Multisource Least-Squares
Extended Reverse-time Migration with Preconditioning Guided Gradient Method

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Outline

1 Background
   - ERTM vs LSERTM
   - Phase Encoding
   - Preconditioning

2 Theory and Implementation
   - ERTM with Phase Encoding
   - Preconditioner: approximated diagonal of Hessian
   - Inversion Scheme: PGG method

3 Numerical tests
   - Salt Model

4 Conclusion and Discussion
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4 Conclusion and Discussion
ERTM vs LSERTM

- ERTM (Extended Reverse-Time Migration):

  - is only adjoint of linearized extended Born Modeling (LEBM) operator
  - provides extended image with limited resolution and imbalanced amplitudes

- LSERTM (Least-Squares ERTM):

  - approximates inverse of LEBM operator using iteration methods
  - provides extended image with high resolution and balanced amplitude
  - is more reliable in velocity analysis and AVO/AVA
  - is more expensive!
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Review of Phase Encoding

Main idea: Encode all of shot gathers into one or several super-shot gathers with designed encoding functions so as to solve a smaller number of wave equations.
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- Plane-wave phase encoding (Zhang, 2005; Liu, 2006; Tang, 2009; ...)
- Amplitude encoding (Godwin el al. 2010)
- Deterministic source encoding (Symes, 2010; Gao, 2010)
- ...
Review of Preconditioning

Main idea: Using preconditioner to accelerate the convergent rate of iteration
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- Approximated diagonal of Hessian (Pratt, 1999; Shin, 2001; Tang, 2010; ...)
- Structure-oriented filter (Prucha, 2002; Clapp, 2005)
- Deblurring filter (Aoki et al., 2009)
- Sparsity promotion (Herrmann et al. 2009)
- Image-guided filter (Ma et al., 2010)
- ...

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Acoustic constant density (ACD) wave equation:

\[(\nabla^2 + \omega^2 m(x, z))u(x, z, w) = -f(w)\delta(x - x_s)\]  \hspace{1cm} (1)

Extended ACD wave equation:

\[\nabla^2 u(x, z, w) + \omega^2 \int dym(x, y, z)u(y, z, w) = -f(w)\delta(x - x_s)\]  \hspace{1cm} (2)

Split the extended model into two parts:

\[m(x, y, z) = b(x, z)\delta(x - y) + r(x, y, z)\]  \hspace{1cm} (3)

Linearized approximation:

\[(\nabla^2 + \omega^2 b(x, z))u_0(x, z, w) = f(w)\delta(x - x_s)\]  \hspace{1cm} (4)

\[(\nabla^2 + \omega^2 b(x, z))\delta u(x, z, w) = -\omega^2 \int dyr(x, y, z)u_0(y, z, w)\]  \hspace{1cm} (5)
LEBM operator and its adjoint

Linearized extended Born modeling (LEBM):

\[ d(x_r, x_s, \omega) = -\omega^2 f(\omega) \int dxdh G(x_r, x + h, \omega)r(x, h)G(x - h, x_s, \omega) \]

Extended reverse time migration (ERTM):

\[ r(x, h) = -\int dx_sdxd\omega \omega^2 f^*(\omega)G^*(x_s, x - h, \omega)G^*(x + h, x_r, \omega)d(x_r, x_s, \omega) \]
Encoded LEBM and ERTM

- **Encoding function:**
  \[ \alpha(x_s, p_s) = \frac{1}{\sqrt{N}} \gamma(x_s, p_s) \]
  
  \((p_s: \text{realization index; } N: \text{realization number; } \gamma \text{ random sequence of signs.})\)

- **Encoded seismic data:**
  \[ \tilde{d}_{obs}(x_r, p_s, \omega) = \int d x_s \alpha(x_s, p_s) d_{obs}(x_r, x_s, \omega) \]

- **Encoded source wavefield:**
  \[ S(x, p_s, \omega) = \int d x_s \alpha(x_s, p_s) f(\omega) G(x, x_s, \omega) \]
Encoded LEBM and ERTM

- Encoded LEBM:
  \[ \tilde{d}(x_r, p_s, \omega) = -\omega^2 \int \! dx dh G(x_r, x + h, \omega) r(x, h) S(x - h, p_s, \omega) \]

- Encoded ERTM:
  \[ r(x, h) = -\int \! dp_s dx_r d\omega \omega^2 S^*(p_s, x - h, \omega) G^*(x + h, x_r, \omega) \tilde{d}(x_r, p_s, \omega) \]
Preconditioner: approximated diagonal of Hessian

Diagonal of Hessian in the subsurface offset domain (Valenciano, 2006):

\[ D(x, h) = \int d\omega \omega^4 |f(\omega)|^2 |G(x - h, x_s, \omega)|^2 |G(x + h, x_r, \omega)|^2. \]

Encode receiver wavefield with \( \beta(x_r, p_r) = \frac{1}{N} \gamma(x_r, p_r) \),

\[ R(x, p_r, \omega) = \int d\omega \beta(x_r, p_r) G(x, x_r, \omega). \]

Encoded Diagonal of Hessian:

\[ \tilde{D}(x, h, p_s, p_r) = \int d\omega \omega^4 |S(x - h, p_s, \omega) R(x + h, p_r, \omega)|^2. \]

More approximation:

\[ \tilde{D}_{SS}(x, h, p_s) = \int \omega \omega^4 |S(x - h, p_s, \omega)|^2 |S(x + h, p_s, \omega)|^2. \]

Observation:

It has shown that this approximation seems to be accurate enough as a preconditioner in least-squares inversion (Tang, 2010). Here we extend this idea into subsurface offset domain.
Encoded LEBM can be written in a compact form:

$$\mathbf{d} = \mathbf{Lm}$$

Encoded Least-squares extended reverse-time migration:

$$\min_{\mathbf{m}} J_{LSM}[\mathbf{m}] = \frac{1}{2} \| \mathbf{Lm} - \mathbf{d}_{obs} \|_p + \frac{\sigma}{2} \| \mathbf{m} \|_p$$

where $\| \cdot \|_p$ denotes $\ell^p$ norm with $1 \leq p \leq 2$. 
Inversion Scheme: PGG Method

Equivalent form:

\[
\min_{\mathbf{m}} J_{LSM}[\mathbf{m}] = \frac{1}{2} \| \mathbf{W}_r (\tilde{\mathbf{L}} \mathbf{m} - \tilde{\mathbf{d}}_{obs}) \|_2 + \frac{\sigma}{2} \| \mathbf{W}_m \mathbf{m} \|_2
\]

Methods:

- **Iteratively Reweighted Least-Squares (IRLS) method** (Claerbout, 1992)
  - Non-linear inverse problem
  - Need to calculate weighting matrix at the outer loop of CG

- **Conjugate Guided Gradient (CGG) method** (Ji, 2006)
  - A variant of IRLS
  - Linear inverse problem
  - Only one calculation of \( \mathbf{L} \) and \( \mathbf{L}^T \) is needed at each iteration

- **Preconditioning Guided Gradient (PGG) method**
  - Updated version of CGG
  - Incorporates preconditioner into CGG
  - Adapts to phase encoding scheme
Inversion Scheme: PGG Method

Algorithm 1 MLSERTM with PGG

1: for $k = 0 \cdots niter$ do
2:    generate random sequence of signs $\gamma$
3:    encode sources and data to get $\tilde{L}$ and $\tilde{d}_{obs}$
4:    $r^k = \tilde{L}m^k - \tilde{d}_{obs}$
5:    compute $\tilde{W}_r^k$
6:    compute $\tilde{W}_m^k$
7:    $dm^k = \tilde{W}_m^T k \hat{D}^{-1}_{SS} \tilde{L}^T \tilde{W}_r^T k r^k$
8:    $dr^k = \tilde{L}dm^k$
9:    $\alpha_k = \frac{\langle dr^k, r^k \rangle}{\langle dr^k, dr^k \rangle}$
10:   $m^{k+1} = m^k + 1 - \alpha_k dm^k$
11:   end for
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Reflectivity imaging test

Figure: Velocity model
Reflectivity imaging test

Figure: Reflectivity
Reflectivity imaging test

Figure: RTM
Reflectivity imaging test

Figure: RTM after Laplacian filter
Reflectivity imaging test

**Figure**: Approximated diagonal of Hessian
Reflectivity imaging test

**Figure:** LSRTM without preconditioning
Reflectivity imaging test

**Figure:** LSRTM with preconditioning
Reflectivity imaging test

Figure: LSRTM with preconditioning and $\ell_{1.5}$ norm on model
Reflectivity imaging test

Figure: LSRTM with preconditioning and $\ell_1$ norm on model
Extended reflectivity imaging test

Figure: Approximated diagonal Hessian in subsurface offset domain
Extended reflectivity imaging test

Figure: LSERTM with preconditioning
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LSERTM provides reflectivity image with higher resolution and more balanced amplitude, which is useful in the following migration velocity analysis and AVO/AVA analysis;

Seismic data is compressed greatly with the help of phase encoding so that the efficiency of seismic imaging is improved dramatically;

Using approximated diagonal of Hessian as preconditioner can improve the convergent rate of LSM;

A modified CGG inversion scheme namely PGG is proposed to solve $\ell_p$ norm problem flexibly and efficiently;

Sparsity seems to be a good prior information in suppressing crosstalk introduced by phase encoding, especially in imaging of extended reflectivity.
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