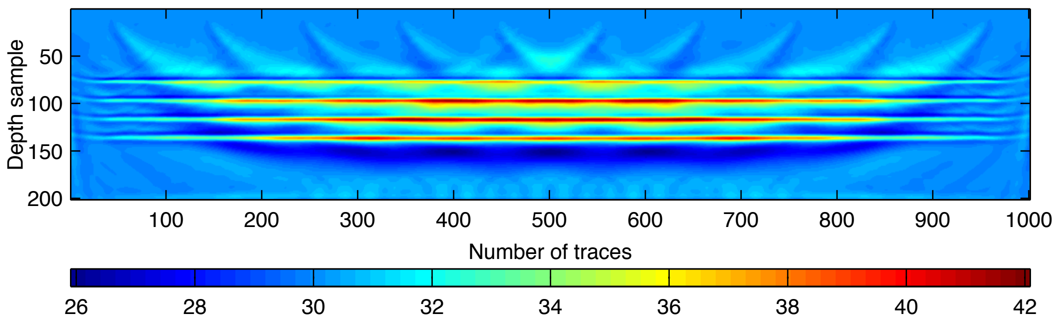
Initial V :smooth

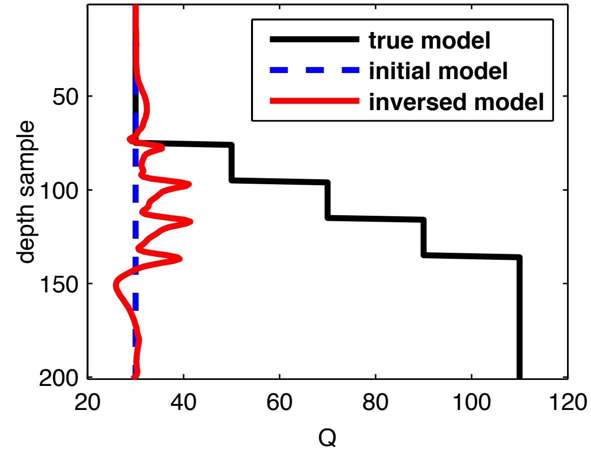
Initial Q : Q=30

Data： only long offset reflection



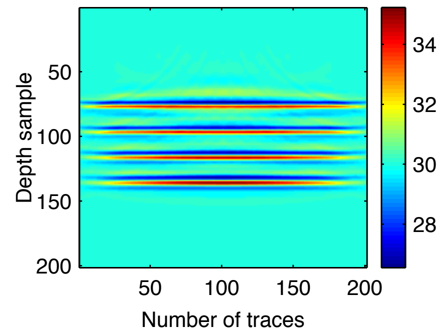


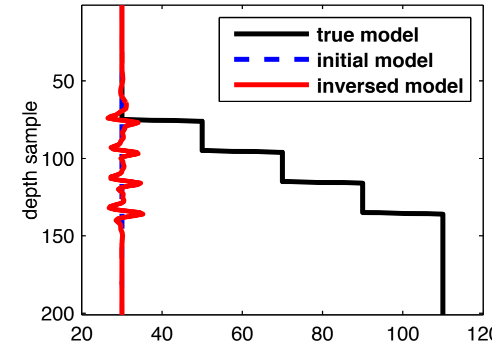




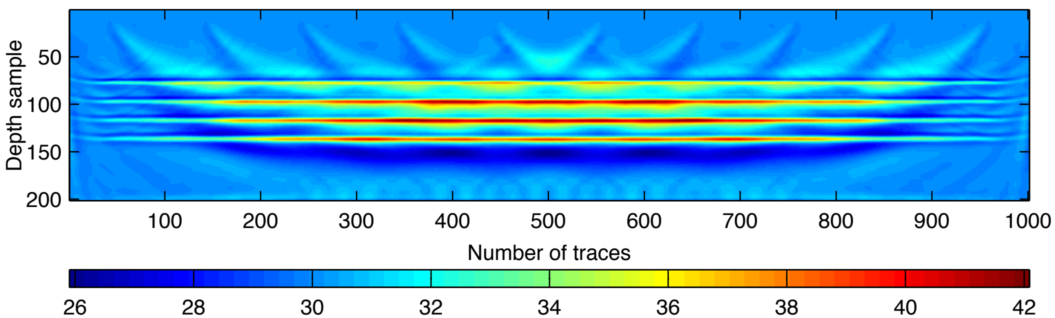
Initial V:smooth initial Q: Q=30

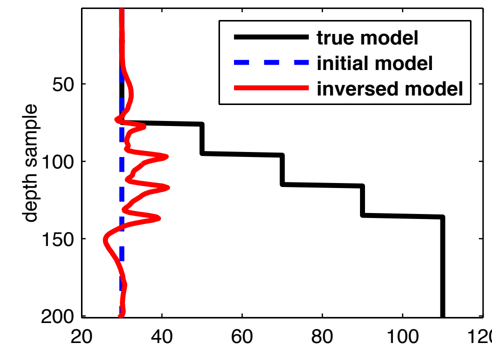
Reflection : small aperture



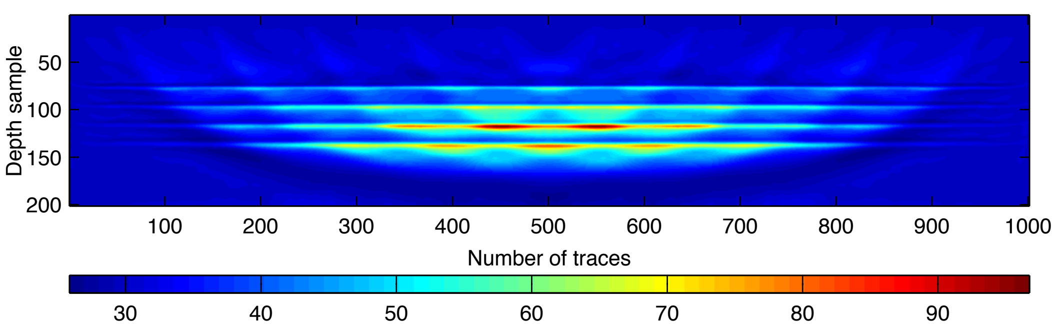


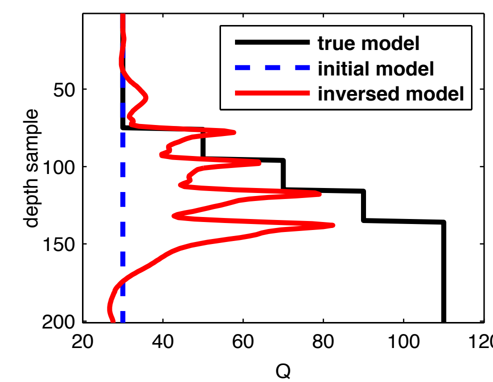
Reflection : large aperture





all: large aperture





Conclusion

- For reflection data, v_p and Q are strongly coupled . Incorrect Q results in incorrect reflector amplitudes. Long wavelength Q structure is not updated. The large aperture reflection contains limited long wavelength Q.

- For refraction data , information about long wavelength Q structure presents in the data, and this permits update of Q.

- The update of Q contains more short scale structure than that of velocity, and is easier to fall into local solution, especially for the case that the velocity is far from the true one.

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Thank you