Yujin Liu

- 2012.08-Present
  - Visiting student at Rice University
  - Advisor: Dr. William W. Symes
  - Project: EFWI, IVA, LSRTM, Sparse optimization
- 2010.09-Present
  - PhD candidate in geophysics at China University of Petroleum (Huadong)
  - Advisor: Dr. Zhenchun Li
  - Project: Wavefield inversion, Data regularization, Image domain denoising
- Awards
  - CSC Scholarship
  - National Scholarship
  - National Scholarship for Encouragement
  - CNPC Scholarship
  - National Excellent PhD Thesis Scholarship of CUP

Linearized Extended Waveform Inversion and Inversion Velocity Analysis

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Annual Meeting 2013

Overview

1. Background
2. Inversion Velocity Analysis
3. Numerical Tests
4. Summary and Future Plan

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4. Summary and Future Plan

WI and MVA

- Waveform Inversion (WI):

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  - Provide model with high resolution

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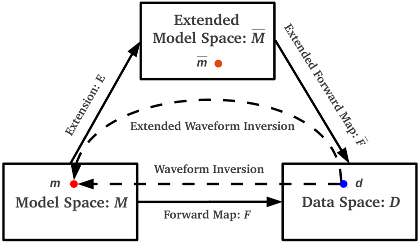
  - Model extension and extended modeling
  - Extended waveform inversion
  - Differential semblance optimization

Waveform Inversion (WI)



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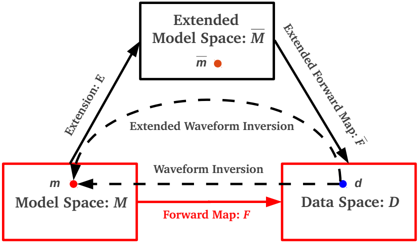

- Model space $M := \{m(\mathbf{x}) = \frac{1}{v^2(\mathbf{x})}\}$; Data space $D := \{d(\mathbf{x}_r, \mathbf{x}_s, \omega)\}$

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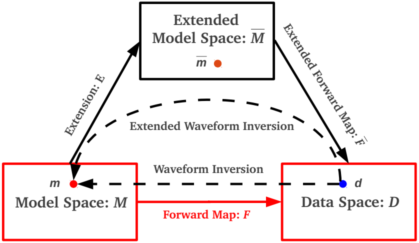

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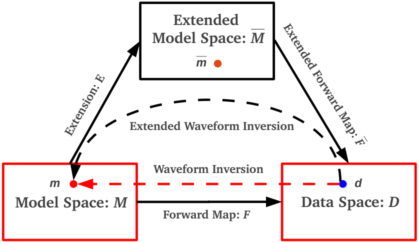
- Forward map F in acoustic constant density medium

$$(\nabla^2 + \omega^2 m(\mathbf{x}))u(\mathbf{x}, \omega) = -f(\omega)\delta(\mathbf{x} - \mathbf{x}_s) \quad (1)$$

$$d(\mathbf{x}_r, \mathbf{x}_s, \omega) = S(\mathbf{x}_r, \mathbf{x}_s)u(\mathbf{x}, \omega) \quad (2)$$

Waveform Inversion (WI)



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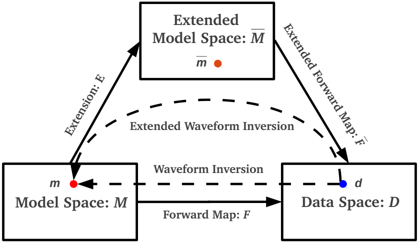
- Waveform inversion

$$min_m J_{WI}[m, d] = \frac{1}{2} \parallel F[m] - d \parallel^2 \quad (3)$$

Extended Waveform Inversion (EWI)



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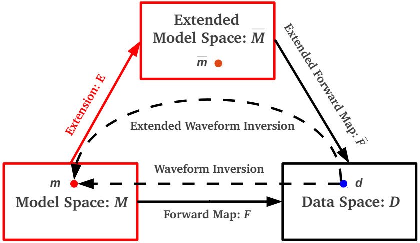
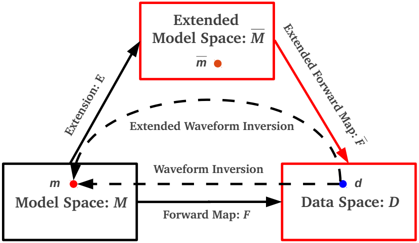


- Extended Model Space $\bar{M} := \{\bar{m}(\mathbf{x}, \mathbf{y}) = \frac{1}{v^2(\mathbf{x},\mathbf{y})}\}$

Extended Waveform Inversion (EWI)

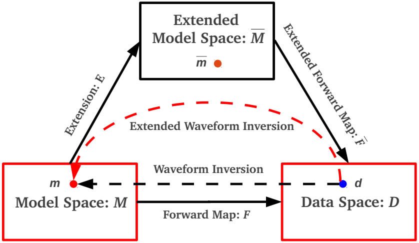


- Extended Model Space $\bar{M} := \{\bar{m}(\mathbf{x}, \mathbf{y}) = \frac{1}{v^2(\mathbf{x},\mathbf{y})}\}$
- Extended forward map $\bar{F}$ in acoustic constant density medium

$$\nabla^2 u(\mathbf{x}, \omega) + \omega^2 \int d\mathbf{y} m(\mathbf{x}, \mathbf{y}) u(\mathbf{y}, \omega) = -f(\omega)\delta(\mathbf{x} - \mathbf{x}_s) \quad (4)$$

$$d(\mathbf{x}_r, \mathbf{x}_s, \omega) = S(\mathbf{x}_r, \mathbf{x}_s) u(\mathbf{x}, \omega) \quad (5)$$

Extended Waveform Inversion (EWI)



- Extended Model Space $\bar{M} := \{\bar{m}(\mathbf{x},\mathbf{y}) = \frac{1}{v^2(\mathbf{x},\mathbf{y})}\}$
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$$\nabla^2 u(\mathbf{x},\omega) + \omega^2 \int d\mathbf{y} m(\mathbf{x},\mathbf{y}) u(\mathbf{y},\omega) = -f(\omega)\delta(\mathbf{x}-\mathbf{x}_s) \quad (4)$$

$$d(\mathbf{x}_r,\mathbf{x}_s,\omega) = S(\mathbf{x}_r,\mathbf{x}_s) u(\mathbf{x},\omega) \quad (5)$$

- Extended waveform inversion

$$min_{\bar{m}} J_{EWI}[\bar{m},d] = \frac{1}{2} \parallel \bar{F}[\bar{m}] - d \parallel^2 + \frac{\sigma}{2} \parallel A[\bar{m}] \parallel^2 \quad (6)$$

Linearized Extended Waveform Inversion (LEWI)

Problems of EWI: computational cost is extremely high!

Solution:

(1) Linearized approximation:

$\bar{m} \simeq m_0 + \delta\bar{m}$; $\bar{F}[\bar{m}] \simeq F[m_0] + D\bar{F}[m_0] * \delta\bar{m}$
where $D\bar{F}[m_0]$ is one order derivative of F to m at m_0

(2) LEWI:

$$min_{m_0,\delta\bar{m}} J_{LEWI}[m_0, \delta\bar{m}] = \frac{1}{2} \parallel D\bar{F}[m_0]\delta\bar{m} - (d - F[m_0]) \parallel^2 + \frac{\sigma}{2} \parallel A\delta\bar{m} \parallel^2$$

Connection with LSM and MVA

when $\sigma = 0$, it limits to migration velocity analysis (MVA); when $\sigma \to \infty$, it limits to least-squares migration (LSM).

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Linearized Extended Waveform Inversion Scheme

LEWI:

$$min_{m_0,\delta\bar{m}} J_{LEWI}[m_0,\delta\bar{m}] = \frac{1}{2} \parallel D\bar{F}[m_0]\delta\bar{m} - F_d) \parallel^2 + \frac{\sigma}{2} \parallel A\delta\bar{m} \parallel^2$$

Solve the above problem with two level of loops:

1. Inner Loop: Invert short scales (i.e. reflectivity) to get $\delta\bar{m}_k[m_0]$

$$min_{\delta\bar{m}} J_{LEWI}[m_0,\delta\bar{m}] = \frac{1}{2} \parallel D\bar{F}[m_0]\delta\bar{m} - F_d) \parallel^2 + \frac{\sigma}{2} \parallel A\delta\bar{m} \parallel^2$$

2. Outer Loop: Invert long scales (i.e. Background velocity)

$$min_{m_0} J_{LEWI}[m_0,\delta\bar{m}_k[m_0]] = \frac{1}{2} \parallel D\bar{F}[m_0]\delta\bar{m}_k[m_0] - F_d) \parallel^2 + \frac{\sigma}{2} \parallel A\delta\bar{m}_k[m_0] \parallel^2$$

Gradient of short scales

The gradient of the objective function $J_{DS}[m_0, \delta\bar{m}]$ with respect to $\delta\bar{m}$:

$$\nabla_{\delta\bar{m}} J_{DS}[m_0, \delta\bar{m}] = D\bar{F}^T[m_0](D\bar{F}[m_0]\delta\bar{m} - F_d) + \sigma A^T A\delta\bar{m} \tag{7}$$

Set the gradient to zero gives the normal equation, i.e.

$$(D\bar{F}^T[m_0]D\bar{F}[m_0] + \sigma A^T A)\delta\bar{m} = D\bar{F}^T[m_0]F_d \tag{8}$$

which can be re-written as:

$$N[m_0]\delta\bar{m} = M[m_0]F_d \tag{9}$$

where $N[m_0]$ is normal operator and $M[m_0]$ is migration operator.

Gradient of long scales

The gradient of the objective function $J_{DS}[m_0, \delta\bar{m}[m_0]]$ with respect to m_0:

$$\nabla_{m_0} J_{DS}[m_0, \delta\bar{m}_k[m_0]] = B[\delta\bar{m}_k, D\bar{F}[m_0]\delta\bar{m}_k - F_d] + B[P(N[m_0])e_k, F_d]$$

where B is bilinear operator, $P(N[m_0])$ is a polynomial in the normal operator $N[m_0]$, $\delta\bar{m}_k$ is the inverted reflectivity and e_k is the normal equation error $D_{\delta\bar{m}} J_{DS}[m_0, \delta\bar{m}]$. The derivation can be found in [Liu, Symes; 2013].

Notes:

This formula is only justified when we use Chebyshev iteration to solve normal equation 9 in the case of depth-oriented model extension, but we can approximate it in different degree, migration velocity analysis is one of the approximations.

Inversion Velocity Analysis Scheme

IVA:

$$min_{m_0,\delta\bar{m}} J_{LEWI}[m_0,\delta\bar{m}] = \frac{1}{2} \parallel D\bar{F}[m_0]\delta\bar{m} - F_d) \parallel^2 + \frac{\sigma}{2} \parallel A\delta\bar{m} \parallel^2$$

Compared with LEWI scheme, we solve the above problem separately:

1. Inner Loop: Invert short scales (i.e. reflectivity) to get $\delta\bar{m}_k[m_0]$

$$min_{\delta\bar{m}} J_{IVA}[m_0,\delta\bar{m}] = \frac{1}{2} \parallel D\bar{F}[m_0]\delta\bar{m} - F_d) \parallel^2$$

2. Outer Loop: Invert long scales (i.e. Background velocity)

$$min_{m_0} J_{IVA}[m_0,\delta\bar{m}_k[m_0]] = \frac{1}{2} \parallel A\delta\bar{m}_k[m_0] \parallel^2$$

Gradient of short scales and long scales

- The gradient of the objective function $J_{IVA}[m_0, \delta\bar{m}]$ with respect to $\delta\bar{m}$:

$$\nabla_{\delta\bar{m}} J_{IVA}[m_0, \delta\bar{m}] = D\bar{F}^T[m_0](D\bar{F}[m_0]\delta\bar{m} - F_d)$$

- The gradient of the objective function $J_{IVA}[m_0, \delta\bar{m}[m_0]]$ with respect to m_0:

$$\nabla_{m_0} J_{IVA}[m_0, \delta\bar{m}_k[m_0]] = B[P(N[m_0])A^T A\delta\bar{m}_k, F_d]$$

Notes:

If the iteration number of inner loop is set to be zero and approximate $P(N[m_0])$ to be identity matrix, the above formula is actually equivalent to the gradient given by WEMVA.

Operator formulas

- Extended linearized Born modeling $d = D\bar{F}[m_0]\delta\bar{m}$:

$$d(\mathbf{x}_r, \mathbf{x}_s, \omega) = -\omega^2 f(\omega) \int \mathrm{d}\mathbf{x}\mathrm{d}\mathbf{h}\, G(\mathbf{x}_r, \mathbf{x}+\mathbf{h}, \omega)\delta m(\mathbf{x}, \mathbf{h}) G(\mathbf{x}-\mathbf{h}, \mathbf{x}_s, \omega)$$

- Extended reverse time migration $\delta\bar{m} = D\bar{F}^T[m_0]d$:

$$\delta m(\mathbf{x}, \mathbf{h}) = -\int \mathrm{d}\mathbf{x_s}\mathrm{d}\mathbf{x_r}\mathrm{d}\omega\, \omega^2 f^*(\omega) G^*(\mathbf{x}_s, \mathbf{x}-\mathbf{h}, \omega) G^*(\mathbf{x}+\mathbf{h}, \mathbf{x}_r, \omega) d(\mathbf{x}_r, \mathbf{x}_s, \omega)$$

- Bilinear operator $\Delta m_0 = B[\delta\bar{m}, \Delta d]$:

$$\Delta m_0(\mathbf{y})$$
$$= \int \mathrm{d}\mathbf{x_s}\mathrm{d}\mathbf{x_r}\mathrm{d}\mathbf{x}\mathrm{d}\mathbf{h}\mathrm{d}\omega\, \left\{G_0(\mathbf{y}, \mathbf{x}_s, \omega)\omega^4 f(\omega)\right\}^* \{G_0^*(\mathbf{y}, \mathbf{x}-\mathbf{h}, \omega)\delta\bar{m}(\mathbf{x}, \mathbf{h}) G_0^*(\mathbf{x}+\mathbf{h}, \mathbf{x}_r, \omega)\Delta d(\mathbf{x}_r, \mathbf{x}_s, \omega)$$
$$+ \int \mathrm{d}\mathbf{x_s}\mathrm{d}\mathbf{x_r}\mathrm{d}\mathbf{x}\mathrm{d}\mathbf{h}\mathrm{d}\omega\, \left\{G_0(\mathbf{y}, \mathbf{x}+\mathbf{h}, \omega)\delta\bar{m}(\mathbf{x}, \mathbf{h}) G_0(\mathbf{x}-\mathbf{h}, x_s, \omega)\omega^4 f(\omega)\right\}^* \{G_0^*(\mathbf{y}, \mathbf{x}_r, \omega)\Delta d(\mathbf{x}_r, \mathbf{x}_s, \omega)$$

Numerical Tests

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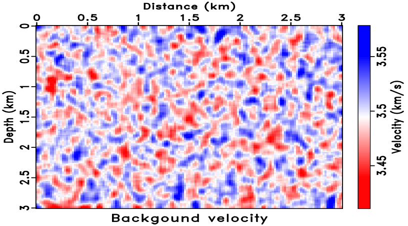
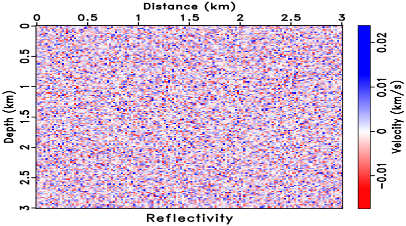
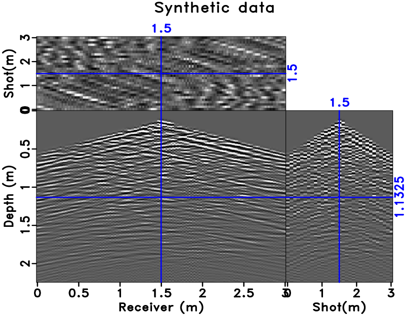
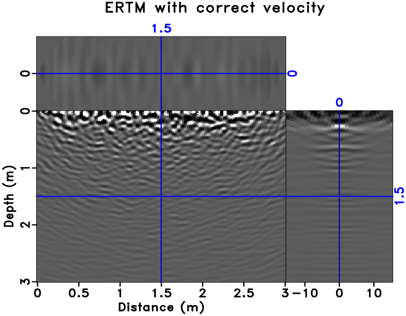
Scan tests of LEWI Objective Function





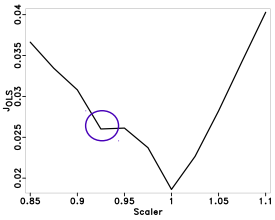


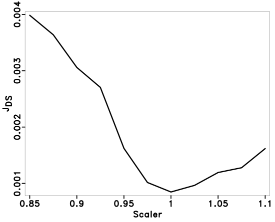


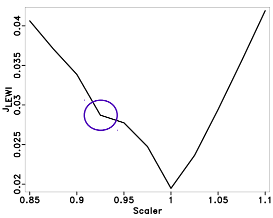
Scan tests of LEWI Objective Function

$$J_{LEWI}[m_0, \delta\bar{m}_k[m_0]] = J_{OLS}[m_0, \delta\bar{m}_k[m_0]] + \sigma J_{DS}[m_0, \delta\bar{m}_k[m_0]]$$
$$= \frac{1}{2} \parallel D\bar{F}[m_0]\delta\bar{m}_k[m_0] - F_d \parallel^2 + \frac{\sigma}{2} \parallel A\delta\bar{m}_k[m_0] \parallel^2$$

- Scan $J_{OLS}[m_0, \delta\bar{m}_k[m_0]]$ along $m_0 = \mu m_0^*$, $\mu \in [0.85, 1.1]$
- Scan $J_{DS}[m_0, \delta\bar{m}_k[m_0]]$ along $m_0 = \mu m_0^*$, $\mu \in [0.85, 1.1]$
- Scan $J_{LEWI}[m_0, \delta\bar{m}_k[m_0]]$ along $m_0 = \mu m_0^*$, $\mu \in [0.85, 1.1]$

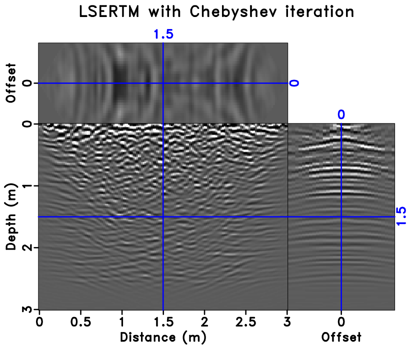


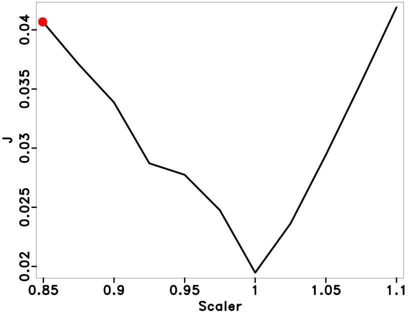




Scan tests of LEWI Objective Function

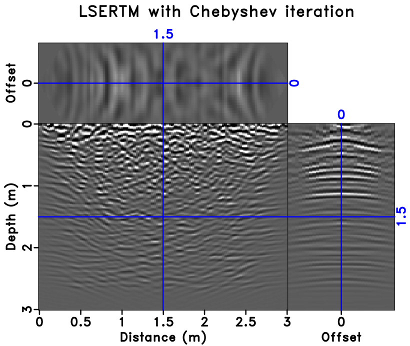
$$J_{LEWI}[m_0, \delta\bar{m}_k[m_0]] \text{ at } m_0 = 0.850 m_0^*$$

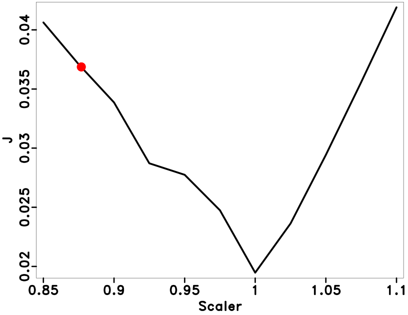




Scan tests of LEWI Objective Function

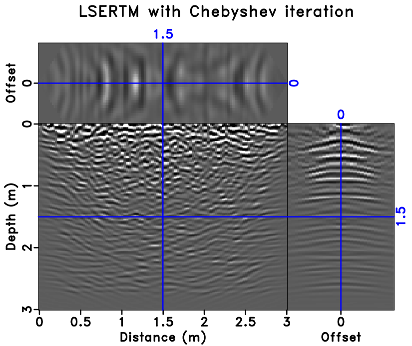
$$J_{LEWI}[m_0, \delta\bar{m}_k[m_0]] \text{ at } m_0 = 0.875 m_0^*$$

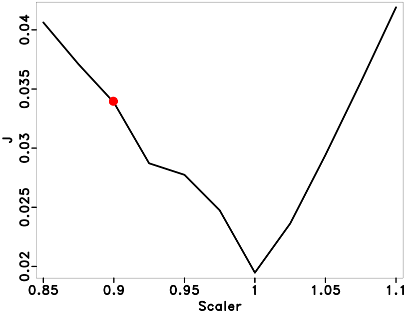




Scan tests of LEWI Objective Function

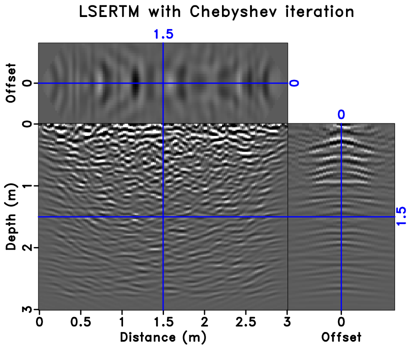
$$J_{LEWI}[m_0, \delta \bar{m}_k[m_0]] \text{ at } m_0 = 0.900 m_0^*$$

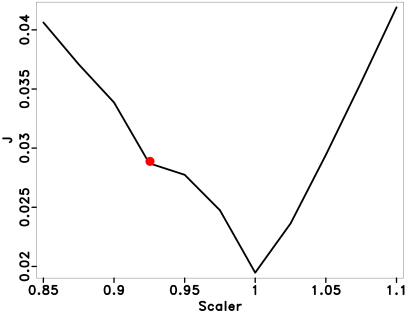




Scan tests of LEWI Objective Function

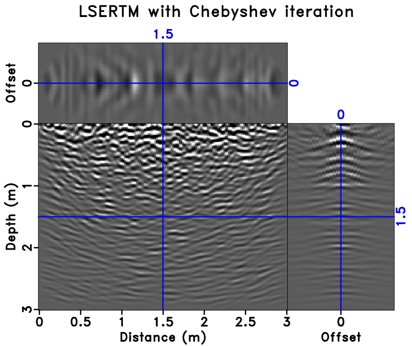
$$J_{LEWI}[m_0, \delta\bar{m}_k[m_0]] \text{ at } m_0 = 0.925 m_0^*$$

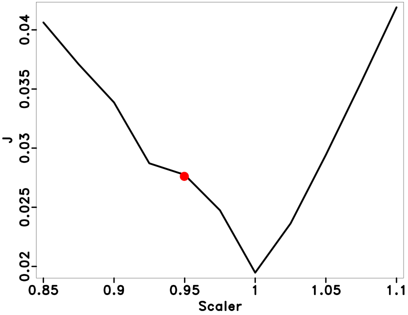




Scan tests of LEWI Objective Function

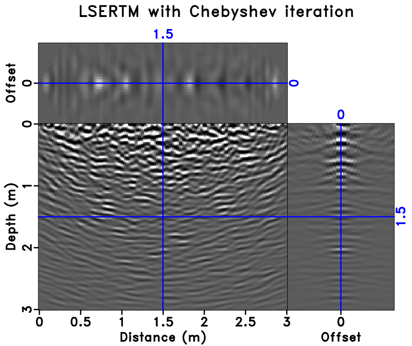
$$J_{LEWI}[m_0, \delta \bar{m}_k[m_0]] \text{ at } m_0 = 0.950 m_0^*$$

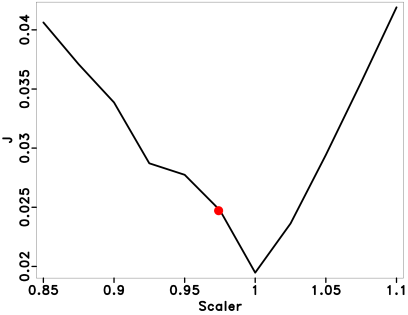




Scan tests of LEWI Objective Function

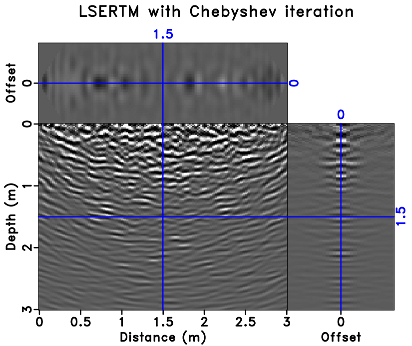
$$J_{LEWI}[m_0, \delta\bar{m}_k[m_0]] \text{ at } m_0 = 0.975 m_0^*$$

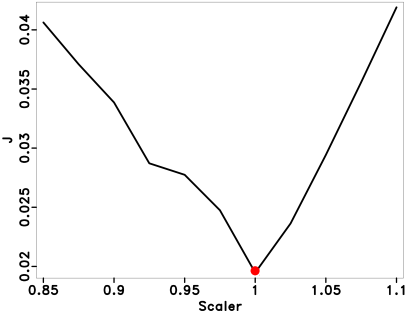




Scan tests of LEWI Objective Function

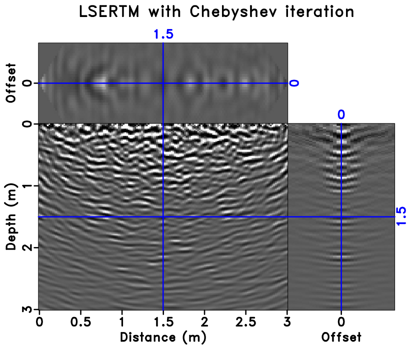
$$J_{LEWI}[m_0, \delta\bar{m}_k[m_0]] \text{ at } m_0 = 1.000 m_0^*$$

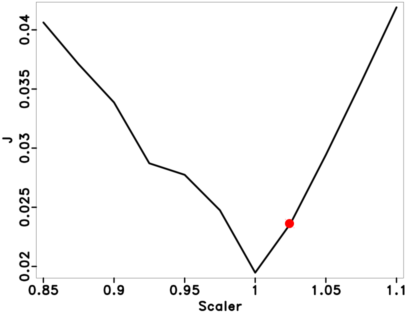




Scan tests of LEWI Objective Function

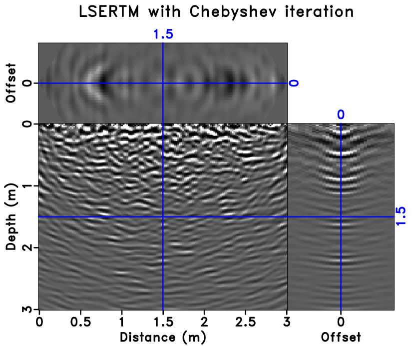
$$J_{LEWI}[m_0, \delta\bar{m}_k[m_0]] \text{ at } m_0 = 1.025m_0^*$$

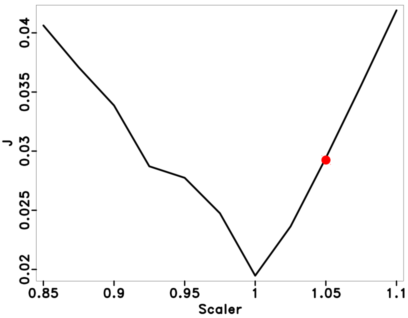




Scan tests of LEWI Objective Function

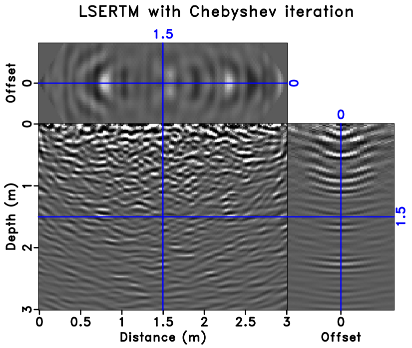
$$J_{LEWI}[m_0, \delta\bar{m}_k[m_0]] \text{ at } m_0 = 1.050 m_0^*$$

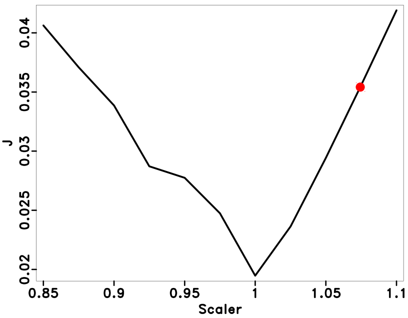




Scan tests of LEWI Objective Function

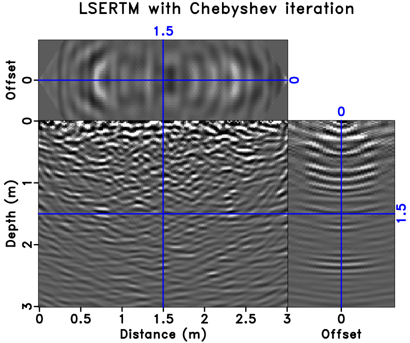
$$J_{LEWI}[m_0, \delta\bar{m}_k[m_0]] \text{ at } m_0 = 1.075m_0^*$$

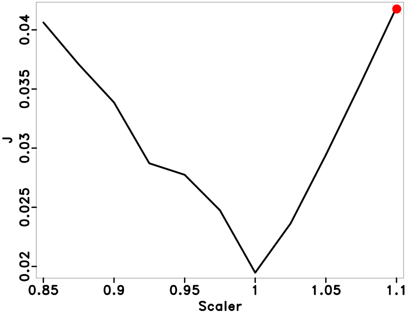




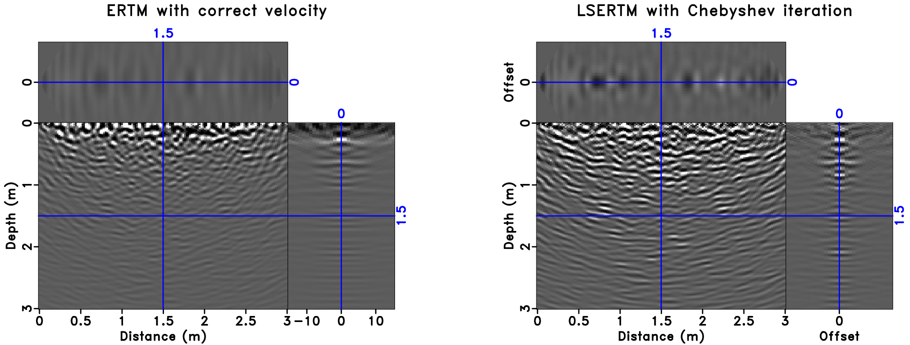
Scan tests of LEWI Objective Function

$$J_{LEWI}[m_0, \delta\bar{m}_k[m_0]] \text{ at } m_0 = 1.100 m_0^*$$





ERTM vs LSERTM



LEWI tests: Gaussian Model

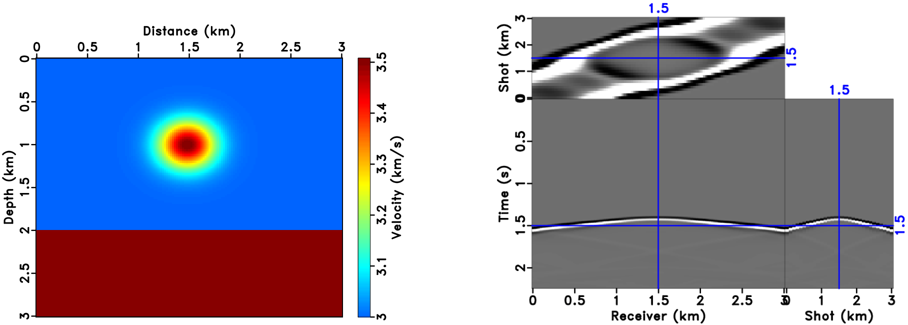
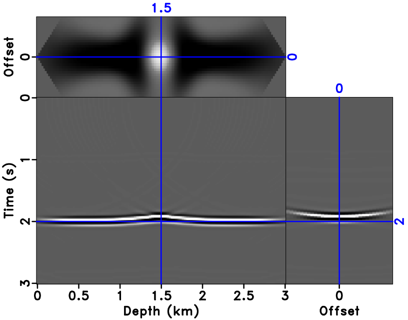


Figure: (a) Gaussian velocity model; (b) Synthetic data.

LEWI tests: Gaussian Model



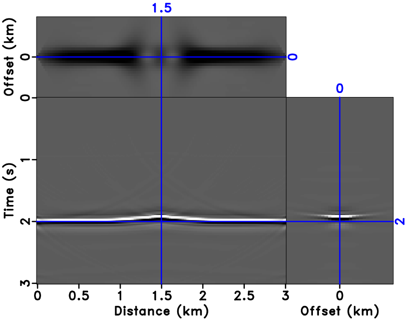


Figure: (a) Reflectivity with extended reverse-time migration; (b) Inverted reflectivity with Chebyshev iteration method.

LEWI tests: Gaussian Model

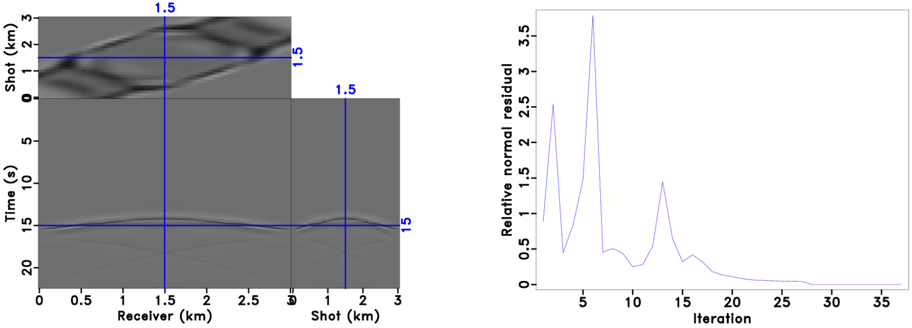


Figure: (a) Data misfit residual of Chebyshev iteration method; (b) Relative normal residual curve.

LEWI tests: Gaussian Model

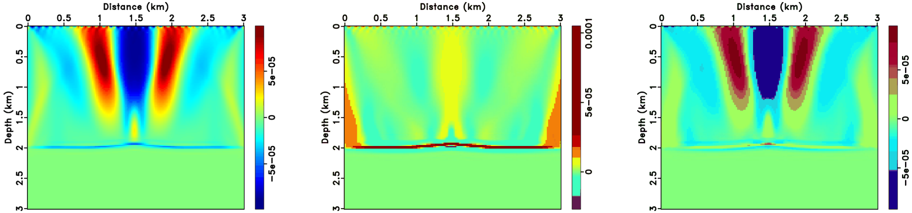
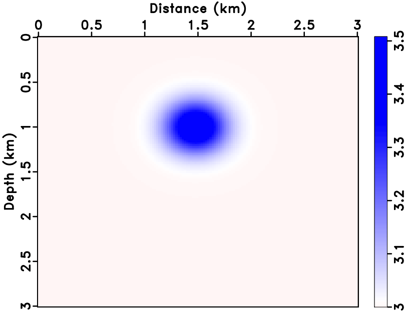
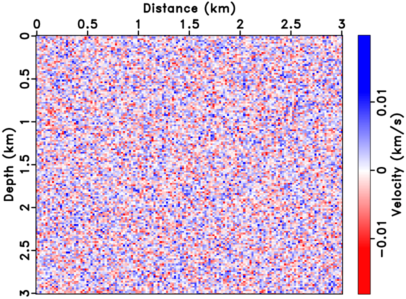


Figure: LEWI gradient of long scales (a) the first term; (b) the second term; (c) total.

IVA vs MVA: Gaussian-Random Model Tests



Backgound velocity



Reflectivity

IVA vs MVA: Gaussian-Random Model Tests

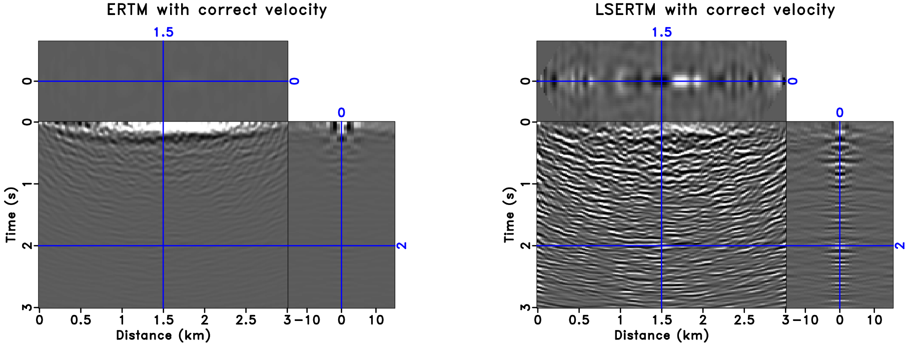


Figure: (a) Extended reverse-time migration; (b) Least-squares extended reverse-time migration when background velocity is correct.

IVA vs MVA: Gaussian-Random Model Tests

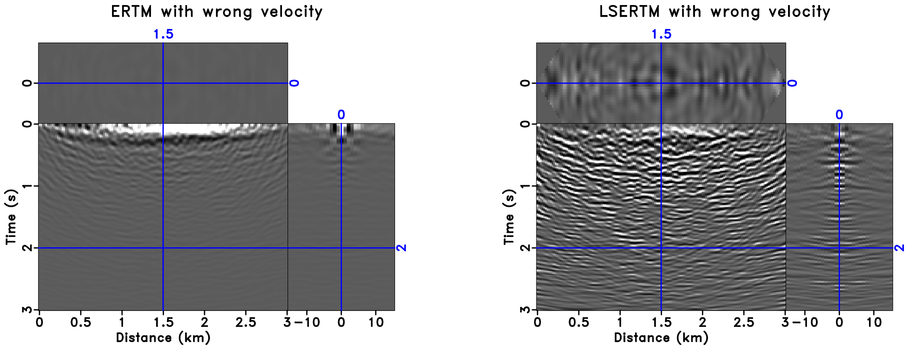


Figure: (a) Extended reverse-time migration; (b) Least-squares extended reverse-time migration when background velocity is 3 km/s.

IVA vs MVA: Gaussian-Random Model Tests

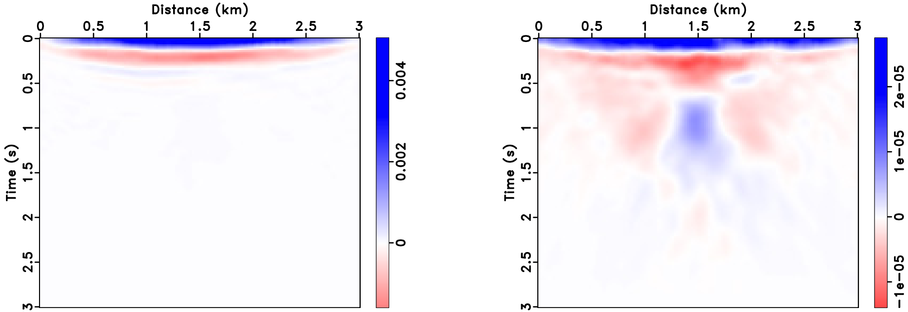


Figure: Gradient computed by (a) extended reverse-time migration; (b) least-squares extended reverse-time migration.

1 Background

2 Inversion Velocity Analysis

3 Numerical Tests

4 Summary and Future Plan

Summary and Future Plan

Conclusion

- The OLS objective function has local minimum problem, which can be solved with the idea of differential semblance optimization.
- The inverted reflectivity has higher resolution and more balanced amplitude, which is also crucial in background velocity inversion.
- As the inverted reflectivity image, instead of prestack migration approximation, is used to adjust velocity model, IVA is more accurate than MVA.

Future plan

- Improve the efficiency of IVA $\Rightarrow$ Preconditioning, Compressive sensing.
- Extend to non-linear case $\Rightarrow$ Plane-wave domain, depth-oriented extension.

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Thank you!