A nonlinear differential-semblance algorithm for waveform inversion

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The Rice Inversion Project

Annual Review Meeting

March 30, 2012

Agenda

1. Full Waveform Inversion
2. Extension and Differential Semblance Optimization (DSO)
3. Examples: Inversion in Layered Medium

Full Waveform Inversion (FWI)

A usual set-up:

- $\mathcal{M}$ = a set of models ($\{m(\mathbf{x})\}$), $\mathcal{D}$ = a space of data
- $\mathcal{F} \; : \mathcal{M} \to \mathcal{D}$ - forward map

FWI (least-squares inversion): given $d_o \in \mathcal{D}$, solve

$$\min_{m \in \mathcal{M}} J_{LS} := \frac{1}{2} \|\mathcal{F}[m] - d_o\|^2 \quad [+ \text{ regularizing term(s)}]$$

Note:

- FWI - globally minimize the objective using local methods
- observed data $d_o = \prod_s d_o(\mathbf{x}_r, t; s)$ - **highly redundant** & **band-limited**
  – possibilities for acquisition parameter s include source position, offset, slowness (plane wave data),...

Full Waveform Inversion (FWI)

Studied since the 80's, becoming feasible in last 10 yrs:

- + accommodates any modeling physics & provides quantitative inferences of subsurface (some spectacular successes)
- ± applicable to long-offset surface data (diving and refracted waves); most of successful inversions so far involve transmitted data
- − **inversion with reflection data still a challenge**

  fundamental obstacles:

  1. spectral data incompleteness - missing low frequencies in data leads to missing long scale components in estimated model
  2. strong nonlinearity, many false local minima - descent methods fail

Full Waveform Inversion (FWI)

Remedies for reflection inversion:

- automatic migration velocity analysis (MVA) (e.g., DSO variants): decompose model into slowly & fast varying parts (background & reflectivity), then alternately
  a. build reflectivity via migration or linearized LS-inversion
  b. update background to reduce inconsistency of reflectivity gathers

  \+ infer global changes in model (long-scale updates)
  − limited by linearization and scale separation assumptions

✓ **nonlinear DSO: import MVA's concepts (image gathering, semblance measuring) into FWI and drop MVA limitations**

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Formulate FWI via DSO

Concept: relax FWI via extension then demand coherence of extended model

FWI - attempt to match all data simultaneously with one model

$$\min_{m \in \mathcal{M}} \frac{1}{2} \|\mathcal{F}[m] - d\|^2 := \frac{1}{2} \sum_s \left(\|\mathcal{F}[m](s) - d(s)\|^2 \right)$$

Extended WI - fit subsets of data with non-physical *extended* models

$$\min_{\bar{m} \in \overline{\mathcal{M}}} \frac{1}{2} \left\|\overline{\mathcal{F}}[\bar{m}] - d\right\|^2 := \frac{1}{2} \sum_s \left(\|\mathcal{F}[\bar{m}(.,s)](s) - d(s)\|^2 \right)$$

Note:

- *extended models* $\overline{\mathcal{M}} = \{\bar{m}(\mathbf{x}, s)\}$ $(\mathcal{M} \subset \overline{\mathcal{M}}$, regarding $\bar{m} = m$ iff $\bar{m}(\mathbf{x}, s) = m(\mathbf{x})$ for all $s)$
- *extended modeling* $\overline{\mathcal{F}} : \overline{\mathcal{M}} \to \mathcal{D}$ by $\overline{\mathcal{F}}[\bar{m}](s) = \mathcal{F}[\bar{m}(\cdot, s)](s)$

Equivalent problem to FWI - find coherent solution to extended WI

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Formulate FWI via DSO

Differential Semblance (with surface oriented extension):

- s finely sampled $\Rightarrow$ coherence criterion is $\partial\bar{m}/\partial s = 0$
- **Differential Semblance Optimization:**

$$
\begin{aligned}
&\min_{\bar{m}\in\overline{\mathcal{M}}} \quad J_{DS}[\bar{m}] := \frac{1}{2}\left\|\frac{\partial}{\partial s}\bar{m}\right\|^2 && \texttt{(coherence measurement)} \\
&\texttt{s.t.} \quad \left\|\overline{\mathcal{F}}[\bar{m}] - d\right\| \approx 0 && \texttt{(data-fitting constraints)}
\end{aligned}
$$

Key to success: need a proper control in order to navigate through the feasible model set

$$\left\{\bar{m} \in \overline{\mathcal{M}} : \left\|\overline{\mathcal{F}}[\bar{m}] - d\right\| \approx 0\right\}$$

Using LF data as control

Concept:

Cannot use independent long-scale model as control, as in MVA: "low spatial frequency" not well defined, depends on velocity.

However, *temporal* passband *is* well-defined, and observed data lacks very low frequency energy (0-3, 0-5,... Hz) with good s/n

Generally, the impulsive inverse problem is solvable: LS inversion leads to "unique" model, if data d is *not* band-limited (good s/n down to 0 Hz)

So: **use low-frequency data as control (analogous to long-scale model in MVA)**

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Using LF data as control

Approach I: use low-frequency data as control

- define low-frequency source complementary to missing passband, *low-frequency modeling op* $\mathcal{F}_l$ and its extension $\overline{\mathcal{F}_l}$
- define complementary *low-frequency data* d_l
- define $\bar{m}[d_l]$ to be minimizer of

$$\|\overline{\mathcal{F}}[\bar{m}] + \overline{\mathcal{F}_l}[\bar{m}] - (d + d_l)]\|^2 + \sigma^2 \left\| \frac{\partial \bar{m}}{\partial s} \right\|^2$$

- nDSO: determine d_l by solving

$$\min_{d_l} \left\| \frac{\partial}{\partial s} \bar{m}[d_l] \right\|^2$$

Initial exploration: D. Sun (2008), D. Sun & W. Symes (2009), for plane wave / layered medium modeling – slices of DS objective are convex, i.e., an enlargement of the domain of attraction of the global minimum is achieved

Further thought: find a way to supply meaningful/consistent low frequency data

Using LF data as control

Approach II: *generate* low-frequency data from model

- Given *low frequency control model* $m_l \in \mathcal{M}$, define extended model $\bar{m} = \bar{m}[m_l]$ by minimizing

$$\|\overline{\mathcal{F}}[\bar{m}] + \overline{\mathcal{F}_l}[\bar{m}] - (d + \mathcal{F}_l[m_l])]\|^2 + \sigma^2 \left\| \frac{\partial \bar{m}}{\partial s} \right\|^2$$

- nDSO: determine m_l by solving

$$\min_{m_l} \left\| \frac{\partial}{\partial s} \bar{m}[m_l] \right\|^2$$

Advantage of this approach: m_l plays same role as migration velocity model, but no linearization, scale separation assumptions required by formulation.

Current project: 2D constant-density acoustics, plane-wave sources (to minimize edge artifacts)

DSO Algorithm

Key step: compute m_l *gradient*:

$$\nabla J_{DS}[m_l] = -D\mathcal{F}_l[m_l]^T D\overline{\mathcal{F}}_l[\bar{m}[m_l]] H[\bar{m}[m_l]]^{-1} \frac{\partial^2}{\partial s^2} \bar{m}[m_l]$$

where

$$H[\bar{m}] = \left(D\overline{F}[\bar{m}] + D\overline{F_l}[\bar{m}]\right)^T \left(D\overline{F}[\bar{m}] + D\overline{\mathcal{F}_l}[\bar{m}]\right)$$

Computational procedure:

1. initialize m_l, etc.
2. solve sub-LS problem for $\bar{m}[m_l]$
3. evaluate $J_{DS}[\bar{m}]$. If stopping criterion satisfied, stop; else, continue.
4. compute updating direction $g = -\nabla J_{DS}[m_l]$
5. update m_l, cycle

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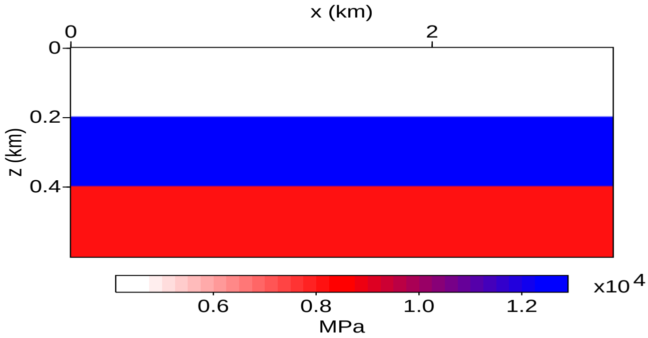
Model

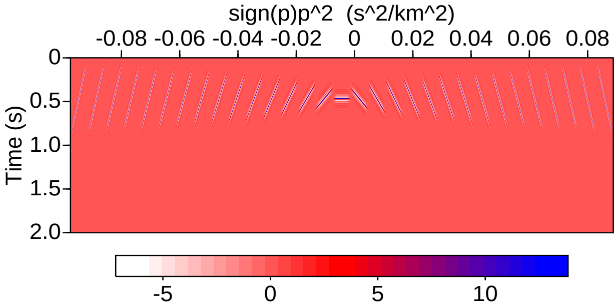


Three layer bulk modulus model.
(acoustic velocity $v = 1.5, 2.5, 2\,\mathrm{km/s}$, density $\rho = 1\,\mathrm{g/cm^3}$)

Two sets of inversion exercises:

- absorbing surface
- free surface

Absorbing Surf: LS Inversion (standard FWI)



Band limited (5-12-36-45 Hz) plane wave source - 31 slowness panels

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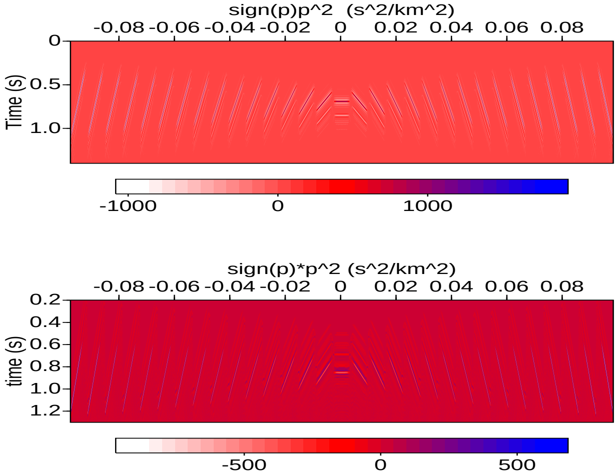
Absorbing Surf: LS Inversion (standard FWI)



Target data (Top), Data residual (Bottom)

: after 40 LBFGS iterations - RMS $\sim$ 32%

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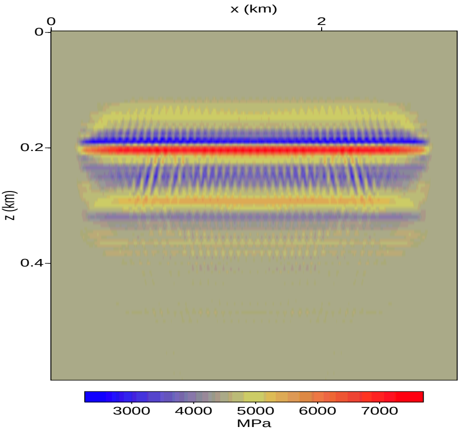
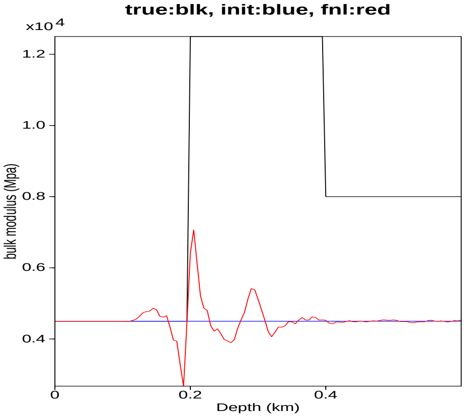
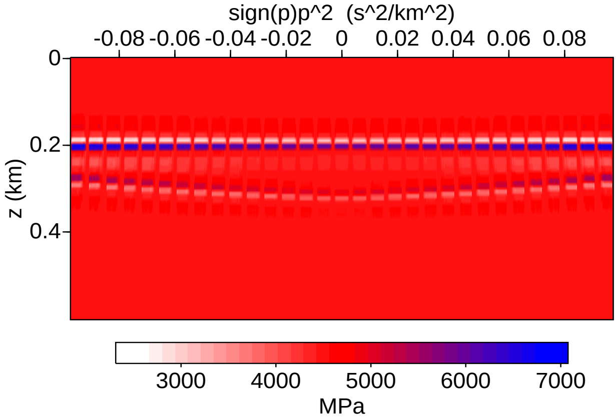
Absorbing Surf: LS Inversion (standard FWI)





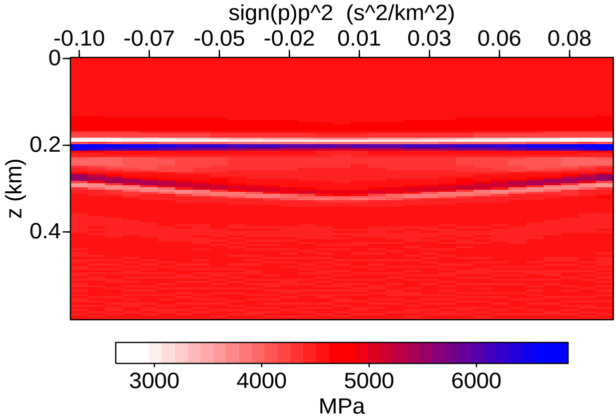
Left: FWI estimate of bulk modulus after 40 LBFGS iterations; Right: final model slice at $x = 1.5(\,\mathrm{km})$

Absorbing Surf: DS Inversion with LF control



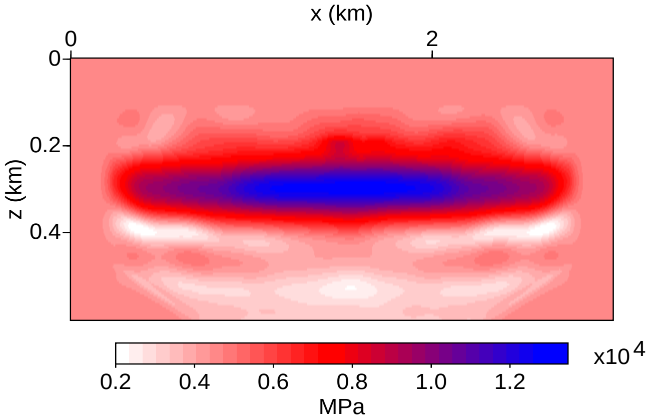
Inverted model $\bar{m}[m_l]$, $m_l =$ homogeneous model

Absorbing Surf: DS Inversion with LF control



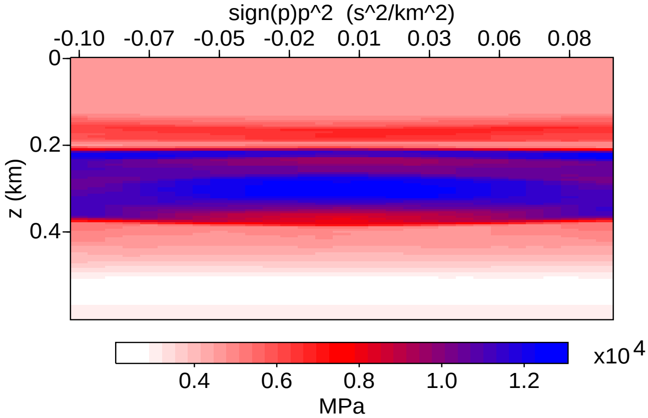
Inverted gather $\bar{m}[m_l]$, m_l = homogeneous model, $x = 1.5$ km

Absorbing Surf: DS Inversion with LF control



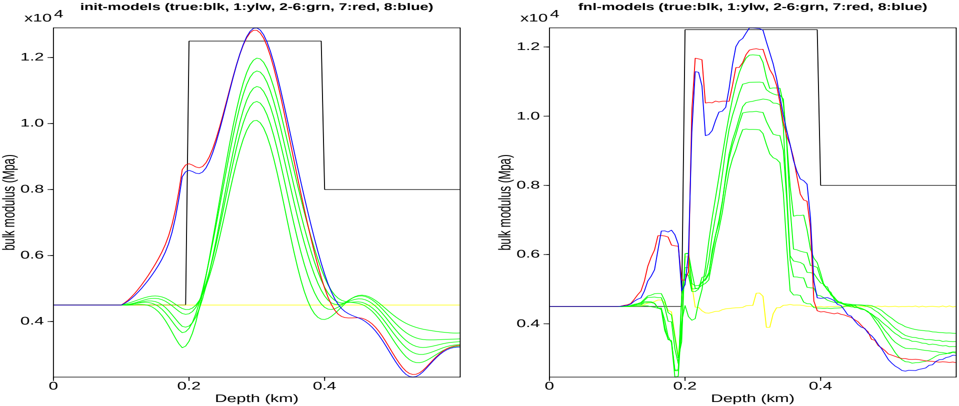
Low frequency control model m_l after 7 updates

Absorbing Surf: DS Inversion with LF control



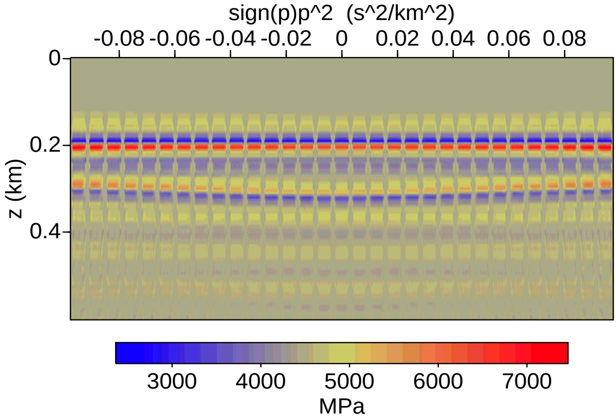
Inverted gather $\bar{m}[m_l]$, 7 updates of m_l, $x = 1.5$ km

Absorbing Surf: DS Inversion with LF control



Left: m_l at $x = 1.5(\,\mathrm{km})$; Right: final model at $x = 1.5(\,\mathrm{km})$

Free Surf: DS Inversion with LF control



Inverted model $\bar{m}[m_l]$, $m_l =$ homogeneous model

! multiple reflections got suppressed during LS inversion

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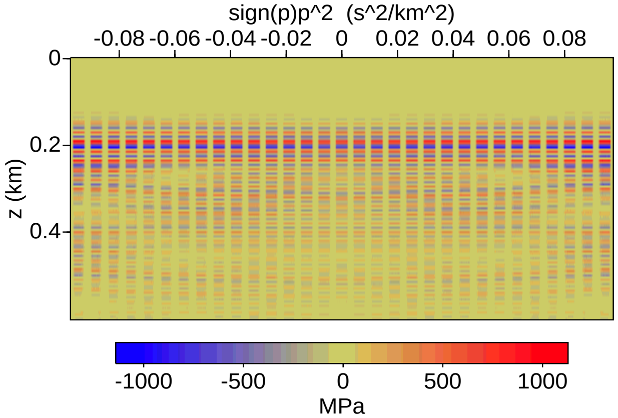
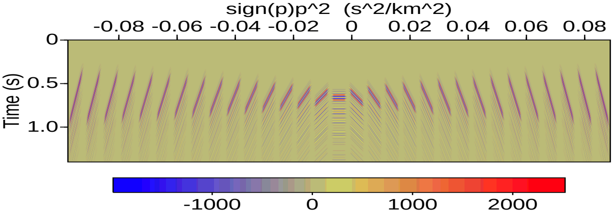
Free Surf: DS Inversion with LF control

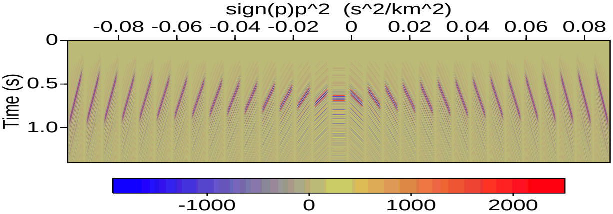


LS gradient in 1st iteration (i.e., residual migration)

! lots of multiple reflections

Free Surf: DS Inversion with LF control

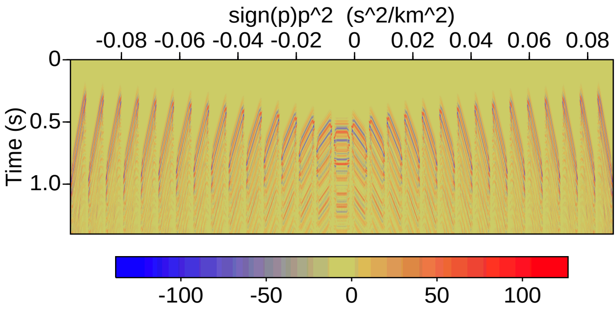




Target Data (Top), Predicted Data (Bottom)

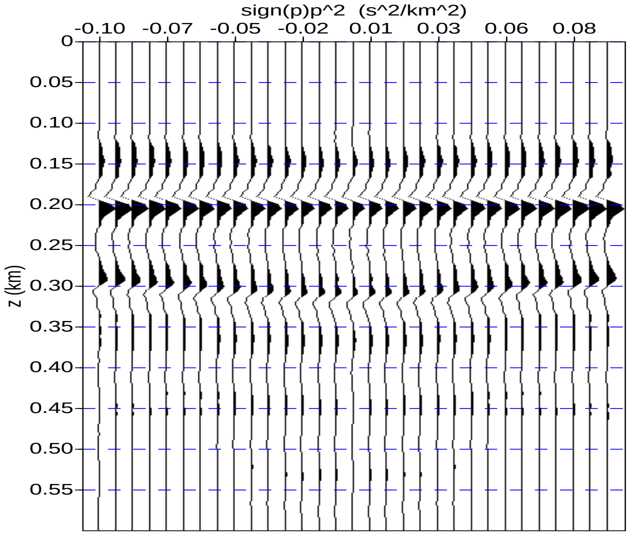
:after 60 LBFGS iterations

Free Surf: DS Inversion with LF control



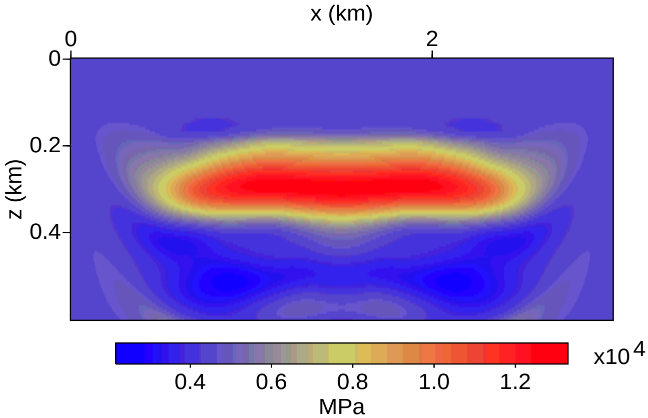
Data residual after 60 LBFGS iterations - RMS $\sim$ 8%

Free Surf: DS Inversion with LF control



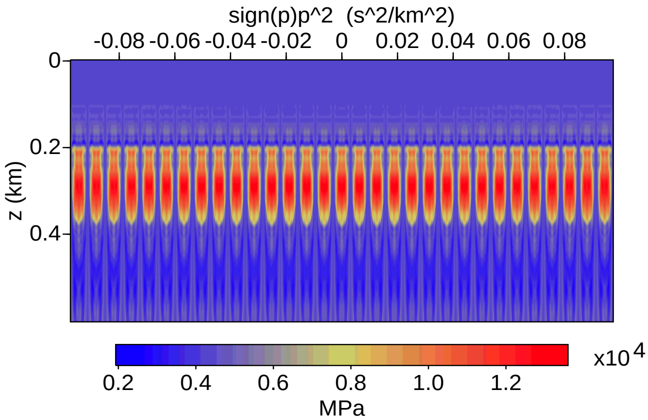
Inverted model gather $\bar{m}[m_l]$, m_l = homogeneous model, $x = 1.5$ km

Free Surf: DS Inversion with LF control



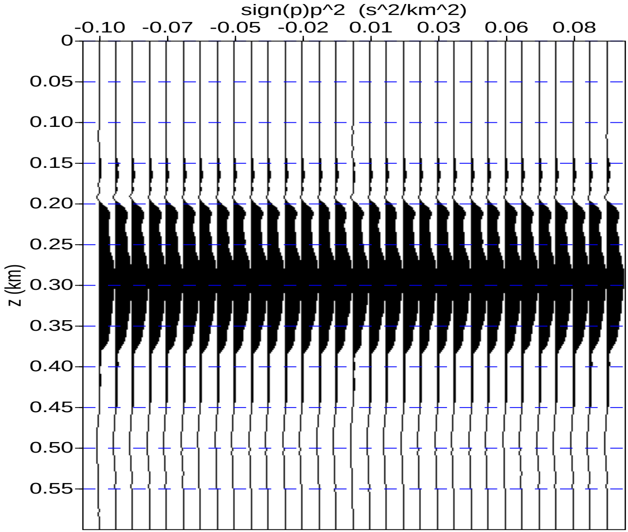
Low frequency control model m_l in the 3rd DS-iteration

Free Surf: DS Inversion with LF control



Inverted model $\bar{m}[m_l]$ in the 3rd DS-iteration

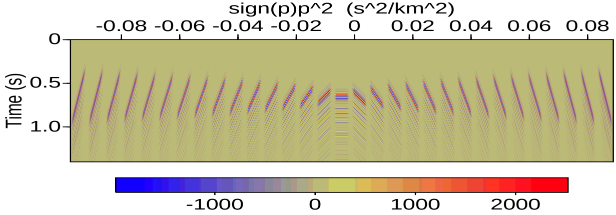
Free Surf: DS Inversion with LF control

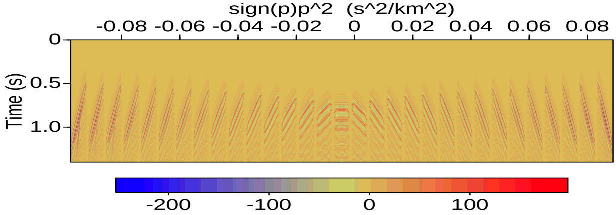


Inverted model gather $\bar{m}[m_l]$ after 3 DS-iterations, $x = 1.5$ km

DS-objective value reduced by 60%

Free Surf: DS Inversion with LF control





Target Data (Top), Data Residual (Bottom)

:after 60 LBFGS iterations - RMS $\sim$ 15% (in 3rd DS iteration)

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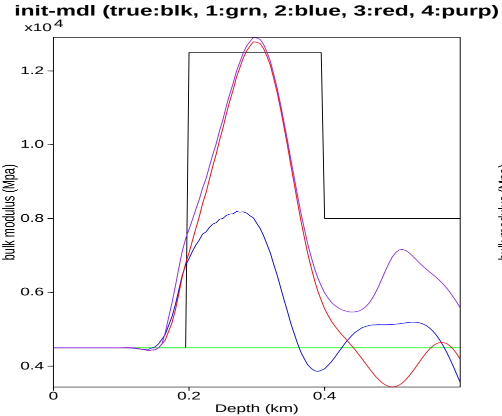
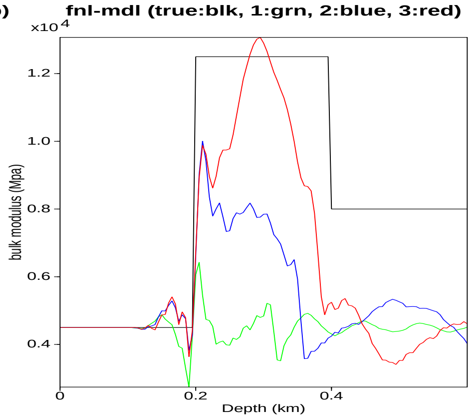
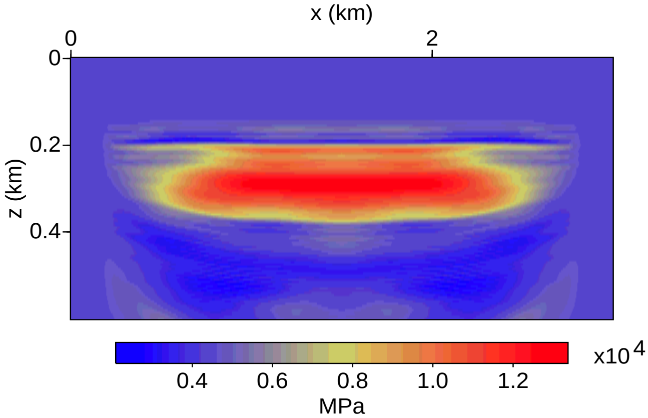
Free Surf: DS Inversion with LF control





Left: m_l at $x = 1.5(\text{ km})$; Right: final model at $x = 1.5(\text{ km})$

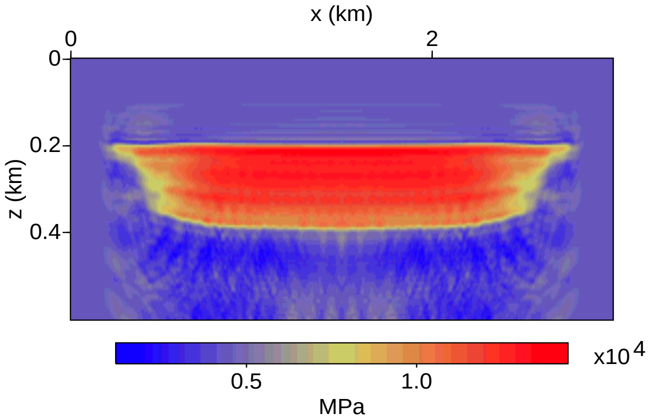
Free Surf: DS Inversion with LF control



Inverted model stack (in 3rd DS iteration)

Free Surf: DS Inversion with LF control

Standard LS inversion staring from the final model stack from nDSO



Inverted model after 153 LBFGS iterations (RMS residual $\sim$ 6%)

(initial RMS res $\sim$ 77%; after 30 LBFGS RMS res $\sim$ 14%)

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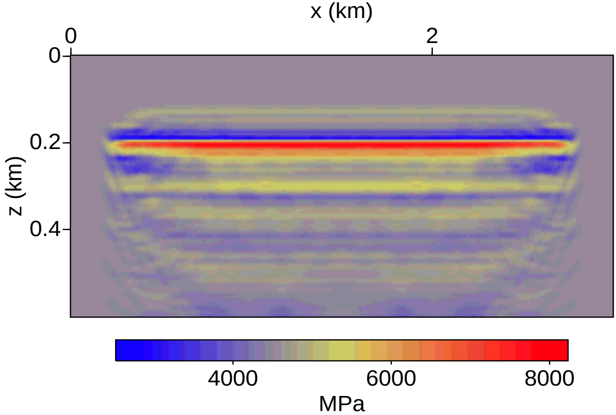
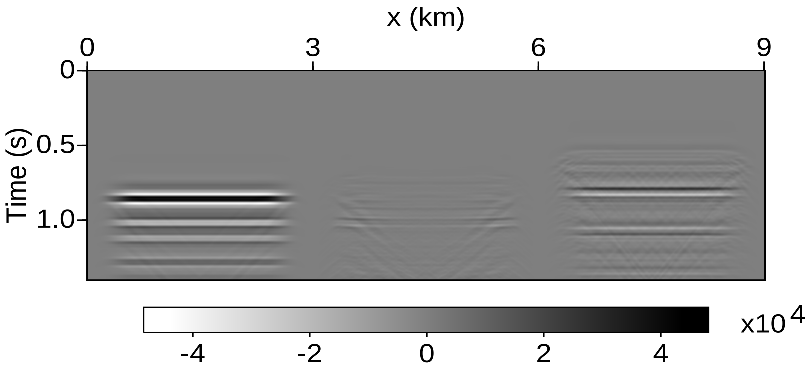
Free Surf: DS Inversion with LF control

Standard LS inversion staring from the homogeneous model



Inverted model after 30 LBFGS iterations (RMS residual $\sim$ 27%)

Free Surf: DS Inversion with LF control



Data fitting (for shot with slowness 0): Left, target data; Middle, residual for LS inv from DS-fnl-stack; Right, residual for LS inv from homogeneous model

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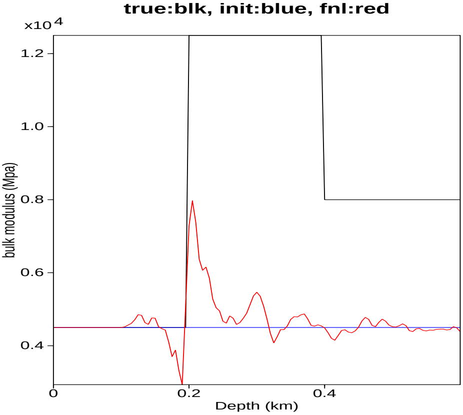
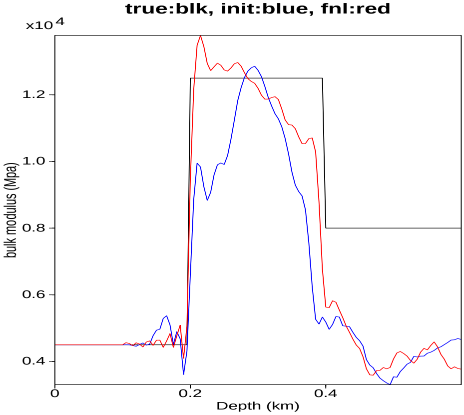
Free Surf: DS Inversion with LF control





Left: LS inv fnl model at $x = 1.5(\,\mathrm{km})$; Right: nDS inv fnl model at $x = 1.5(\,\mathrm{km})$

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Remarks

Pros:

- successfully infers global model updates for both absorbing surface and free surface

- stack at least gives a good initial model for LS-FWI

Remarks

Cost: 3-7 DS-iterations, mainly consisting of:

- 1 LS inversion (for $\bar{m}[m_l]$)
- direction computation $g = D\mathcal{F}_l[m_l]^T D\overline{\mathcal{F}}_l[\bar{m}[m_l]] H[\bar{m}[m_l]]^{-1} \frac{\partial^2}{\partial s^2}\bar{m}[m_l]$
  - solve $H[\bar{m}[m_l]]q = \frac{\partial^2}{\partial s^2}\bar{m}[m_l]$ for q (~ 30 CG iterations)

    to reduce cost: (a) pre-conditioning; (b) scaling strategies; (c) replacing H with I (might work) ...
  - compute $g = D\mathcal{F}_l[m_l]^T D\overline{\mathcal{F}}_l[\bar{m}[m_l]]\, q$ (1 Born Sim + 1 Adj Comp)
- step-length computation (1 CG or Lin-LS + 1 Born Sim):

$$\alpha = -\frac{\langle \psi, \frac{\partial}{\partial s}\bar{m}[m_l] \rangle}{\langle \psi, \psi \rangle}$$

, where $\psi = \frac{\partial}{\partial s}\left(D\overline{F}[\bar{m}[m_l]]^{\dagger}\; DF[m_l]g\right)$

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Summary

This approach may:

- combine best features of MVA and FWI (no linearization, scale separation assumptions required),
- address the spectral data incompleteness and local-minima issue,
- infer global changes in model and provide good initial model for FWI.

Next to-do: explore this strategy with more tests, improve efficiency, ...

Acknowledgements

Great thanks to

- my Ph.D. committee:
  William Symes, Matthias Heinkenschloss, Yin Zhang, Colin Zelt
- Present and former TRIP team members:
  Xin Wang, Marco Enriquez, Igor Terentyev, Tanya Vdovina, Rami Nammour
- ExxonMobil URC FWI team
  especially Dave Hinkley, Jerry Krebs
- WesternGeco FWI Management for support and permission to present here
- Sponsors of The Rice Inversion Project
- NSF DMS 0620821

Thank you!