Analyzing the space-shift differential semblance gradient - interim progress report

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TRIP 2011 Annual Review

Agenda

Semblance gradient artifacts

Where they come from

What to do about them

That is not all...

Space-shift Differential Semblance

Space-shift gather / HOCIG via shot record migration:(IEI, Biondi, Sava, Fomel,...)

$$I(x,z,h) = \sum_{x_s} \int dt\, S(x-h,z,t;x_s) R(x+h,z,t;x_s)$$

S = source wavefield, R = receiver wavefield - computed anyhow (depth extrapolation, two-way plus time reversal,...)

2D for convenience only!

DIfferential semblance MVA objective, in simplest form:

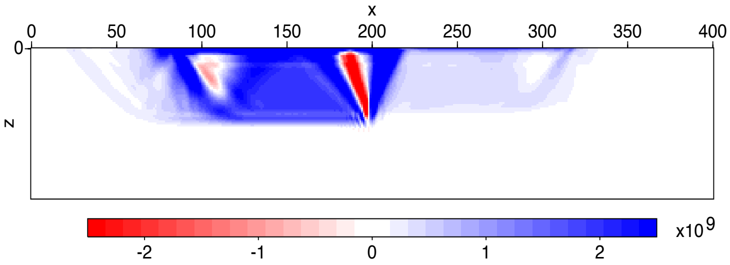
$$J[v] = \int\int\int dxdzdh\, h^2 \, |I(x,z,h)|^2$$

Shen's thesis 04, others (Shen & coauthors 03, 05, 07, Shen & S. 08, Kabir 07, Fei 09, 10)

Space-shift Differential Semblance

Upshot, it works, but...

Gradient tends to oscillate horizonally along upward paths from reflector singularities (truncations, corners) - Biondi 08, Fei 10, Vyas 10. Not just for DS:



Stack power gradient, courtesy R.-E. Plessix

Space-shift Differential Semblance

Besides being ugly, these oscillations inhibit convergence of optimization algorithms - seem like poor search directions

Discussions with Rene-Edouard Plessix, Hervé Chauris, and Maarten de Hoop at Newton Institute, Cambridge, December 2011

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Ray-theoretic expression for the gradient

For outline of this computation, see 2010 TRIP AR - here, result + interpretation

Assess effect on gradient at "wrong" velocity: assume that d is Born data for "target" velocity v^*, reflectivity $r(z_d, x_d)$. Presume that both v, v^* are homogeneous, so rays are straight lines. Write

$$d(x_s, x_r, t) =$$

$$\int\int dx_d dz_d \, (...)\delta(t - T^*(x_r, x_d, z_d) - T^*(x_s, x_d, z_d))r(z_d, x_d)$$

["(...)" = amplitude and frequency factors]

Ray-theoretic expression for the gradient

Insert into expression for J, get

$$J[v] = \int\int\int\int dx_d dz_d dx'_d dz'_d \, r(z_d, x_d) r(z'_d, x'_d) K(z_d, x_d; z'_d, x'_d)$$

in which K has explicit expression in terms of ray-theoretic quantities and oscillatory integrals

Via stationary phase analysis, obtain

$$\delta J[v]\delta v = \int\int\int\int dx_d dz_d dx'_d dz'_d \, r(z_d, x_d) r(z'_d, x'_d) \delta K(z_d, x_d; z'_d, x'_d)$$

Ray-theoretic expression for the gradient

$$\delta K = \int \int \int d\theta_s d\theta_r d\omega \, e^{i\Phi} A$$

$$\times \int_0^{Bz_d} dz' \, (Bz_d - z') \left[V_r \cdot \nabla \frac{\delta v}{v}(z', x_d + z_d \tan \Theta_r + z' \tan \theta_r) \right.$$

$$\left. + \; V_s \cdot \nabla \frac{\delta v}{v}(z', x_d + z_d \tan \Theta_s + z' \tan \theta_s) \right]$$

$\Phi = (x_d - x'_d)\Phi_x + (z_d - z'_d)\Phi_z$, $\Phi_x, \Phi_z, V_s, V_r, B, A =$ messy functions of $\theta_r, \Theta_r = \arcsin\left(\frac{v}{v^*} \sin\theta_r\right), \ldots$

Ray-theoretic expression for the gradient

Always true: V_s, V_r not parallel to rays

Consequence for *conormal* reflectors, that is, for some vector field W,

$$W \cdot \nabla r = \alpha r$$

in which α is a smooth function (or somewhat more general op)

Can express $V_s = a_s W + b_s R_s$, where R_s is velocity vector of source ray, similar for V_r

Ray-theoretic expression for the gradient

Then integrate by parts: transfer W to r, R_s, R_r to traveltime-dependent coefficients:

$$\delta J[v]\delta v = \int g \delta v \;\Rightarrow\; g = \nabla J[v]$$

is smooth function = "nice" gradient

Ray-theoretic expression for the gradient

If r is *not* conormal - for any choice of W, $W \cdot \nabla r$ is more singular than r - then $\delta J[v]\delta v$ depends on derivatives of δv, dependence localized near rays from singularities in $W \cdot \nabla r$

Manifestation in computations: oscillations across ray fan from corners, truncations to surface

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Proposed Remedy

Conventionally: "The Gradient" = Riesz representer of derivative via L^2 inner product ("continuous dot product") and discrete approximations

Non-existence of gradient not a new phenomenon - conventional reflection traveltime tomography gradient does not exist, either! (Delprat-Jannaud & Lailly, GJR 1993).

Morally: rate of change of objective (traveltime misfit, DS,...) depends on *derivatives* of velocity perturbation, really only makes sense for *smooth* $v, \delta v$

$\Rightarrow$ must use inner product / norm that *controls* derivatives

Proposed Remedy

Natural family of norms for this application: L^2 Sobolev family

$$\|\delta v\|_k^2 = \int d\mathbf{x}\,[\delta v(I - \sigma^2\nabla^2)^k \delta v]$$

Comparison: "ordinary" gradient $\nabla_0 J$, Sobolev k-norm gradient $\nabla_k J$

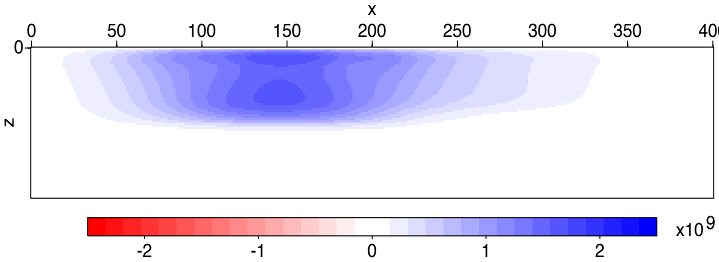
$$\nabla_k J = (I - \sigma^2\nabla^2)^{-k}\nabla_0 J$$

obtain k gradient from ordinary gradient by application of *smoothing* operator - large horizontal oscillations suppressed - isotropic smoothing applies also to VOCIG-based DS

Proposed Remedy

Example: use Sobolev 1-norm to construct regularized gradient of Stack Power gradient

Free parameter in norm $\sim$ length scale of smoothing by inverse Helmholtz.



H^1 stack power gradient, smoothing length = 100 m vertical, 1 km horizontal

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Loose ends, follow-ons

- How to choose weights in Sobolev norm, aka smoothing lengths? Have to do better than "so it looks nice"...
- How to understand (apparently) different fixes proposed by Fei and Williamson, and Albertin? Appear to *decouple* reflector, gradient regularity and produce good descent direction - but then "gradient" is gradient of ???

Further work by Gang of Four anticipated, and students are being recruited...

Loose ends, follow-ons

Plans:

- sabbatical leave AY 2012-13
- "The Book" - review with new TRIP students as part of getting them started on inversion-related projects
- space-shift DS - are stationary points global mins?
- rational basis for nDS, or extended FWI in general - reconcile smoothness sense of forward map (uniform norm) with appropriate & *computable* defn of gradient - already a hint from adjoint state method!
- extend IWAVE++ to encompass space-shift imaging, WEMVA
- move towards a rigorous inversion exercise with field data: does *actually fitting data* (few % RMS) teach us anything useful about the subsurface? Must do it to find out!

RICE