Jie Hou

Education

Rice University 09/2012—Present
Ph.D. in Geophysics, Earth Science

China University of Petroleum(East China) 09/2008—06/2012
B.S. in Exploration Geophysics

Research

- An approximate inverse to extended born modeling operator
- Elastic wave modeling based on the decoupled equation
- Target-oriented wave-equation least-squares migration

An Approximate Inverse to Extended Born Modeling Operator

Jie Hou,William Symes

The Rice Inversion Project
Annual Review Meeting

April 19,2013

Outline

The Usual Set-up

- $\mathcal{M}$ =a set of models
- $\mathcal{D}$ =a Hilbert space of (potential) data
- Forward Map $\mathcal{F} : \mathcal{M} \rightarrow \mathcal{D}$

Inverse Problem:

Given $d \in \mathcal{D}$, find $m \in \mathcal{M}$ so that $\mathcal{F}[m] \simeq d$

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Wave equation:

$$\frac{1}{\rho(\mathbf{x}) v^2(\mathbf{x})} \frac{\partial^2 u}{\partial t^2}(\mathbf{x}, t) - \nabla \cdot \frac{1}{\rho(\mathbf{x})} \nabla u(\mathbf{x}, t) = f(\mathbf{x_s}, t)$$

Born Approximation(Linearized Inverse Problem)
Given smooth background velocity $v(x, y, z) = v(\mathbf{x})$, seismic data $d(\mathbf{x_r}, t; \mathbf{x_s})$, find oscillatory reflectivity $r(\mathbf{x}) = \frac{\delta v(\mathbf{x})}{v(\mathbf{x})}$ to fit the data:

$$F[v]r \simeq d$$

Born Modeling(Acoustic Forward Operator $F[v]$)

$$\left(\frac{1}{v^2} - \nabla^2\right) G = \delta(t)\delta(\mathbf{x} - \mathbf{x_s}); \left(\frac{1}{v^2} - \nabla^2\right) \delta u = \frac{2r}{v^2} G$$

$$F[v]r(\mathbf{x_r}, t; \mathbf{x_s}) = \delta u(\mathbf{x_r}, t; \mathbf{x_s})$$

Assumption: Single scattering at points of discontinuity of impedance in the subsurface(No multiple scattering!)

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Migration is an approximate solution of this linearized inverse problem

Adjoint of Born Modeling = Imaging Operator

- Migration operator (producing image) is adjoint or transpose of modeling operator(Lailly, Tarantola, Claerbout(80's)).
- Migration operator can position reflectors correctly but with possibly incorrect amplitudes.
  - \+ Due to the symmetry of wave-propagation with respect to time-reversal, migrating with the adjoint operator treats event kinematics correctly, and produces structurally correct images of the subsurface. It is robust to the presence of noise, or missing or inconsistent data.
  - \- The migration with the adjoint doesn't treat seismic amplitudes correctly. It focus on kinematics rather than amplitudes, amplitude terms are usually completely ignored, or artificially constructed so that $F^* F \approx I$
- True amplitude migration is (pseudo) inverse

Born Modeling

$$F[v]r(\mathbf{x_r}, t; \mathbf{x_s}) = \frac{\partial^2}{\partial t^2} \int d\mathbf{x} \int d\tau \frac{2r(\mathbf{x})}{v^2(\mathbf{x})} G(\mathbf{x}, \mathbf{t} - \tau; \mathbf{x_r}) G(\mathbf{x}, \tau; \mathbf{x_s})$$

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The adjoint of F is a prestack migration operator. It is defined by

$$\int d\mathbf{x_s} d\mathbf{x_r} dt (Fr)(\mathbf{x_r}, t; \mathbf{x_s}) d(\mathbf{x_r}, t; \mathbf{x_s}) = \int d\mathbf{x} r(\mathbf{x})(F^* d)(\mathbf{x})$$

Born Modeling

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Do the integral by parts shows that

$$F^* d(\mathbf{x}) = \frac{2}{v^2(\mathbf{x})} \int d\mathbf{x_s} d\mathbf{x_r} dt d\tau \, G(\mathbf{x}, t - \tau; \mathbf{x_r}) \frac{\partial^2 G(\mathbf{x}, \tau; \mathbf{x_s})}{\partial \tau^2} d(\mathbf{x_r}, t; \mathbf{x_s})$$

Least Squares Inversion:

Given d, find m to minimize

$$J_{LS}[m] = ||F[m] - d||^2 [+\mathit{Regularizing\ terms}]$$

Due to the local minima problem, extended model was introduced.

- Definition: the modeling of wavefields is extended to nonphysical models depending on redundant parameters.
- Extended Model $\bar{F} : \bar{\mathcal{M}} \to \mathcal{D}$ where $\bar{\mathcal{M}}$ is a larger model space $=$ models depending on $\mathbf{x}$ and $\mathbf{h}$

Extended Modeling(subsurface common offset):
In integral representation of $F[v]r$, permit r to depend on (half) offset $\mathbf{h}$.

Extended Modeling and Migration

$$\bar{F}[v]r = \frac{\partial^2}{\partial t^2}\int d\mathbf{x}d\mathbf{h}d\tau\, G(\mathbf{x}+\mathbf{h}, t-\tau; \mathbf{x_r})\frac{2r(\mathbf{x},\mathbf{h})}{v^2(\mathbf{x})}G(\mathbf{x}-\mathbf{h},\tau;\mathbf{x_s})$$

$$\bar{F}^* d = \frac{2}{v^2(\mathbf{x})}\int d\mathbf{x_s}d\mathbf{x_r}dtd\tau\frac{\partial^2 G}{\partial\tau^2}(\mathbf{x}-\mathbf{h},\tau;\mathbf{x_s})\partial\tau^2 G(\mathbf{x}+\mathbf{h},t-\tau;\mathbf{x_r})d(\mathbf{x_r},t;\mathbf{x_s})$$

When we use it to solve the inverse problem, it needs many iterations. So we need an approximate inverse to do the preconditioning.

$$\tilde{K}i = \frac{1}{2\pi}\int d\mathbf{x}d\mathbf{h}d\omega e^{-i\omega t} G(\mathbf{x_r}, \mathbf{x}+\mathbf{h}, \omega)\frac{\partial i(\mathbf{x},\mathbf{h})}{\partial z} G(\mathbf{x}-\mathbf{h}, \mathbf{x_s}, \omega)$$

$$\tilde{I}d = \frac{32}{\pi v^2(\mathbf{x})}\int d\mathbf{x_r} d\mathbf{x_s} d\omega(-i\omega)\frac{\partial G^*(\mathbf{x}+\mathbf{h}, \mathbf{x_r}, \omega)}{\partial z_r} d(\mathbf{x_r}, \mathbf{x_s}, \omega)\frac{\partial G^*(\mathbf{x_s}, \mathbf{x}-\mathbf{h}, \omega)}{\partial z_s}$$

Result 3(Fons ten Kroode,2012)

$\tilde{K}$ and $\tilde{I}$ are the Fourier integral operators of order 1 and -1 respectively.There exist order zero pseudo-differential operators Ψ_X and Ψ_Y, such that

$$\tilde{I} \circ \tilde{K} = \Psi_X$$

$$\tilde{K} \circ \tilde{I} = \Psi_Y$$

The operator Ψ_X acts as identity on focused space-shift-extended images. The operators Ψ_Y acts as identity on primary reflection data.

(http://iopscience.iop.org/0266-5611/28/11/115013)

$$(\tilde{K}i)(\mathbf{x_s}, \mathbf{x_r}, t) = \frac{1}{2\pi}\int dx dh d\omega e^{-i\omega t} G(\mathbf{x_r}, \mathbf{x}+\mathbf{h}, \omega)\frac{\partial i}{\partial z}(\mathbf{x}, \mathbf{h}) G(\mathbf{x}-\mathbf{h}, \mathbf{x_s}, \omega)$$

$$= \int dx dh d\tau\, G(\mathbf{x}+\mathbf{h}, t-\tau; \mathbf{x_r})\frac{\partial i(\mathbf{x}, \mathbf{h})}{\partial z} G(\mathbf{x}-\mathbf{h}, \tau; \mathbf{x_s})$$

$$\bar{F}[v]r = \frac{\partial^2}{\partial t^2}\int dx dh d\tau\, G(\mathbf{x}+\mathbf{h}, t-\tau; \mathbf{x_r})\frac{2r(\mathbf{x}, \mathbf{h})}{v^2(\mathbf{x})} G(\mathbf{x}-\mathbf{h}, \tau; \mathbf{x_s})$$

If we assume $i(\mathbf{x}, \mathbf{h}) = \frac{2r(\mathbf{x},\mathbf{h})}{v^2(x)}$, then we have

$$\bar{F} \circ \frac{\partial}{\partial z} \circ \frac{2r}{v^2} = \frac{\partial^2}{\partial t^2} \circ \tilde{K} \circ i$$

$$\tilde{I}d = \frac{32}{\pi v^2(\mathbf{x})} \int d\mathbf{x_r} d\mathbf{x_s} d\omega (-i\omega) \frac{\partial G^*(\mathbf{x}+\mathbf{h}, \mathbf{x_r}, \omega)}{\partial z_r} d(\mathbf{x_r}, \mathbf{x_s}, \omega) \frac{\partial G^*(\mathbf{x_s}, \mathbf{x}-\mathbf{h}, \omega)}{\partial z_s}$$

$$= \frac{64}{v^2(\mathbf{x})} \frac{\partial^2}{\partial z_r \partial z_s} \int dx_s dx_r dt d\tau \frac{\partial G(\mathbf{x}-\mathbf{h}, \tau; \mathbf{x_s})}{\partial \tau} G(\mathbf{x}+\mathbf{h}, t-\tau; \mathbf{x_r}) d(\mathbf{x_s}, \mathbf{x_r}, t)$$

$$\bar{F}^* d = \frac{2}{v^2(\mathbf{x})} \int d\mathbf{x_s} d\mathbf{x_r} dt d\tau \frac{\partial^2 G}{\partial \tau^2} (\mathbf{x}-\mathbf{h}, \tau; \mathbf{x_s}) \partial \tau^2 G(\mathbf{x}+\mathbf{h}, t-\tau; \mathbf{x_r}) d(\mathbf{x_r}), t; \mathbf{x_s})$$

$$= -\frac{2}{v^2(\mathbf{x})} \int dx_s dx_r dt d\tau \frac{\partial G(\mathbf{x}-\mathbf{h}, \tau; \mathbf{x_s})}{\partial \tau} G(\mathbf{x}+\mathbf{h}, t-\tau; \mathbf{x_r}) \frac{\partial d(\mathbf{x_r}, t; \mathbf{x_s})}{\partial t}$$

$$-\frac{64}{v^2(\mathbf{x})} \frac{\partial^2}{\partial z_s \partial z_r} \circ \bar{F}^* \circ \int_t = \tilde{I}$$

Meaning of Result 3

If we apply $\tilde{K}$ on $\tilde{I}$, the result is in terms of a ratio of some slowness (s, s_+, s_-) at different places, where s, s_+, s_- are the slowness at the points $\mathbf{x}, \mathbf{x}+\mathbf{h}, \mathbf{x}-\mathbf{h}$ respectively. So s_+, s_- are the slowness values at the ends of two different rays of geometric optics.

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$$\boxed{\tilde{K} \circ \tilde{I} \simeq I}$$

From the analysis above, we can get the following results:

$$\begin{aligned} \bar{F} \circ \frac{\partial}{\partial z} \circ \frac{2r}{v^2} &= \frac{\partial^2}{\partial t^2} \circ \tilde{K} \circ i \\ -\frac{64}{v^2(\mathbf{x})} \frac{\partial^2}{\partial z_s \partial z_r} \circ \bar{F}^* \circ \int_t &= \tilde{I} \\ \tilde{K} \circ \tilde{I} &\simeq I \end{aligned}$$

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Now we can get the approximate inverse

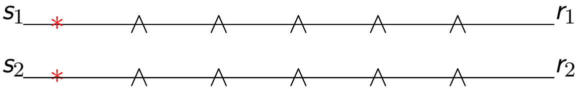
$$\bar{F}^{-1} \simeq \frac{\partial}{\partial z} \circ \frac{-64}{v^2} \frac{\partial^2}{\partial z_s \partial z_r} \circ \bar{F}^* \circ \int\int\int_t$$

Implementation Process

TRIP

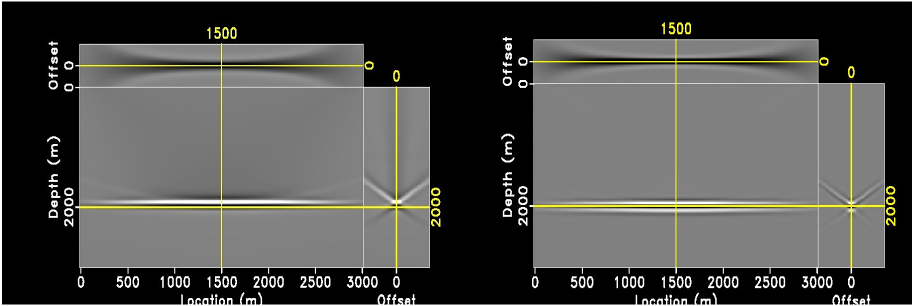
$$r(\mathbf{x},\mathbf{h}) = \bar{F}^{-1} d \simeq \frac{\partial}{\partial z} \circ \frac{-64}{v^2} \frac{\partial^2}{\partial z_s \partial z_r} \circ \bar{F}^* \circ \int\int\int_t d(\mathbf{x_r}, t; \mathbf{x_s})$$

1. **Data Preparation:** Since we need to take derivative to z_s, z_r, we need four data($d(\mathbf{x}_{s1}, \mathbf{x}_{r1}), d(\mathbf{x}_{s1}, \mathbf{x}_{r2}), d(\mathbf{x}_{s2}, \mathbf{x}_{r1}), d(\mathbf{x}_{s2}, \mathbf{x}_{r2})$)



2. **Integrate the data:** Just do the sums
3. **Do prestack depth migration:** Use normal RTM code
4. **Derivative:** Do differential three times

$$\frac{\partial^2 i(\mathbf{x},\mathbf{h};\mathbf{x_s},\mathbf{x_r})}{\partial z_s \partial z_r} = \frac{i(\mathbf{x_{s1}},\mathbf{x_{r1}}) + i(\mathbf{x_{s2}},\mathbf{x_{r2}}) - i(\mathbf{x_{s1}},\mathbf{x_{r2}}) - i(\mathbf{x_{s2}},\mathbf{x_{r1}})}{\Delta z^2}$$



Migration Result

Approximate Inverse Result

Thanks for Yujin's RTM code

Summary

If it's true

- Approximate Inverse
- Get the amplitude right
- It's not expensive

Possible Problems

- Ten Kroode(2012) $\rightarrow$ 3D How about 2D?
  Green Function: 3D form $\xlongequal{Mod.}$ 2D form
  (Creation of GRT inversion formula, William Symes,1998)
- Blow up the low frequency and kill the high frequency.
  (A lot of sums and differences)
- Numerical Errors

Future Plans

- Go through the proof, make any necessary modification for 2D.
- Implement the operator in 2D and 3D
- Replace four migrations with respect to one-way operator

  replace $\frac{\partial}{\partial z_r}$ with the one way operator

  according to the reciprocal principle, same to $\frac{\partial}{\partial z_s}$
- Apply this operator as a preconditioner

Acknowledgement

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- TRIP Members
- TRIP Sponsors

- Thank you for listening