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- EDUCATION
  *PhD*. candidate in geophysics, Rice University 2011 - present
  *MS* in geophysics, University of Utah 2009 - 2011
  *BS* in geophysics, University of Science and Technology of China 2005 - 2009

- RESEARCH
  Depth-oriented extended seismic full waveform inversion (EFWI)
  Implementing Perfect Marched Layer (PML) in EFWI
  Investigating the subsurface offset range in EFWI

# Depth-oriented extended Born modeling operator and its adjoint

Lei Fu, William W. Symes

The Rice Inversion Project (TRIP)

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# Outline

# Introduction

## Objective

Solving the local minima problem in the data-fitting waveform inversion.

## Solution

Depth-oriented extended FWI

Surface-oriented ...

# Introduction

The extended forward map $\bar{F} : \bar{M} \to D$ is defined by:

$$\bar{F}[E[m]] = \bar{F}[\bar{m}] = d = F[m] \quad (1)$$

where $E$ is the extension operator.

- $F$: modeling operator
- $m$: model (velocity $v(\mathbf{x})$)
- $d$: data (time derivative of the acoustic potential $\frac{\partial u}{\partial t}(t, \mathbf{x}; \mathbf{x_s})$)

# Depth-oriented extended forward modeling

The constant-density acoustic wave equation:

$$\left(\frac{1}{m(\mathbf{x})}\frac{\partial^2}{\partial t^2} - \bigtriangledown^2\right) u(t, \mathbf{x}; \mathbf{x_s}) = f(t, \mathbf{x}; \mathbf{x_s}) \tag{2}$$

$$u(t, \cdot; \cdot) \equiv 0, t \ll 0 \tag{3}$$

where $f(t, \mathbf{x}; \mathbf{x_s}) = w(t)\delta(\mathbf{x} - \mathbf{x_s})$

The depth-oriented extended version:

$$\left(\int dy \frac{1}{\bar{m}(\mathbf{x}, \mathbf{y})}\frac{\partial^2}{\partial t^2} - \bigtriangledown^2\right) u(t, \mathbf{y}; \mathbf{x_s}) = f(t, \mathbf{x}; \mathbf{x_s}) \tag{4}$$

$$\partial_t^2 u - \bar{m}\bigtriangledown^2 u = \bar{m} f \tag{5}$$

# Extended Born operator, $D\bar{F}[\bar{m}]$

Born approximation about physical background model

$$\bar{m}(\mathbf{x},\mathbf{y}) = m(\mathbf{x})\delta(\mathbf{x}-\mathbf{y}) + \delta\bar{m}(\mathbf{x},\mathbf{y}) \tag{6}$$

then $\bar{u} \simeq u + \delta u$,

$$\partial_t^2 \delta u - m\nabla^2 \delta u = \int dy \delta\bar{m}(\mathbf{x},\mathbf{y})\nabla^2 u(\mathbf{y},t) \tag{7}$$

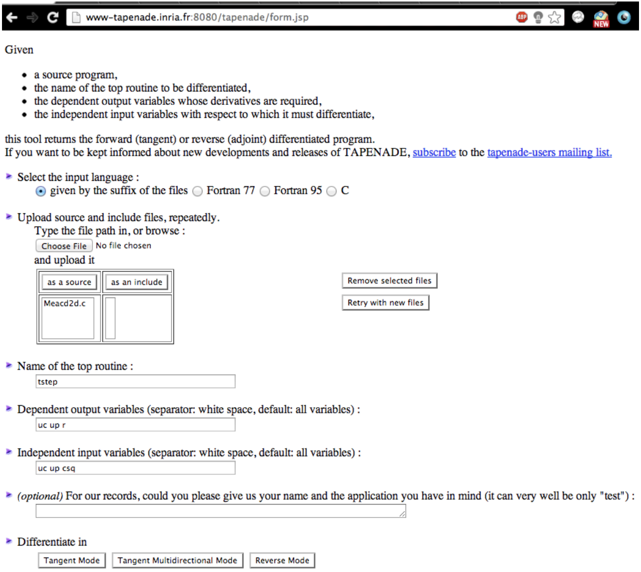
The extended Born scattering operator $D\bar{F}$ is defined by

$$D\bar{F}[\bar{m}]\delta\bar{m} = \delta u \tag{8}$$

# Adjoint of extended Born operator, $D\bar{F}[\bar{m}]^*$

"The least-squares gradient $DF[m]^*(d - F[m])$ is an image of $\delta\bar{v}$, in the sense of having the oscillatory components or locations of rapid change (reflectors), except for dip-dependent scaling and filtering". (Symes, 2008)

# TAPENADE is an Automatic Differentiation Engine.



www-tapenade.inria.fr:8080/tapenade/form.jsp

Given

- a source program,
- the name of the top routine to be differentiated,
- the dependent output variables whose derivatives are required,
- the independent input variables with respect to which it must differentiate,

this tool returns the forward (tangent) or reverse (adjoint) differentiated program.
If you want to be kept informed about new developments and releases of TAPENADE, subscribe to the tapenade-users mailing list.

- Select the input language :
  (•) given by the suffix of the files ( ) Fortran 77 ( ) Fortran 95 ( ) C
- Upload source and include files, repeatedly.
  Type the file path in, or browse :
  [Choose File] No file chosen
  and upload it
  [as a source] [as an include]
  Meacd2d.c
  [Remove selected files]
  [Retry with new files]
- Name of the top routine :
  tstep
- Dependent output variables (separator: white space, default: all variables) :
  uc up r
- Independent input variables (separator: white space, default: all variables) :
  uc up csq
- *(optional)* For our records, could you please give us your name and the application you have in mind (it can very well be only "test") :
- Differentiate in
  [Tangent Mode] [Tangent Multidirectional Mode] [Reverse Mode]

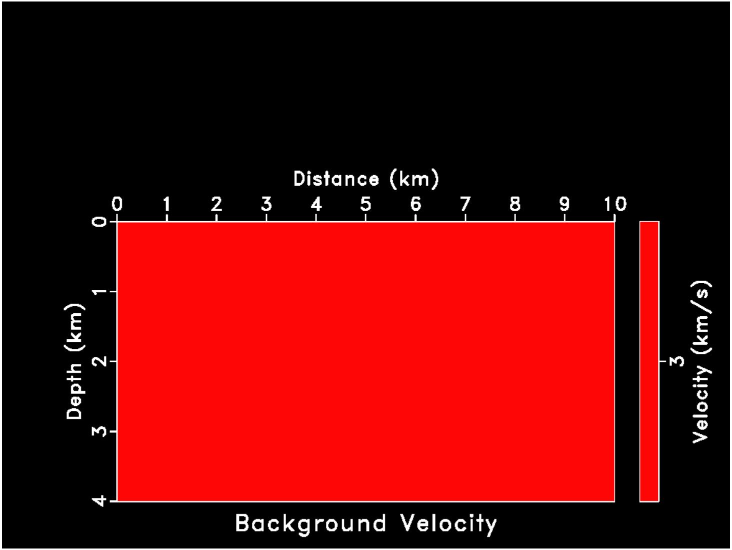
# TAPENADE

- Used for single step
- Adjoint is “global”. It could be difficult.
- TAPENADE is a nice tool to compute derivative and adjoint.

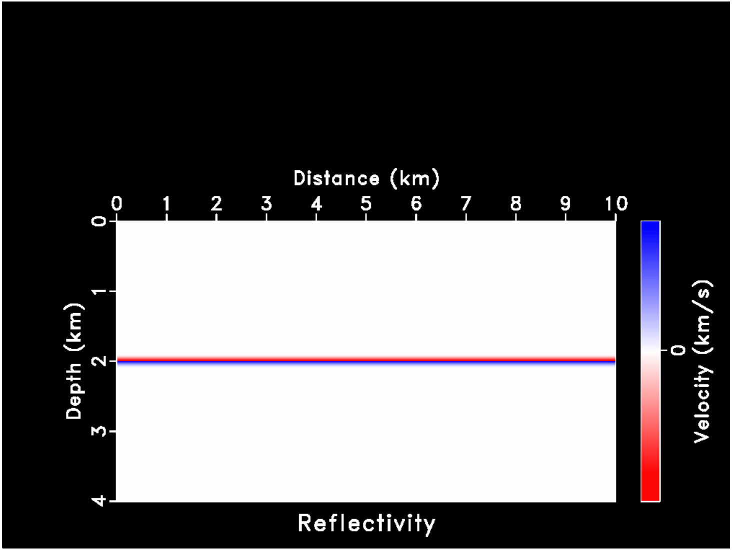
# Numerical experiment

- two-layers velocity model: $c_1 = 3, c_2 = 4$
- Grid spacing: $\Delta x = \Delta z = 0.01$
- Domain size: $10 \times 4$
- Source: Ricker wavelet at $(5, 0.6)$ .($f_{peak} = 10Hz$)
- Receivers: $1 : 0.05 : 9$ at depth of $0.1$
- Thickness of PML: $L_x = 1$ and $L_z = 0.4$
- The extension is horizontal, $-0.15 \leqslant h \leqslant 0.15$.

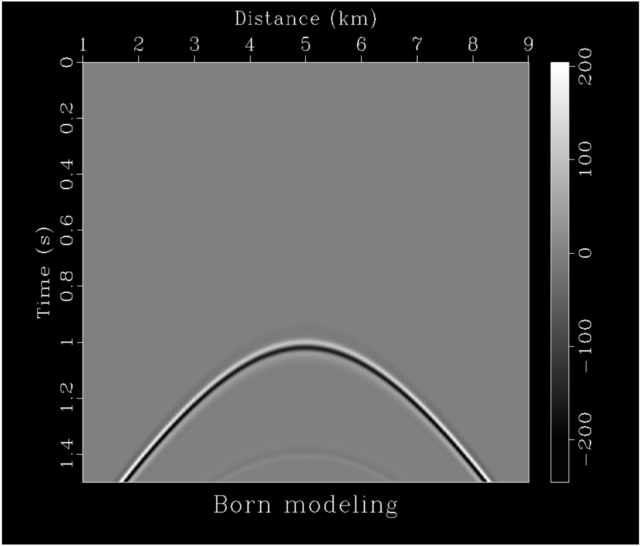
# Reference model



# Model perturbation $\delta\bar{m}(\mathbf{x})$



# Born modeling $D\bar{F}[\bar{m}]\delta\bar{m}$



# Adjoint of Born modeling operator $D\bar{F}[\bar{m}]^*\delta u$

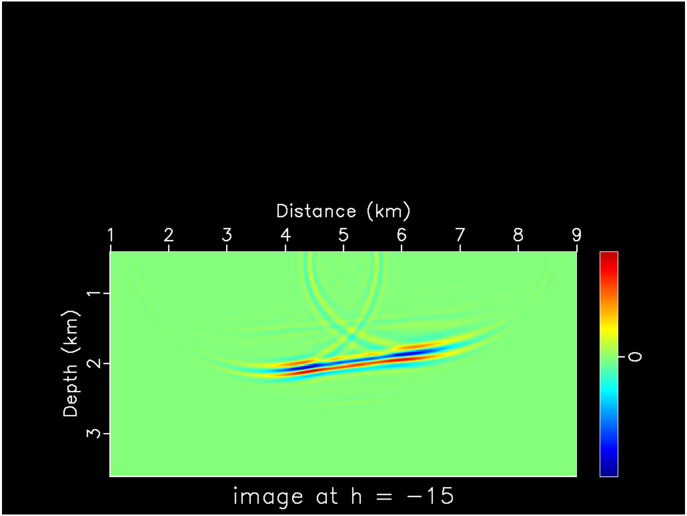



Figure : Single shot, image of $\delta\bar{m}$

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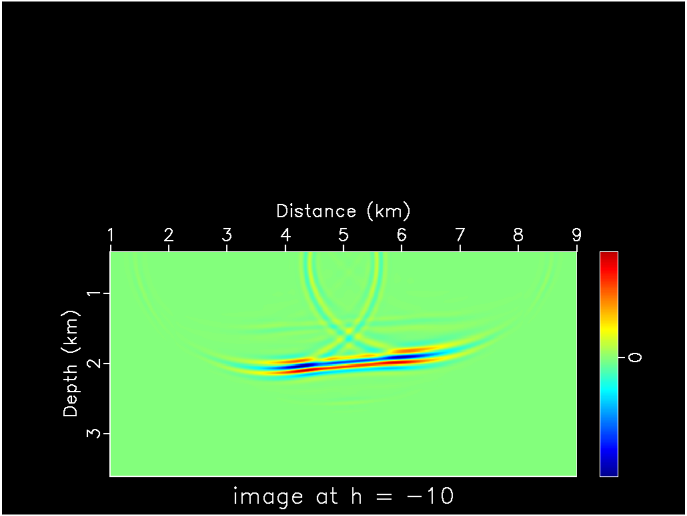


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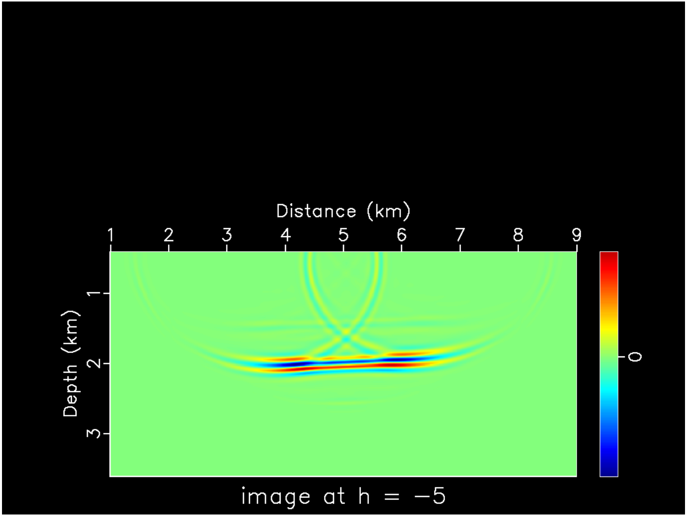


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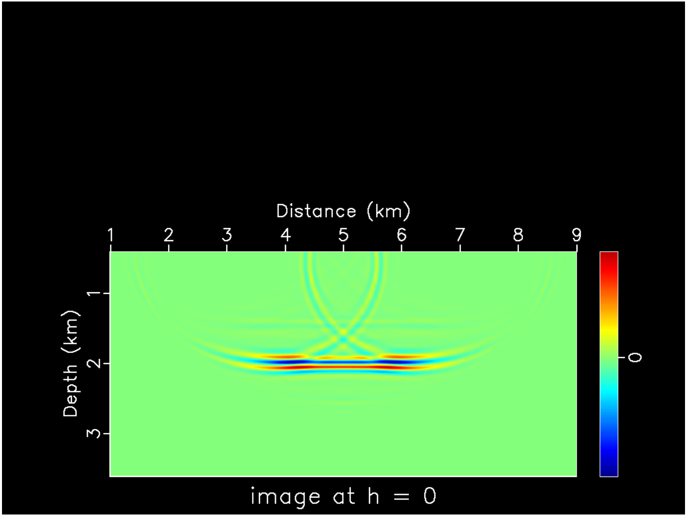


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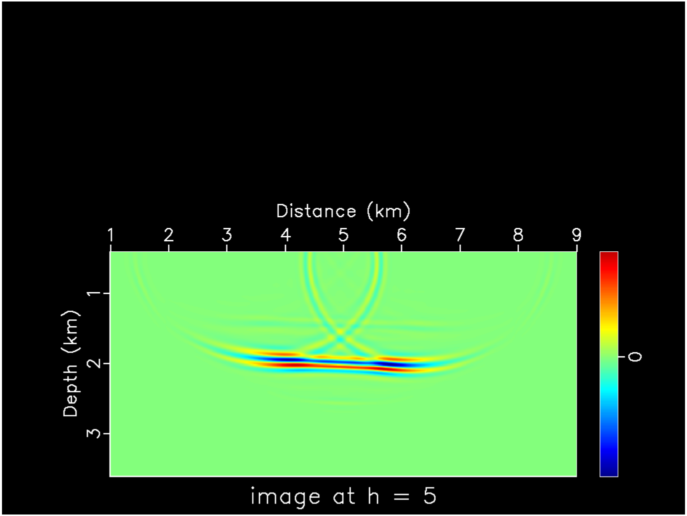


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# Adjoint of Born modeling operator $D\bar{F}[\bar{m}]^*\delta u$

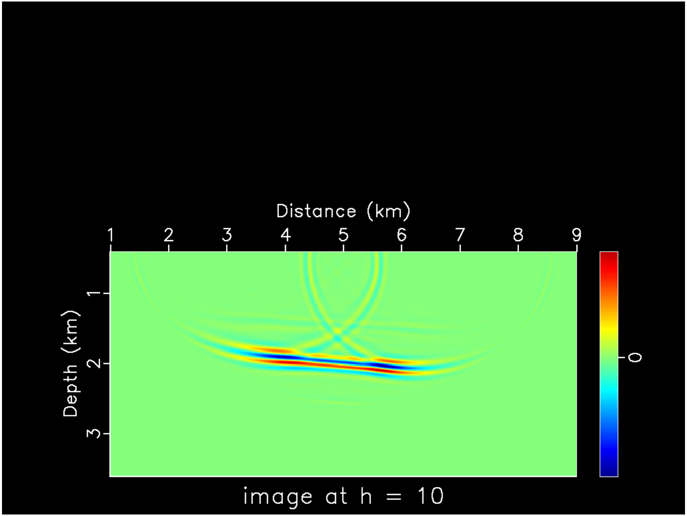


Figure : Single shot, image of $\delta\bar{m}$

# Adjoint of Born modeling operator $D\bar{F}[\bar{m}]^*\delta u$

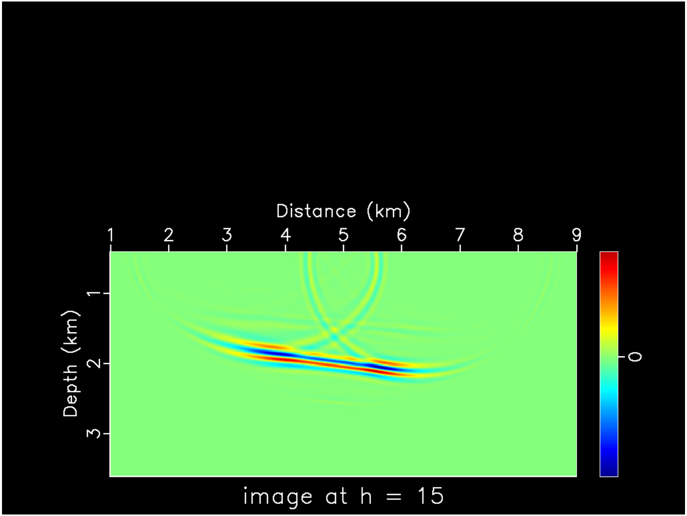


Figure : Single shot, image of $\delta\bar{m}$

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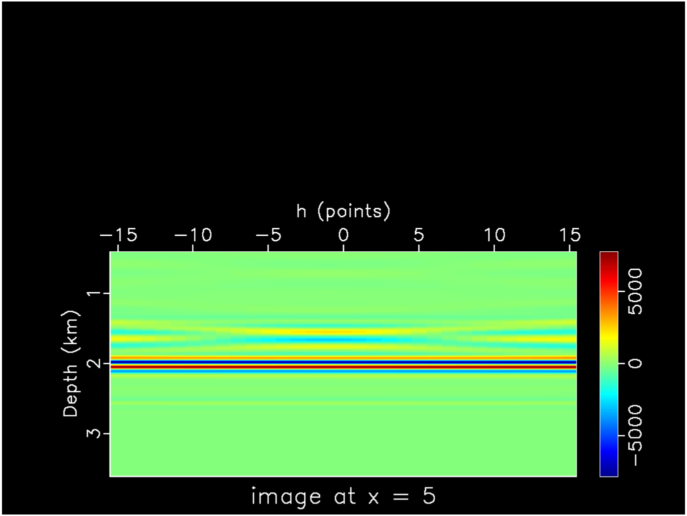


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# Adjoint of Born modeling operator $D\bar{F}[\bar{m}]^*\delta u$

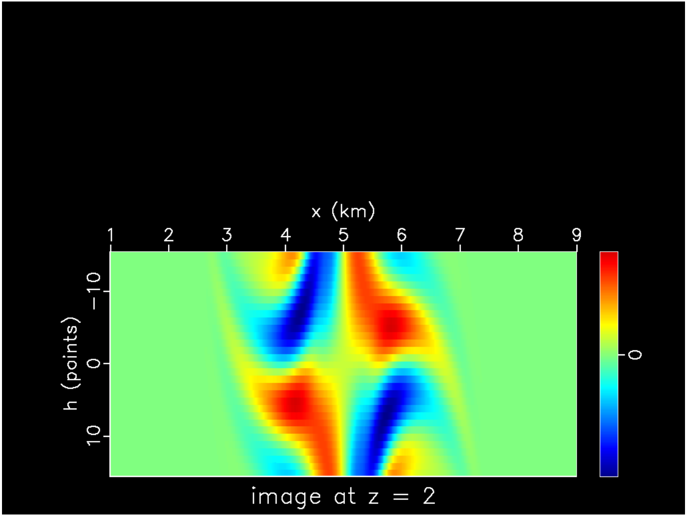


Figure : Single shot, image of $\delta\bar{m}$

# Summary

- In this study, we implemented depth-oriented extended Born modeling operator and its adjoint by using TAPENADE.
- They will be very useful in the next step - inverison.
- PML is available in current version.
- Future work
  - Integrate it into IWAVE
  - Improve the efficiency of our code.
  - Depth-oriented extended FWI.
  - Investigate energy focusing to determinie the range of extended dimension.

# Acknowledgement

**Thanks to**

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