Yin Huang

Education

- Ph.D. Candidate, Rice University, Houston, TX, USA, 08/2010 - Present
  - Dissertation Topic: Nonlinear Extended Waveform Inversion
  - M.A. with Master Thesis: " Transparency property of one dimensional acoustic wave equations"
  - Relevant courses: Numerical Differential Equations; Optimization; High Performance Computing; Geophysical Data Analysis
- M.S, Shanghai Jiao Tong University, Shanghai, China, 09/2006 - 03/2009
  - Dissertation topic: Comparision of many numerical methods for saddle point system arising from the mixed finite element method of elliptic problems with nonsmooth coefficients

Research Interests

- Seismic waveform inversion
- Forward and inverse problems for non-homogeneous medium
- High performace computing

Linearized Extended Waveform Inversion: Reduced Objective Function and Its Gradient

Yin Huang
Advisor: William Symes

The Rice Inversion Project

Annual Review Meeting

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Introduction

Seismic reflection inversion

- Over-determined: highly redundant in the observed data;
  (Gauthier et al., 1986; Santosa & Symes, 1989; Virieux & Operto, 2009)

Extended model fitting

- Under-determined: model has more degree of freedom than data.

Data misfit + extended modeling + differential semblance
⇒ smooth objective function of model parameter.
(Symes & Kern 1994, Symes, 1999, Stolk & Symes 2003)

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Waveform Inversion and Extended Modeling

Extended Waveform Inversion: Given data $d \in D$, find $\bar{m} \in \bar{M}$ so that (Symes, 1986, 1991; Biondi and Almomin, 2012)

$$\bar{F}[\bar{m}] \simeq d.$$

- $M = \{m(\mathbf{x})\}$ physical model space: velocity, density, bulk modulus, ...
- $\bar{M} = \{m(\mathbf{x}, h)\}$ extended model space, $M \subset \bar{M}$.
- D data space.
- $F : M \mapsto D$ forward map: acoustic, elastic ...
- $\bar{F} : \bar{M} \mapsto D$ extended forward map.

Extended model separation: $\bar{m} \simeq m_l + \delta\bar{m}$.

- Background model is physical.
- Reflectivity is extended.

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Linearized Inversion

Given data $d \in D$, find m_l, $\delta \bar{m}$ so that

$$D\bar{F}[m_l]\delta\bar{m} \simeq d - F[m_l].$$

$D\bar{F}$ is the derivative, or Born approximation.
(Symes and Carazzone, 1991; Chauris and Noble, 2001; Mulder and ten Kroode, 2002; Shen and Symes, 2008; Symes 2008.)

Objective function:

$$J[m_l, \delta\bar{m}] = \frac{1}{2}\|D\bar{F}[m_l]\delta\bar{m} - (d - F[m_l])\|^2 + \frac{\alpha^2}{2}\|A\delta\bar{m}\|^2$$

- A annihilator, $A\delta m = 0$ for all $\delta m \in M$.

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Reduced Objective Function

The value of the reduced objective function is the minimum value of J for fixed m_l

$$\tilde{J}[m_l] = \min_{\delta\bar{m}} J[m_l, \delta\bar{m}].$$

Let

$$N[m_l] = D\bar{F}[m_l]^T D\bar{F}[m_l] + \alpha^2 A^T A$$

Minimizer of J

$$\delta\bar{m}[m_l] = N[m_l]^{-1} D\bar{F}[m_l]^T (d - F[m_l]).$$

Gradient of $\tilde{J}[m_l]$

Directional derivative

$$D\tilde{J}[m_l]dm_l = D_{m_l}J[m_l,\delta\bar{m}]dm_l + D_{\delta\bar{m}}J[m_l,\delta\bar{m}]D_{m_l}\delta\bar{m}dm_l.$$

Second term is 0, if $\delta\bar{m}$ is solved exactly.

Direct computation gives gradient

$$\nabla\tilde{J}[m_l] = D^2\bar{F}[m_l]^T[\delta\bar{m}, D\bar{F}[m_l]\delta\bar{m} - (d - F[m_l])]$$

$$+DF[m_l]^T(D\bar{F}[m_l]\delta\bar{m} - (d - F[m_l])).$$

- Second term is zero if apply an appropriate cutoff function to the data.
- $D^2\bar{F}[m_l]^T : \bar{M} \times D$ such that for $dm_l \in M$, $q \in \bar{M}$ and $\phi \in D$

$$\langle D^2\bar{F}[m_l][dm_l, q], \phi\rangle = \langle dm_l, D^2\bar{F}[m_l]^T[q,\phi]\rangle.$$

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Example: Computation of $D^2\bar{F}[m_l]^T$

For acoustic constant density wave equation: $q \in \bar{M}$, $\phi \in D$.
Let ω solve the zero final value problem

$$\left(\frac{1}{m_l^2}\frac{\partial^2}{\partial t^2} - \Delta\right)\omega = \int dx_r \phi \delta(x - x_r).$$

$$D\bar{F}[m_l]^T\phi = \frac{2}{m_l}\int dt \omega \Delta u.$$

Let ω_0 solve the zero final value problem

$$\left(\frac{1}{m_l^2}\frac{\partial^2}{\partial t^2} - \Delta\right)\omega_0 = 2\Delta\left(\frac{q}{m_l}\omega\right).$$

$$D^2\bar{F}^T[m_l][q,\phi] = 2\frac{dm_l}{m_l}\int dh \int dt \left(\left(\Delta\bar{u} + \frac{q}{m_l}\Delta u\right)\omega + \omega_0 \Delta u\right).$$

with $\bar{u}$ the Born approximation wave field.

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Gradient needs a correction term

Why need correction term?

- $\nabla_{\delta\bar{m}} J[m_l, \delta\bar{m}]$ is not zero numerically.
- $D_{m_l}\delta\bar{m}$ could be large.
- Neglect of the second term leads to large errors (Symes and Kern, 1994).

Iteration method is used to solve for $\delta\bar{m}$, P_k polynomial with degree k.

$$\delta\bar{m}_k[m_l] = P_k(N[m_l])D\bar{F}[m_l]^T(d - F[m_l]).$$

$$\begin{aligned}\Rightarrow D_{m_l}\delta\bar{m}_k[m_l]dm_l = D(P_k(N[m_l]))dm_l D\bar{F}[m_l]^T(d - F[m_l]) \\ + P_k(N[m_l])D^2\bar{F}[m_l]^T[dm_l, d - F[m_l]] \\ - P_k(N[m_l])D\bar{F}[m_l]^T DF[m_l]dm_l.\end{aligned}$$

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Chebyshev iteration

Chebyshev is preferred.

- Coefficients of P_k does not depend on $N[m_l]$, once spectrum bound of $N[m_l]$ is known in advance. Easy to analyze $D_{m_l}\delta\bar{m}_k[m_l]$.
- Coefficients in conjugate gradient depends on $N[m_l]$. Bound on derivative is not obvious to calculate.

For operator A, want $P_k(A) \approx A^{-1} \Rightarrow I - AP_k(A) \approx 0$

Fix k, find P_k with leading coefficient 1, such that

$$P_k = \arg\min_{p_k}\max_{\lambda}|1 - \lambda p_k(\lambda)|$$

with $\lambda \in [\lambda_{\text{min}}, \lambda_{\text{max}}]$, i.e. spectrum bound of operator A.

NOTE: Chebyshev polynomial is the unique optimal solution. (Varga 1962)

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Gradient with a correction term

$\Rightarrow P'_k(N[m_l])$ is of order k^2.
$\Rightarrow D(P_k(N[m_l])) \approx D_{m_l}N[m_l]P'_k(N[m_l])$ is of order k^2.
$\Rightarrow \|D(P_k(N[m_l]))r_k\| = O(k^2\|r_k\|)$.

Normal residual $r_k = N[m_l]\delta\bar{m}_k - D\bar{F}[m_l]^T(d - F[m_l])$ decrease exponentially in k.

$$\begin{aligned} D_{\delta\bar{m}}J[m_l,\delta\bar{m}]D_{m_l}\delta\bar{m}dm_l &= \langle D_{m_l}(P_k(N[m_l]))dm_l D\bar{F}[m_l]^T(d-F[m_l]), r_k\rangle \\ &+\langle P_k(N[m_l])D^2\bar{F}[m_l]^T[dm_l, d - F[m_l]], r_k\rangle \\ &\simeq \langle dm_l, D^2\bar{F}[m_l]^T[P_k(N[m_l])r_k, d - F[m_l]]\rangle. \end{aligned}$$

Gradient with correction term

$$\begin{aligned} \nabla\tilde{J}[m_l] &\simeq D^2\bar{F}[m_l]^T[\delta\bar{m}, D\bar{F}[m_l]\delta\bar{m} - (d - F[m_l])] \\ &+D^2\bar{F}[m_l]^T[P_k(N[m_l])r_k, d - F[m_l]]. \end{aligned}$$

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&\quad +\langle P_k(N[m_l])D^2\bar{F}[m_l]^T[dm_l, d - F[m_l]], r_k\rangle \\
&\simeq \langle dm_l, D^2\bar{F}[m_l]^T[P_k(N[m_l])r_k, d - F[m_l]]\rangle.
\end{aligned}
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Gradient with correction term

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\begin{aligned}
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& + D^2\bar{F}[m_l]^T[P_k(N[m_l])r_k, d - F[m_l]].
\end{aligned}
$$

Restarting Chebyshev iteration

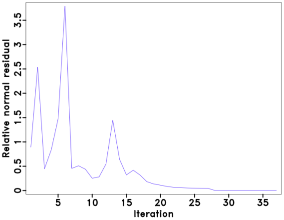
Simple scheme:

- Compute no. of iterations k and Chebyshev coefficients
- Initialize the iteration, compute Rayleigh Quotient (RQ) to get spectrum bound λ
- Run Chebyshev iteration until
  - Get bigger λ, restart Chebyshev
  - Or run k steps and return.

NOTE: Performance depends on condition number. Good preconditioner is needed.

Implementation is available in RVL.

Restarting curve for depth-oriented extended waveform inversion (Liu et al. 2013).



Summary

Reduced objective function

- Smooth in long scale model
- $D^2\bar{F}[m_l]^T$ of extended acoustic constant density forward operator
- Gradient
- Correction term to the gradient

Chebyshev iteration

- Better than CG for operator with moderate condition number and uniformly distributed spectrum
- Restarting scheme for operators without prior information of spectrum
- Available in RVL

Future work

Short term

- Fill in detailed mathematical analysis
- Find good preconditioner for shot-coordinate extension

Long term

- Extend to nonlinear waveform inversion (D. Sun's PhD thesis)

Acknowledgements

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- Sponsors of The Rice Inversion Project.
- Wonderful audience.