Yujin Liu

- **2012.08-Present**
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  - Advisor: Dr. William W. Symes
  - Project: EFWI, IVA, LSRTM, Sparse optimization

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- **Awards**
  - CSC Scholarship
  - National Scholarship
  - National Scholarship for Encouragement
  - CNPC Scholarship
  - National Excellent PhD Thesis Scholarship of CUP
Linearized Extended Waveform Inversion and Inversion Velocity Analysis

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Annual Meeting 2013
Overview

1. Background

2. Inversion Velocity Analysis

3. Numerical Tests

4. Summary and Future Plan
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1. Background

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4. Summary and Future Plan
Waveform Inversion (WI):

- Provide model with high resolution
- Tend to get trapped in local minimum

Migration Velocity Analysis (MVA):

- Provide background velocity robustly
- Only take single scattering into account
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Connection between WI and MVA?
- Model extension and extended modeling
- Extended waveform inversion
- Differential semblance optimization
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Waveform Inversion (WI)

Model Space: \( M \)

\[ m \]

Data Space: \( D \)

\[ d \]

Forward Map: \( F \)

Extended Model Space: \( \bar{M} \)

\[ \bar{m} \]

Extended Forward Map: \( E \)

Extended Waveform Inversion

Waveform Inversion

\[ \min_{m} J_{\text{WI}}[m,d] = \frac{1}{2} \| F[m] - d \|_2^2 \]
Model space $M := \{m(x) = \frac{1}{\nu^2(x)}\}$; Data space $D := \{d(x_r, x_s, \omega)\}$
Waveform Inversion (WI)

- Model space $M := \{m(x) = \frac{1}{v^2(x)}\}$; Data space $D := \{d(x_r, x_s, \omega)\}$
- Forward map $F$ in acoustic constant density medium
  \[
  (\nabla^2 + \omega^2 m(x))u(x, \omega) = -f(\omega)\delta(x - x_s)
  \]
  \[
  d(x_r, x_s, \omega) = S(x_r, x_s)u(x, \omega)
  \]
Waveform Inversion (WI)

Model space $M := \{m(x) = \frac{1}{v^2(x)}\}$; Data space $D := \{d(x_r, x_s, \omega)\}$

Forward map $F$ in acoustic constant density medium

$$ (\nabla^2 + \omega^2 m(x))u(x, \omega) = -f(\omega)\delta(x - x_s) $$

$$ d(x_r, x_s, \omega) = S(x_r, x_s)u(x, \omega) $$

Waveform inversion

$$ \min_m J_{WI}[m, d] = \frac{1}{2} \parallel F[m] - d \parallel^2 $$
Extended Waveform Inversion (EWI)

Model Space: $\overline{M}$

Data Space: $D$

Forward Map: $F$

Extended Model Space: $\overline{m}$

Extended Forward Map: $\overline{F}$

Extended Waveform Inversion

Waveform Inversion

Model Space: $M$

Data Space: $D$

Forward Map: $F$
Extended Model Space $\bar{M} := \{ \bar{m}(x, y) = \frac{1}{u^2(x,y)} \}$
Extended Model Space $\bar{M} := \{ \bar{m}(x, y) = \frac{1}{v^2(x,y)} \}$

Extended forward map $\bar{F}$ in acoustic constant density medium

$$\nabla^2 u(x, \omega) + \omega^2 \int dy m(x, y) u(y, \omega) = -f(\omega)\delta(x - x_s) \quad (4)$$

$$d(x_r, x_s, \omega) = S(x_r, x_s) u(x, \omega) \quad (5)$$
Extended Model Space \( \bar{M} := \{ \bar{m}(x, y) = \frac{1}{v^2(x, y)} \} \)

Extended forward map \( \bar{F} \) in acoustic constant density medium

\[
\nabla^2 u(x, \omega) + \omega^2 \int dy m(x, y) u(y, \omega) = -f(\omega)\delta(x - x_s)
\]  

(4)

\[
d(x_r, x_s, \omega) = S(x_r, x_s)u(x, \omega)
\]  

(5)

Extended waveform inversion

\[
\min_{\bar{m}} J_{EWI}[\bar{m}, d] = \frac{1}{2} \| \bar{F}[\bar{m}] - d \|^2 + \frac{\sigma}{2} \| A[\bar{m}] \|^2
\]  

(6)
Problems of EWI: computational cost is extremely high!

Solution:

(1) Linearized approximation:

\[ \bar{m} \simeq m_0 + \delta \bar{m}; \quad \bar{F}[\bar{m}] \simeq F[m_0] + D\bar{F}[m_0] \cdot \delta \bar{m} \]

where \( D\bar{F}[m_0] \) is one order derivative of \( F \) to \( m \) at \( m_0 \)

(2) LEWI:

\[ \min_{m_0, \delta \bar{m}} J_{LEWI}[m_0, \delta \bar{m}] = \frac{1}{2} \| D\bar{F}[m_0] \delta \bar{m} - (d - F[m_0]) \|^2 + \frac{\sigma}{2} \| A\delta \bar{m} \|^2 \]

Connection with LSM and MVA

when \( \sigma = 0 \), it limits to migration velocity analysis (MVA); when \( \sigma \to \infty \), it limits to least-squares migration (LSM).
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4. Summary and Future Plan
LEWI:

\[ \min_{m_0, \delta \bar{m}} J_{LEWI}[m_0, \delta \bar{m}] = \frac{1}{2} \| D\bar{F}[m_0]\delta \bar{m} - F_d \|_2^2 + \frac{\sigma}{2} \| A\delta \bar{m} \|_2^2 \]

Solve the above problem with two level of loops:

1. **Inner Loop**: Invert short scales (i.e. reflectivity) to get \( \delta \bar{m}_k[m_0] \)

\[ \min_{\delta \bar{m}} J_{LEWI}[m_0, \delta \bar{m}] = \frac{1}{2} \| D\bar{F}[m_0]\delta \bar{m} - F_d \|_2^2 + \frac{\sigma}{2} \| A\delta \bar{m} \|_2^2 \]

2. **Outer Loop**: Invert long scales (i.e. Background velocity)

\[ \min_{m_0} J_{LEWI}[m_0, \delta \bar{m}_k[m_0]] = \frac{1}{2} \| D\bar{F}[m_0]\delta \bar{m}_k[m_0] - F_d \|_2^2 + \frac{\sigma}{2} \| A\delta \bar{m}_k[m_0] \|_2^2 \]
The gradient of the objective function $J_{DS}[m_0, \delta \bar{m}]$ with respect to $\delta \bar{m}$:

$$\nabla_{\delta \bar{m}} J_{DS}[m_0, \delta \bar{m}] = D\bar{F}^T[m_0](D\bar{F}[m_0]\delta \bar{m} - F_d) + \sigma A^T A \delta \bar{m}$$  \hspace{1cm} (7)

Set the gradient to zero gives the normal equation, i.e.

$$(D\bar{F}^T[m_0]D\bar{F}[m_0] + \sigma A^T A)\delta \bar{m} = D\bar{F}^T[m_0]F_d$$  \hspace{1cm} (8)

which can be re-written as:

$$N[m_0] \delta \bar{m} = M[m_0] F_d$$  \hspace{1cm} (9)

where $N[m_0]$ is normal operator and $M[m_0]$ is migration operator.
Gradient of long scales

The gradient of the objective function $J_{DS}[m_0, \delta \bar{m}[m_0]]$ with respect to $m_0$:

$$\nabla_{m_0} J_{DS}[m_0, \delta \bar{m}_k[m_0]] = B[\delta \bar{m}_k, D\bar{F}[m_0]\delta \bar{m}_k - F_d] + B[P(N[m_0])e_k, F_d]$$

where $B$ is bilinear operator, $P(N[m_0])$ is a polynomial in the normal operator $N[m_0]$, $\delta \bar{m}_k$ is the inverted reflectivity and $e_k$ is the normal equation error $D_{\delta \bar{m}} J_{DS}[m_0, \delta \bar{m}]$. The derivation can be found in [Liu, Symes; 2013].

Notes:

This formula is only justified when we use Chebyshev iteration to solve normal equation 9 in the case of depth-oriented model extension, but we can approximate it in different degree, migration velocity analysis is one of the approximations.
IVA:

\[
\min_{m_0, \delta \tilde{m}} J_{LEWI}[m_0, \delta \tilde{m}] = \frac{1}{2} \|D \tilde{F}[m_0] \delta \tilde{m} - F_d\|_2^2 + \frac{\sigma}{2} \|A \delta \tilde{m}\|_2^2
\]

Compared with LEWI scheme, we solve the above problem separately:

1. **Inner Loop**: Invert short scales (i.e. reflectivity) to get \(\delta \tilde{m}_k[m_0]\)

\[
\min_{\delta \tilde{m}} J_{IVA}[m_0, \delta \tilde{m}] = \frac{1}{2} \|D \tilde{F}[m_0] \delta \tilde{m} - F_d\|_2^2
\]

2. **Outer Loop**: Invert long scales (i.e. Background velocity)

\[
\min_{m_0} J_{IVA}[m_0, \delta \tilde{m}_k[m_0]] = \frac{1}{2} \|A \delta \tilde{m}_k[m_0]\|_2^2
\]
The gradient of the objective function $J_{IVA}[m_0, \delta \tilde{m}]$ with respect to $\delta \tilde{m}$:

$$\nabla_{\delta \tilde{m}} J_{IVA}[m_0, \delta \tilde{m}] = DF^T[m_0](DF[m_0]\delta \tilde{m} - F_d)$$

The gradient of the objective function $J_{IVA}[m_0, \delta \tilde{m}[m_0]]$ with respect to $m_0$:

$$\nabla_{m_0} J_{IVA}[m_0, \delta \tilde{m}_k[m_0]] = B[P(N[m_0])A^T\delta \tilde{m}_k,F_d]$$

**Notes:**

*If the iteration number of inner loop is set to be zero and approximate $P(N[m_0])$ to be identity matrix, the above formula is actually equivalent to the gradient given by WEMVA.*
Operator formulas

- Extended linearized Born modeling \( d = D\bar{F}[m_0]\delta\bar{m} \):

\[
d(x_r, x_s, \omega) = -\omega^2 f(\omega) \int dxdh \, G(x_r, x + h, \omega) \delta m(x, h) G(x - h, x_s, \omega)
\]

- Extended reverse time migration \( \delta\bar{m} = D\bar{F}^T[m_0]d \):

\[
\delta m(x, h) = -\int dxdx_r d\omega \, \omega^2 f^*(\omega) G^*(x_s, x - h, \omega) G^*(x + h, x_r, \omega) d(x_r, x_s, \omega)
\]

- Bilinear operator \( \Delta m_0 = B[\delta\bar{m}, \Delta d] \):

\[
\Delta m_0(y) = \int dx_s dx_r dx dhd\omega \, \left\{ G_0(y, x_s, \omega) \omega^4 f(\omega) \right\}^* \left\{ G_0^*(y, x - h, \omega) \delta\bar{m}(x, h) G_0^*(x + h, x_r, \omega) \Delta d(x_r, x_s, \omega) \right\}
\]

\[
\quad + \int dx_s dx_r dx dhd\omega \, \left\{ G_0(y, x + h, \omega) \delta\bar{m}(x, h) G_0(x - h, x_s, \omega) \omega^4 f(\omega) \right\}^* \left\{ G_0^*(y, x_r, \omega) \Delta d(x_r, x_s, \omega) \right\}
\]
Numerical Tests

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Scan tests of LEWI Objective Function

SYNTHETIC DATA

ERTM with correct velocity
Scan tests of LEWI Objective Function

\[ J_{LEWI}[m_0, \delta \bar{m}_k[m_0]] = J_{OLS}[m_0, \delta \bar{m}_k[m_0]] + \sigma J_{DS}[m_0, \delta \bar{m}_k[m_0]] \]

\[ = \frac{1}{2} \| D \bar{F}[m_0] \delta \bar{m}_k[m_0] - F_d \|^2 + \frac{\sigma}{2} \| A \delta \bar{m}_k[m_0] \|^2 \]

- Scan \( J_{OLS}[m_0, \delta \bar{m}_k[m_0]] \) along \( m_0 = \mu m_0^*, \mu \in [0.85, 1.1] \)
- Scan \( J_{DS}[m_0, \delta \bar{m}_k[m_0]] \) along \( m_0 = \mu m_0^*, \mu \in [0.85, 1.1] \)
- Scan \( J_{LEWI}[m_0, \delta \bar{m}_k[m_0]] \) along \( m_0 = \mu m_0^*, \mu \in [0.85, 1.1] \)
Scan tests of LEWI Objective Function

\[ J_{LEWI}[m_0, \delta \tilde{m}_k[m_0]] \text{ at } m_0 = 0.850m^*_0 \]
Scan tests of LEWI Objective Function

\[ J_{LEWI}[m_0, \delta\bar{m}_k[m_0]] \text{ at } m_0 = 0.875m_0^* \]
Scan tests of LEWI Objective Function

\[ J_{LEWI}[m_0, \delta \bar{m}_k[m_0]] \text{ at } m_0 = 0.900m_0^* \]
Scan tests of LEWI Objective Function

\[ J_{LEWI}[m_0, \delta \bar{m}_k[m_0]] \text{ at } m_0 = 0.925 m_0^* \]
Scan tests of LEWI Objective Function

\[ J_{LEWI}[m_0, \delta \bar{m}_k[m_0]] \text{ at } m_0 = 0.950m_0^* \]
Scan tests of LEWI Objective Function

\[ J_{LEWI} [m_0, \delta \tilde{m}_k [m_0]] \text{ at } m_0 = 0.975m^*_0 \]
Scan tests of LEWI Objective Function

\[ J_{LEWI}[m_0, \delta \bar{m}_k[m_0]] \text{ at } m_0 = 1.000m_0^* \]
Scan tests of LEWI Objective Function

\[ J_{LEWI}[m_0, \delta \bar{m}_k[m_0]] \text{ at } m_0 = 1.025m_0^* \]
Scan tests of LEWI Objective Function

\[ J_{LEWI}[m_0, \delta \bar{m}_k[m_0]] \] at \( m_0 = 1.050m^* \)
Scan tests of LEWI Objective Function

\[ J_{\text{LEWI}}[m_0, \delta \tilde{m}_k[m_0]] \text{ at } m_0 = 1.075m_0^* \]
Scan tests of LEWI Objective Function

\[ J_{\text{LEWI}}[m_0, \tilde{m}_k[m_0]] \text{ at } m_0 = 1.100m_0^* \]
LEWI tests: Gaussian Model

Figure: (a) Gaussian velocity model; (b) Synthetic data.
Figure: (a) Reflectivity with extended reverse-time migration; (b) Inverted reflectivity with Chebyshev iteration method.
Figure: (a) Data misfit residual of Chebyshev iteration method; (b) Relative normal residual curve.
**LEWI tests: Gaussian Model**

*Figure:* LEWI gradient of long scales (a) the first term; (b) the second term; (c) total.
IVA vs MVA: Gaussian-Random Model Model Tests

![Graph showing distance vs depth and reflectivity vs depth with color scales for velocity.](image-url)
Figure: (a) Extended reverse-time migration; (b) Least-squares extended reverse-time migration when background velocity is correct.
Figure: (a) Extended reverse-time migration; (b) Least-squares extended reverse-time migration when background velocity is 3 km/s.
Figure: Gradient computed by (a) extended reverse-time migration; (b) least-squares extended reverse-time migration.
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Conclusion

- The OLS objective function has local minimum problem, which can be solved with the idea of differential semblance optimization.
- The inverted reflectivity has higher resolution and more balanced amplitude, which is also crucial in background velocity inversion.
- As the inverted reflectivity image, instead of prestack migration approximation, is used to adjust velocity model, IVA is more accurate than MVA.

Future plan

- Improve the efficiency of IVA ⇒ Preconditioning, Compressive sensing.
- Extend to non-linear case ⇒ Plane-wave domain, depth-oriented extension.
References

Migration velocity analysis and waveform inversion
*Geophysical Prospecting* 56(6), 765–790.

William W. Symes (2009)
The seismic reflection inverse problem
*Inverse problems* 25(12), 123008.

Yujin Liu, William W. Symes, Yin Huang, Zhenchun Li (2013)
Inversion Velocity Analysis via Differential Semblance Optimization in the Depth-oriented Extension
*83nd SEG Annual International Meeting, Expanded Abstract*, submitted

Inversion of reflection seismograms by differential semblance analysis: algorithm structure and synthetic examples
*Geophysical Prospecting* 42(6), 565–614.
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