Multisource Least-Squares Extended Reverse-time Migration with Preconditioning Guided Gradient Method

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Outline

1. Background
   - ERTM vs LSERTM
   - Phase Encoding
   - Preconditioning
2. Theory and Implementation
   - ERTM with Phase Encoding
   - Preconditioner: approximated diagonal of Hessian
   - Inversion Scheme: PGG method
3. Numerical tests
   - Salt Model
4. Conclusion and Discussion

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  - is more expensive!

Review of Phase Encoding

Main idea: Encode all of shot gathers into one or several super-shot gathers with designed encoding functions so as to solve a smaller number of wave equations.

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- Random phase encoding (Morton,1998; Romero,2000; Kreb et.al 2009; Tang, 2009; ...)
- Plane-wave phase encoding (Zhang, 2005; Liu, 2006; Tang, 2009; ...)
- Amplitude encoding (Godwin el al. 2010)
- Deterministic source encoding (Symes, 2010; Gao, 2010)
- ...

Review of Preconditioning

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- Approximated diagonal of Hessian (Pratt, 1999; Shin, 2001; Tang, 2010; ...)
- Structure-oriented filter (Prucha, 2002; Clapp, 2005)
- Deblurring filter (Aoki et al., 2009)
- Sparsity promotion (Herrmann et al. 2009)
- Image-guided filter (Ma et al., 2010)
- ...

Outline

Linearized Born approximation

Acoustic constant density(ACD) wave equation:

$$(\nabla^2 + \omega^2 m(\mathbf{x}, z))u(\mathbf{x}, z, w) = -f(w)\delta(\mathbf{x} - \mathbf{x}_s) \tag{1}$$

Extended ACD wave equation:

$$\nabla^2 u(\mathbf{x}, z, w) + \omega^2 \int \mathrm{d}\mathbf{y} m(\mathbf{x}, \mathbf{y}, z)u(\mathbf{y}, z, w) = -f(w)\delta(\mathbf{x} - \mathbf{x}_s) \tag{2}$$

Split the extended model into two parts:

$$m(\mathbf{x}, \mathbf{y}, z) = b(\mathbf{x}, z)\delta(\mathbf{x} - \mathbf{y}) + r(\mathbf{x}, \mathbf{y}, z) \tag{3}$$

Linearized approximation:

$$(\nabla^2 + \omega^2 b(\mathbf{x}, z))u_0(\mathbf{x}, z, w) = f(w)\delta(\mathbf{x} - \mathbf{x}_s) \tag{4}$$

$$(\nabla^2 + \omega^2 b(\mathbf{x}, z))\delta u(\mathbf{x}, z, w) = -\omega^2 \int \mathrm{d}\mathbf{y} r(\mathbf{x}, \mathbf{y}, z)u_0(\mathbf{y}, z, w) \tag{5}$$

LEBM operator and its adjoint

Linearized extended Born modeling (LEBM):

$$d(\mathbf{x}_r, \mathbf{x}_s, \omega) = -\omega^2 f(\omega) \int \mathrm{d}\mathbf{x}\mathrm{d}\mathbf{h}\, G(\mathbf{x}_r, \mathbf{x}+\mathbf{h}, \omega) r(\mathbf{x}, \mathbf{h}) G(\mathbf{x}-\mathbf{h}, \mathbf{x}_s, \omega)$$

Extended reverse time migration (ERTM):

$$r(\mathbf{x}, \mathbf{h}) = -\int \mathrm{d}\mathbf{x_s}\mathrm{d}\mathbf{x_r}\mathrm{d}\omega\, \omega^2 f^*(\omega) G^*(\mathbf{x}_s, \mathbf{x}-\mathbf{h}, \omega) G^*(\mathbf{x}+\mathbf{h}, \mathbf{x}_r, \omega) d(\mathbf{x}_r, \mathbf{x}_s, \omega)$$

Encoded LEBM and ERTM

- Encoding function:

$$\alpha(\mathbf{x_s}, p_s) = \frac{1}{\sqrt{N}}\gamma(\mathbf{x_s}, p_s)$$

(p_s: realization index; N: realization number; γ random sequence of signs.)

- Encoded seismic data:

$$\tilde{d}_{obs}(\mathbf{x}_r, p_s, \omega) = \int \mathrm{d}\mathbf{x_s}\, \alpha(\mathbf{x}_s, p_s) d_{obs}(\mathbf{x}_r, \mathbf{x}_s, \omega)$$

- Encoded source wavefield:

$$S(\mathbf{x}, p_s, \omega) = \int \mathrm{d}\mathbf{x_s} \alpha(\mathbf{x}_s, p_s) f(\omega) G(\mathbf{x}, \mathbf{x}_s, \omega)$$

Encoded LEBM and ERTM

- Encoded LEBM:

$$\tilde{d}(\mathbf{x}_r, p_s, \omega) = \quad -\omega^2 \int \mathrm{d}\mathbf{x}\mathrm{d}\mathbf{h} G(\mathbf{x}_r, \mathbf{x}+\mathbf{h}, \omega) r(\mathbf{x}, \mathbf{h}) S(\mathbf{x}-\mathbf{h}, p_s, \omega)$$

- Encoded ERTM:

$$r(\mathbf{x}, \mathbf{h}) = \quad -\int \mathrm{d}p_s \mathrm{d}\mathbf{x_r} \mathrm{d}\omega\, \omega^2 S^*(p_s, \mathbf{x}-\mathbf{h}, \omega) G^*(\mathbf{x}+\mathbf{h}, \mathbf{x}_r, \omega) \tilde{d}(\mathbf{x}_r, p_s, \omega)$$

Preconditioner: approximated diagonal of Hessian

Diagonal of Hessian in the subsurface offset domain (Valenciano, 2006):

$$D(\mathbf{x},\mathbf{h}) = \int \mathrm{d}\mathbf{x}_s \mathrm{d}\mathbf{x}_r \mathrm{d}\omega\, \omega^4 |f(\omega)|^2\, |G(\mathbf{x}-\mathbf{h},\mathbf{x}_s,\omega)|^2 |G(\mathbf{x}+\mathbf{h},\mathbf{x}_r,\omega)|^2.$$

Encode receiver wavefield with $\beta(\mathbf{x_r}, p_r) = \frac{1}{N}\gamma(\mathbf{x_r}, p_r)$,

$$R(\mathbf{x}, p_r, \omega) = \int \mathrm{d}\mathbf{x}_r\, \beta(\mathbf{x}_r, p_r) G(\mathbf{x}, \mathbf{x}_r, \omega).$$

Encoded Diagonal of Hessian:

$$\widetilde{D}(\mathbf{x},\mathbf{h},p_s,p_r) = \int \mathrm{d}\omega\, \omega^4 |S(\mathbf{x}-\mathbf{h},p_s,\omega) R(\mathbf{x}+\mathbf{h},p_r,\omega)|^2.$$

More approximation:

$$\widetilde{D}_{SS}(\mathbf{x},\mathbf{h},p_s) = \int \omega\, \omega^4 |S(\mathbf{x}-\mathbf{h},p_s,\omega)|^2 |S(\mathbf{x}+\mathbf{h},p_s,\omega)|^2.$$

Observation:

It has shown that this approximation seems to be accurate enough as a preconditioner in least-squares inversion (Tang, 2010). Here we extend this idea into subsurface offset domain.

Inversion Scheme: PGG Method

Encoded LEBM can be written in a compact form:

$$\tilde{\mathbf{d}} = \tilde{\mathbf{L}}\mathbf{m}$$

Encoded Least-squares extended reverse-time migration:

$$min_{\mathbf{m}} J_{LSM}[\mathbf{m}] = \frac{1}{2} \parallel \tilde{\mathbf{L}}\mathbf{m} - \tilde{\mathbf{d}}_{obs} \parallel_p + \frac{\sigma}{2} \parallel \mathbf{m} \parallel_p$$

where $\| \cdot \|_p$ denotes ℓ^p norm with $1 \leq p \leq 2$.

Inversion Scheme: PGG Method

Equivalent form:

$$min_{\mathbf{m}} J_{LSM}[\mathbf{m}] = \frac{1}{2} \parallel \mathbf{W_r}(\tilde{\mathbf{L}}\mathbf{m} - \tilde{\mathbf{d}}_{obs}) \parallel_2 + \frac{\sigma}{2} \parallel \mathbf{W_m m} \parallel_2$$

Methods:

- Iteratively Reweighted Least-Squares (IRLS) method (Claerbout, 1992)
  - Non-linear inverse problem
  - Need to calculate weighting matrix at the outer loop of CG
- Conjugate Guided Gradient (CGG) method (Ji, 2006)
  - A variant of IRLS
  - Linear inverse problem
  - Only one calculation of $\mathbf{L}$ and $\mathbf{L}^T$ is needed at each iteration
- Preconditioning Guided Gradient (PGG) method
  - Updated version of CGG
  - Incorporates preconditioner into CGG
  - Adapts to phase encoding scheme

Inversion Scheme: PGG Method

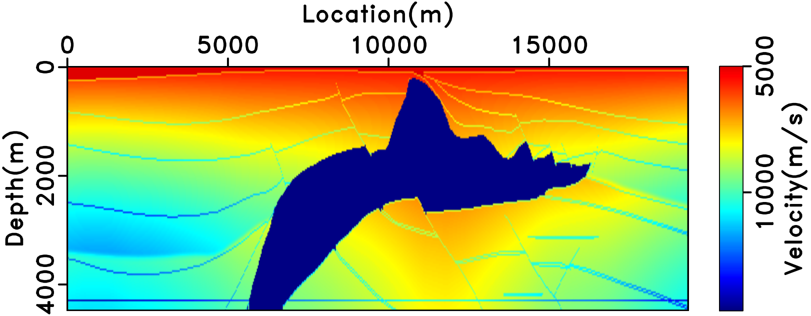
Algorithm 1 MLSERTM with PGG

1: **for** $k = 0 \cdots niter$ **do**
2: generate random sequence of signs γ
3: encode sources and data to get $\tilde{\mathbf{L}}$ and $\tilde{\mathbf{d}}_{obs}$
4: $\mathbf{r}^k = \tilde{\mathbf{L}}\hat{\mathbf{m}}^k - \tilde{\mathbf{d}}_{obs}$
5: compute $\hat{\mathbf{W}}_r^k$
6: compute $\hat{\mathbf{W}}_m^k$
7: $\mathbf{dm}^k = \hat{\mathbf{W}}_m^{T,k}\tilde{\mathbf{D}}_{SS}^{-1}\tilde{\mathbf{L}}^T\hat{\mathbf{W}}_r^{T,k}\mathbf{r}^k$
8: $\mathbf{dr}^k = \tilde{\mathbf{L}}\mathbf{dm}^k$
9: $\alpha_k = \frac{\langle \mathbf{dr}^k, \mathbf{r}^k \rangle}{\langle \mathbf{dr}^k, \mathbf{dr}^k \rangle}$
10: $\mathbf{m}^{k+1} = \mathbf{m}^{k+1} - \alpha_k \mathbf{dm}^k$
11: **end for**

Outline

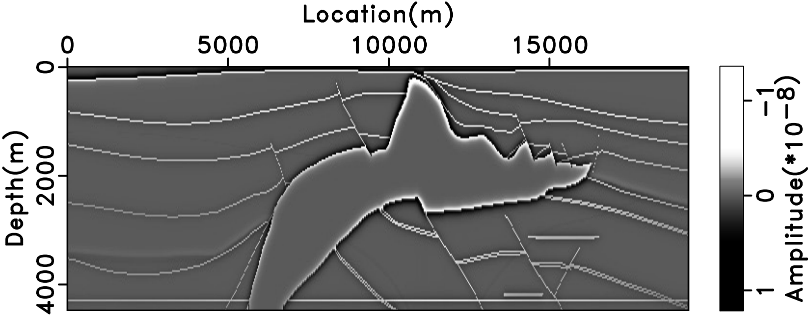
Reflectivity imaging test

Figure: Velocity model



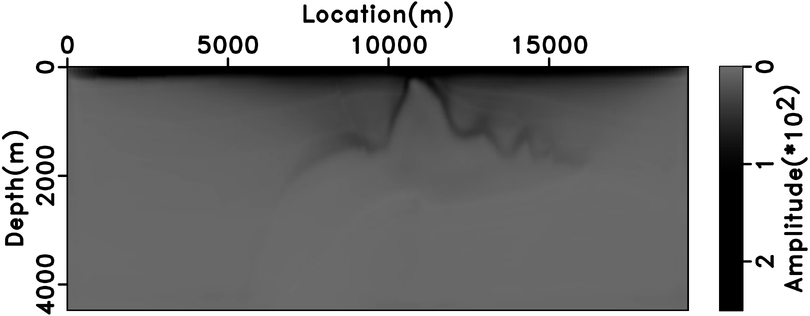
Reflectivity imaging test

Figure: Reflectivity



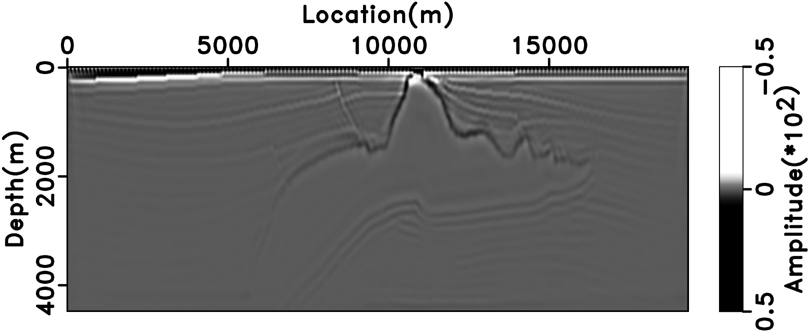
Reflectivity imaging test

Figure: RTM



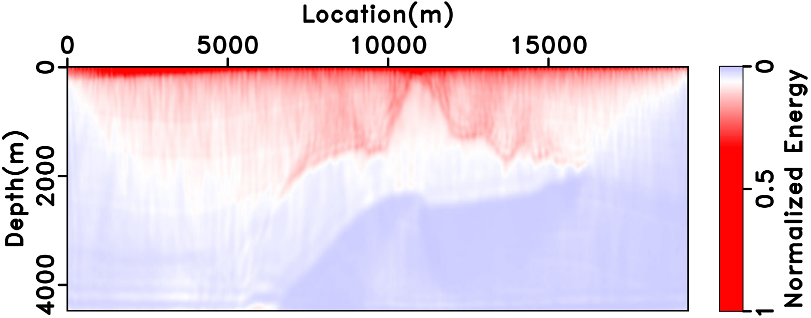
Reflectivity imaging test

Figure: RTM after Laplacian filter



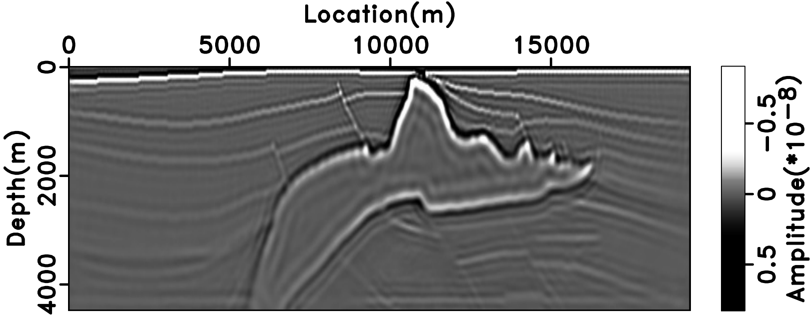
Reflectivity imaging test

Figure: Approximated diagonal of Hessian



Reflectivity imaging test

Figure: LSRTM without preconditioning



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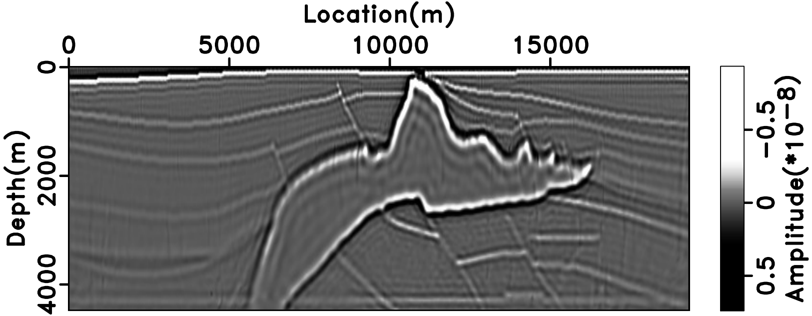
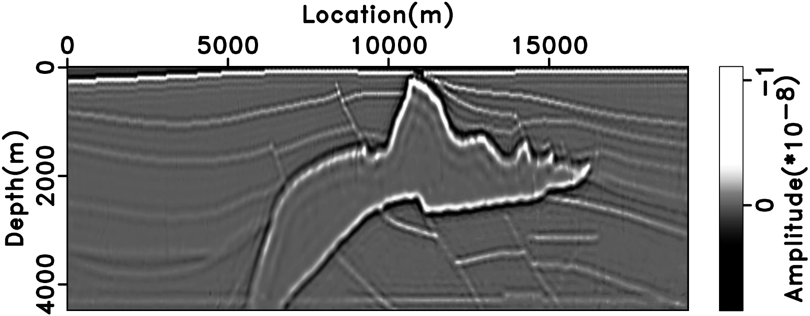


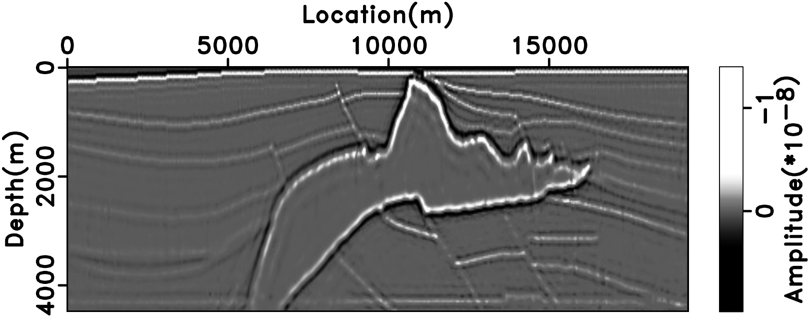
Reflectivity imaging test

Figure: LSRTM with preconditioning and $\ell_{1.5}$ norm on model



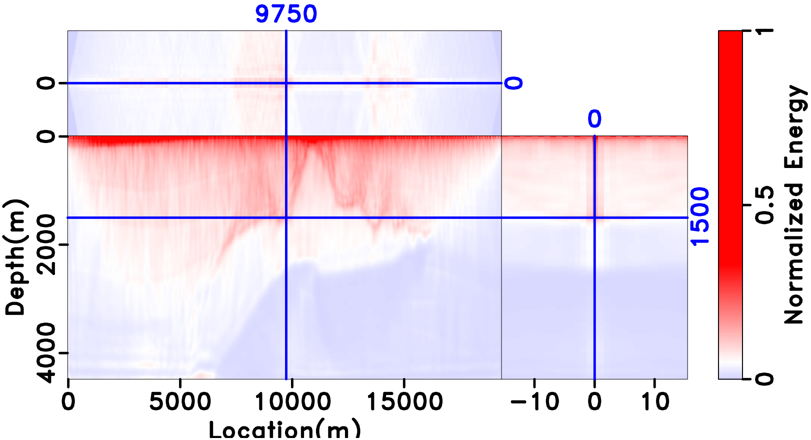
Reflectivity imaging test

Figure: LSRTM with preconditioning and ℓ_1 norm on model



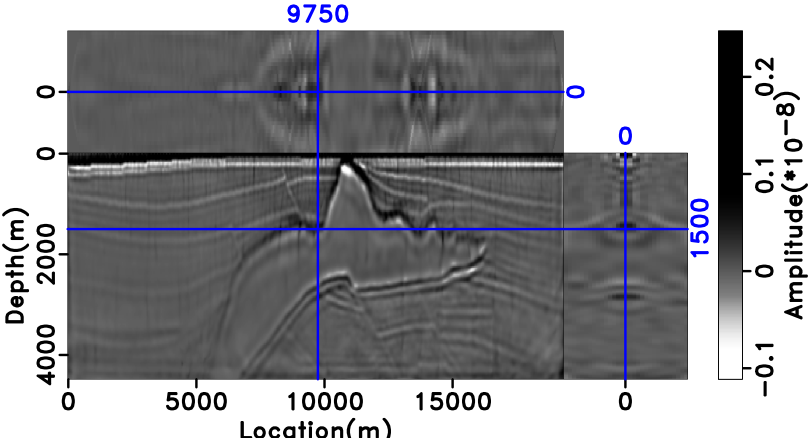
Extended reflectivity imaging test

Figure: Approximated diagonal Hessian in subsurface offset domain



Extended reflectivity imaging test

Figure: LSERTM with preconditioning



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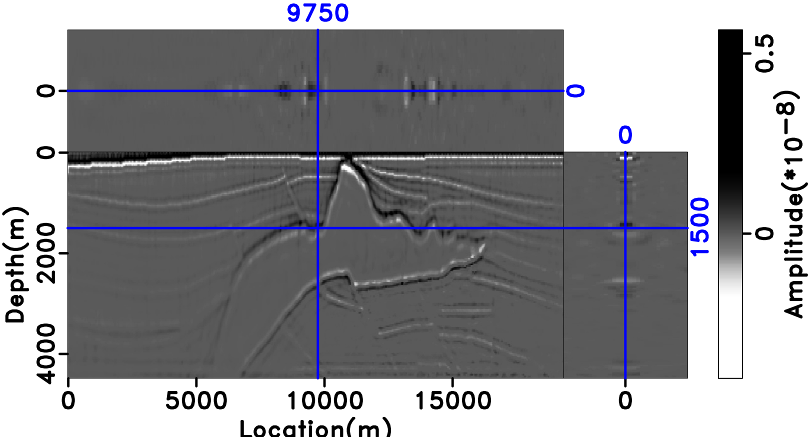

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Conclusion and Discussion

- LSERTM provides reflectivity image with higher resolution and more balanced amplitude, which is useful in the following migration velocity analysis and AVO/AVA analysis;
- Seismic data is compressed greatly with the help of phase encoding so that the efficiency of seismic imaging is improved dramatically;
- Using approximated diagonal of Hessian as preconditioner can improve the convergent rate of LSM;
- A modified CGG inversion scheme namely PGG is proposed to solve ℓ_p norm problem flexibly and efficiently;
- Sparsity seems to be a good prior information in suppressing crosstalk introduced by phase encoding, especially in imaging of extended reflectivity.

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Thank you!