Gradient artifacts in space-shift differential semblance

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Agenda

Background

Ray Theory

The Fix

Space-shift Differential Semblance

Space-shift gather / HOCIG via shot record migration:(IEI, Biondi, Sava, Fomel,...)

$$I(x,z,h) = \sum_{x_s} \int dt\, S(x-h,z,t;x_s) R(x+h,z,t;x_s)$$

S = source wavefield, R = receiver wavefield - computed anyhow (depth extrapolation, two-way plus time reversal,...)

2D for convenience only!

DIfferential semblance MVA objective, in simplest form:

$$J[v] = \int\int\int dxdzdh\, h^2 \, |I(x,z,h)|^2$$

Shen's thesis 04, others (Shen & coauthors 03, 05, 07, Shen & S. 08, Kabir 07, Fei 09, 10)

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Space-shift Differential Semblance

Upshot, it works, but...

Gradient tends to oscillate horizonally - side lobes inhibit convergence (Biondi 08, Fei 10, Vyas 10)

This talk: where the oscillations come from, and how they might be removed.

Goal: redefine the gradient so that it is *still a gradient*, but suppress oscillations.

Approach: explicit ray theory computation suggests a fix

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Ray-theoretic Expression of SSDS

Assume band-unlimited data (Green's function) - then data $d(x_s, x_r, t)$ and image $I(x, z, h)$ related by

$$I(x,z,h) = \int\int\int\int dx_s dx_r dt d\tau$$

$$d(x_s, x_r, t) G(x_r, x+h, z, t-\tau) G(x_s, x-h, z, \tau)$$

$$= (F^T d)(x, z, h)$$

F = extended Born modeling operator

Ray-theoretic Expression of SSDS

Simplest form: assume

- rays obey "DSR" condition - no significant amount of energy on rays turning horizontal.
- rays also obey "simple ray geometry" condition - no multipathing

Also: ignore amplitudes, (more or less) uniform factors of frequency (powers of Laplacian) throughout talk.

Then $G(x_s, x, z, t) \simeq \delta(t - T(x_s, x, z))$ where $T(x_s, x, z) =$ (unique) traveltime $(x_s, z_s) \to (x, z)$

More: F^T, F are *invertible* in a large region of phase space containing most reflection energy (see de Hoop - Stolk - S. 2009)

Ray-theoretic Expression of SSDS

Rewrite using *image space* and *data space* dot products

$$J[v] = \langle hI, hI \rangle_I = \langle F^T d, h^2 F^T d \rangle_I$$

$$= \langle d, Fh^2 F^T d \rangle$$

$$= \int \int \int dx_s dx_r dt \, d(x_s, x_r, t) \int \int \int dk_s dk_r d\omega$$

$$\exp(i(\omega t + k_s x_s + k_r x_r)) \bar{H}^2(x_s, x_r, t, k_s, k_r, \omega) \hat{d}(k_s, k_r, \omega)$$

Egorov's Thm: symbol $\bar{H}$ = function in data phase space which maps, under ray tracing, to h^2 in image phase space.

Ray-theoretic Expression of SSDS

$\bar{H}$ homogenous of degree 0 in phase space coords, so write in terms of function H of phase angles θ_s, θ_r:

$$\omega \sin\theta_r = k_r,\ \omega \sin\theta_s = k_s$$

- receiver ray; starts at x_r, z_r, takeoff angle θ_r: $z \mapsto X(x_r, \theta_r, z), T(x_r, \theta_r, z)$
- source ray; starts at x_s, z_s, takeoff angle θ_s: $z \mapsto X(x_s, \theta_s, z), T(x_s, \theta_s, z)$
- two-way time condition: $t = T(x_s, \theta_s, z) + T(x_r, \theta_r, z)$ - determines $z(x_s, x_r, t, \theta_s, \theta_r)$
-

$$\begin{aligned} H(x_s, x_r, t, \theta_s, \theta_r) &= X(x_r, \theta_r, z) - X(x_s, \theta_s, z), \\ z &= z(x_s, x_r, t, \theta_s, \theta_r) \end{aligned}$$

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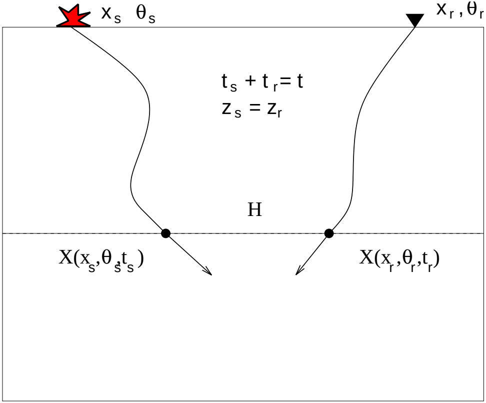
Ray-theoretic Expression of SSDS



Homogeneous medium, velocity v:

$$
\begin{aligned}
z(x_s, x_r, t, \theta_s, \theta_r) &= \frac{vt}{\sec\theta_r + \sec\theta_s} \\
H(x_s, x_r, t, \theta_s, \theta_r) &= x_r - x_s + z(\tan\theta_r - \tan\theta_s)
\end{aligned}
$$

Computing the gradient

Key observation: only v-dependent quantity in oscillatory integral for $J[v]$ is symbol H.

$$\delta H = \delta X(x_r, ...) - \delta X(x_s, ...) + \frac{\partial X}{\partial z}(x_r, ..)\delta z - \frac{\partial X}{\partial z}(x_s, ..)\delta z$$

Compute δz by differentiating $t = T(x_s, \theta_s, z) + T(x_r, \theta_r, z)$ implicitly, use ray perturbation equations - obtain

$$\delta H = \int_0^z dz'\,(z - z')\left[V_r \cdot \nabla \frac{\delta v}{v}(z', x_r + z' \tan\theta_r)\right.$$

$$\left. + V_s \cdot \nabla \frac{\delta v}{v}(z', x_s + z' \tan\theta_s)\right]$$

$V_s, V_r =$ messy functions of θ_s, θ_r.

Computing the gradient

$$\delta H = \int_0^z dz'\,(z - z')\left[V_r \cdot \nabla \frac{\delta v}{v}(z', x_r + z' \tan\theta_r) + V_s \cdot \nabla \frac{\delta v}{v}(z', x_s + z' \tan\theta_s)\right]$$

- tomographic: $\delta H =$ integral along ray pair, like traveltime perturbation
- sensitive to oscillations: unlike traveltime perturbation, involves $\nabla \delta v$

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Computing the gradient

Assess effect on gradient at "wrong" velocity: assume that d is Born data for "target" velocity v^*, reflectivity $r(z_d, x_d)$. Ignoring amplitude and frequency factors,

$$d(x_s, x_r, t) = \int\int dx_d dz_d \, \delta(t - T^*(x_r, x_d, z_d) - T^*(x_s, x_d, z_d)) r(z_d, x_d)$$

Insert into expression for J, get

$$J[v] = \int\int\int\int dx_d dz_d dx'_d dz'_d \, r(z_d, x_d) r(z'_d, x'_d) K(z_d, x_d; z'_d, x'_d)$$

in which K represented by same integral as J above with $t = T^*(x_r, x_d, z_d) + T^*(x_s, x_d, z_d))$ in expression for H, and oscillatory phase Φ.

Important: δJ rep'd by *same* "double diffraction" integral with δH in place of H.

Computing the gradient

Use stationary phase to eliminate 4 of 11 integrals, ignore resulting frequency and amplitude factors, obtain

$$\delta J[v]\delta v = \int\int\int\int dx_d dz_d dx'_d dz'_d \, r(z_d, x_d) r(z'_d, x'_d) \delta K(z_d, x_d; z'_d, x'_d)$$

$$\delta K = \int\int\int d\theta_s d\theta_r d\omega \, e^{i\Phi} A$$

$$\times \int_0^{Bz_d} dz' \, (Bz_d - z') \left[V_r \cdot \nabla \frac{\delta v}{v}(z', x_d + z_d \tan \Theta_r + z' \tan \theta_r) \right.$$

$$\left. + \; V_s \cdot \nabla \frac{\delta v}{v}(z', x_d + z_d \tan \Theta_s + z' \tan \theta_s) \right]$$

$\Phi = (x_d - x'_d)\Phi_x + (z_d - z'_d)\Phi_z$, $\Phi_x, \Phi_z, B, A =$ messy functions of $\theta_r, \Theta_r = \arcsin\left(\frac{v}{v^*} \sin\theta_r\right), \ldots.$

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Computing the gradient

Can extract explicit multiple integral expression for ∇J from this formula - but not so enlightening

integral along ray is trivial if δv oscillates in perp direction:
$\delta v(z,x) = \chi(z,x) e^{ik(x - z\tan\theta_s)} \Rightarrow$

- $\nabla \delta v \simeq k(-\tan\theta_s, 1)^T \delta v$ for large k
- $\delta v(z', x_d + z_d \tan\Theta_s + z' \tan\theta_s) = \chi(...) e^{ik(x_d + z_d \tan\Theta_s)}$ - *approx. independent of ray coord* z' for large k
- so integral along ray $\simeq O(k)\delta v$
- remains approximately true if θ_s perturbed, sim. for θ_r - $O(k)$ growth for near-horizontal oscillation

Computing the gradient

Upshot: $\delta J[v]\delta v = O(k)$ if $\partial_x \delta v = O(k)$

$\Rightarrow$ x-Fourier components of $\nabla J[v]$ must be large (Plancherel)

$\Rightarrow$ gradient must generally (square-integrable r) oscillate in near-horizontal directions, as observed

Finer analysis; if reflectivity $r(x_d, z_d)$ is *smooth in* x_d, then can integrate by parts to absorb growth - however at x-direction singularities this is impossible, leading to *vertical diffraction side-lobes* observed in numerics (Biondi 08, Fei 10, Vyas 10).

Mathematical expression: for general (non-smooth) r, band-unlimited data, *gradient does not exist!*

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Proposed Remedy

Conventionally: "The Gradient" = Riesz representer of derivative via L^2 inner product ("continuous dot product") and discrete approximations

Non-existence of gradient not a new phenomenon - conventional reflection traveltime tomography gradient does not exist, either! (Delprat-Jannaud & Lailly, GJR 1993).

Morally: rate of change of objective (traveltime misfit, DS,...) depends on *derivatives* of velocity perturbation, really only makes sense for *smooth* $v, \delta v$

$\Rightarrow$ must use inner product / norm that *controls* derivatives

Proposed Remedy

Natural family of norms for this application: L^2 Sobolev family

$$\|\delta v\|_k^2 = \int d\mathbf{x}\,[\delta v(I - \sigma^2\nabla^2)^k \delta v]$$

Comparison: "ordinary" gradient $\nabla_0 J$, Sobolev k-norm gradient $\nabla_k J$

$$\nabla_k J = (I - \sigma^2\nabla^2)^{-k}\nabla_0 J$$

obtain k gradient from ordinary gradient by application of *smoothing* operator - large horizontal oscillations suppressed - isotropic smoothing applies also to VOCIG-based DS

Proposed Remedy

Program:

- construct DS ("ordinary") gradient computation via RTM
- implement Helmholtz operators powers in rect. geom. using FFTs, sparse matrix methods
- compute k-norm gradient, use in optimization (for 2D, k=2)

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