Differential semblance MVA via RTM

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Agenda

Background

Theory

Implementation

Space-shift differential semblance

Space-shift gather / HOCIG via shot record migration:(IEI, Biondi, Sava, Fomel,...)

$$I(x,z,h) = \sum_{x_s} \int dt\, S(x-h,z,t;x_s)R(x+h,z,t;x_s)$$

S = source wavefield, R = receiver wavefield - computed anyhow (depth extrapolation, two-way plus time reversal,...)

2D for convenience only!

Space-shift differential semblance

$I(x, z, h)$ implicitly dependent on migration velocity v - minimize

$$J[v] = \sum_{x,z,h} |hI(x, z, h)|^2$$

Concept: small $\Rightarrow$ energy in I *focused* near $h = 0$ - Claerbout's coincident sunken source and receiver principle.

Leads to optimization method for velocity - *space-shift differential semblance* MVA

First implementation - Shen's thesis (2005) (many others since)

Space-shift differential semblance

Reverse-time method for computing $I(x,z,h)$ - Biondi & Shan 2002, S. 2002, generalizes RTM.

Claerbout, Tarantola: image formation $\sim$ application of *adjoint modeling operator* to data.

Q: what is the linear modeling operator whose adjoint outputs $I(x,z,h)$? Is this linear op the derivative of a full waveform modeling op?

Interest:

- positive answers $\Rightarrow$ another approach to FWI (S. 2008)
- facilitates implementation - IWAVE++

Agenda: identify modeling complex related to space-shift MVA, propose IWAVE++ implementation, resolve computational complexity issue

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From Image to Model

"Source wavefield" and "receiver wavefield" are a bit vague...

Source field: $S = \partial p / \partial t$ (time derivative for dimensional reasons)

$$\begin{aligned} \frac{\partial p}{\partial t} + \kappa \nabla \cdot \mathbf{v} &= f \\ \rho \frac{\partial \mathbf{v}}{\partial t} + \nabla p &= 0 \end{aligned}$$

causal: $p, \mathbf{v} = 0$ for $t << 0$

From Image to Model

Sampling operator G extracts traces $\{d(x_r, t; x_s)\}$ from pressure field $p(x, z, t; x_s)$

Adjoint sampling operator G^T inserts traces into field.

Receiver field: $R = q =$ backpropagated pressure field,

$$\begin{aligned} \frac{\partial q}{\partial t} + \kappa \nabla \cdot \mathbf{w} &= G^T d \\ \rho \frac{\partial \mathbf{w}}{\partial t} + \nabla q &= 0 \end{aligned}$$

anticausal: $q, \mathbf{w} = 0$ for $t >> 0$.

From Image to Model

What data is d?

Recall lesson of Claerbout, Tarantola, Lailly: imaging is dual to *Born modeling*.

$\Rightarrow$ d is (treated as) *Born data*, that is, $d = G\delta p$, where

$$\begin{aligned} \frac{\partial \delta p}{\partial t} + \kappa \nabla \cdot \delta \mathbf{v} + \text{Born Source} &= 0 \\ \rho \frac{\partial \delta \mathbf{v}}{\partial t} + \delta\rho \frac{\partial \mathbf{v}}{\partial t} + \nabla \delta p &= 0 \end{aligned}$$

causal: $\delta p, \delta \mathbf{v} = 0$ for $t << 0$; for convenience only, $\delta\rho = 0$.

From Image to Model

Modeling op: $F[\text{Born Source params}] = d$, then $F^T d = I$.

Notation: Born Source params $= K =$ input of $F \sim$ output of F^T - must be *image-like*: $K(x,z,h)$.

Idealize to continuous sampling: *image space dot product* is

$$\langle K, I \rangle_I = \int \int \int dx\, dz\, dh\, K(x,z,h) I(x,z,h)$$

The adjoint relation is

$$\langle FK, d \rangle_D = \langle K, F^T d \rangle_I$$

($\langle \cdot, \cdot \rangle_D =$ data space dot product)

From Image to Model

$$\langle K, I\rangle_I = \int\int\int dx\, dz\, dh\, K(x,z,h)$$

$$\times \int dt \int dx_s \frac{\partial p}{\partial t}(x-h,z,t;x_s)q(x+h,z,t;x_s)$$

$$= \int\int\int\int dx\, dz\, dt\, dx_s q(x,z,t;x_s)$$

$$\times \left[\int dh\, K(x-h,z,h)\frac{\partial p}{\partial t}(x-2h,z,t;x_s)\right]$$

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From Image to Model

Inspiration (pattern recognition!): suppose quantity in square brackets is Born source:

$$\left(\frac{\partial \delta p}{\partial t} + \kappa \nabla \cdot \delta \mathbf{v}\right)(x, z, t; x_s)$$

$$+ \int dh\, K(x - h, z, h) \frac{\partial p}{\partial t}(x - 2h, z, t; x_s) = 0,$$

$$\rho \frac{\partial \delta \mathbf{v}}{\partial t} + \nabla \delta p = 0$$

From Image to Model

Substitute LHS of 1st (pressure) eqn in dot product:

$$\langle K, I \rangle_I = \int \int \int dx\, dz\, dt\, dx_s \left(\frac{\partial \delta p}{\partial t} + \kappa \nabla \cdot \delta \mathbf{v} \right) q$$

$$= ... = \langle G \delta p, d \rangle_D$$

(standard computation - see Sun & S TR 10-06, or Gauthier et al *Geophys.* 1986) - so $FK = G\delta p$ and

$$\langle K, I \rangle_I = \langle K, F^T d \rangle_I = \langle FK, d \rangle_D$$

From Image to Model

Physical significance of space-shift Born Source term:

$$\int dh\, K(x-h,z,h)\frac{\partial p}{\partial t}(x-2h,z,t;x_s)$$

- $h =$ subsurface half-offset (space shift)
- $x =$ "sunken receiver" position
- $x-2h =$ "sunken source" position
- $x-h =$ "sunken midpoint"
- $K =$ perturbational bulk modulus acting over distance $2h$
- *physical* perturbation: $K(x,z,h)=\delta\kappa(x,z)\delta(h)$ - applied to I $=$ Claerbout's focusing principle

From Image to Model

Shift the shift: write K in terms of sunken receiver x, source $x - 2h$, rather than midpoint & offset: Born source term is

$$\int dh\, K(x,z,h) \frac{\partial p}{\partial t}(x-2h,z,t;x_s)$$

Corresponding change in imaging principle: produces sunken receiver gather

$$I(x,z,h) = \int dt \int dx_s \frac{\partial p}{\partial t}(x-2h,z,t;x_s) q(x,z,t;x_s)$$

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From Image to Model

Another question: what is "space shift model" leading to space shift Born model? [Other wise put: what is F the derivative of?]

Needed for (1) nonlinear space-shift DS, (2) IWAVE++ procedure: start with model in IWAVE

Answer: acoustic *extended model* with nonlocal bulk modulus $\mathcal{K}$

$$\begin{aligned}\frac{\partial p}{\partial t}(x,z,t:x_s) + \int do\, \mathcal{K}(x,z,o)\nabla\cdot\mathbf{v}(x-o,z,t;x_s) &= f\\ \rho\frac{\partial \mathbf{v}}{\partial t} + \nabla p &= 0\end{aligned}$$

causal: $p,\mathbf{v} = 0$ for $t << 0$; $o = 2h$

From Image to Model

(Nonlinear) forward map: $\mathcal{F}[\mathcal{K}] = Gp$

Relation with Born calculation = *linearization at physical model*: $F = D\mathcal{F}$, with identifications

$$\begin{aligned} \mathcal{K}(x,z,o) &= \kappa(x,z)\delta(o) \\ \delta\mathcal{K}(x,z,o) &= K(x,z,o/2)\kappa(x-o,z) \end{aligned}$$

NB: $D\mathcal{F}[\mathcal{K}]^T d$ is a *sunken receiver*, rather than midpoint gather - focusing principle still holds

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Implementation in IWAVE++

Discretized pressure equation for $p^n_{j,k} \simeq p(j\Delta x, k\Delta z, n\Delta t; x_s)$:

$$p^{n+1}_{j,k} = p^n_{j,k} + \Delta t \sum_{o_{\min}}^{o_{\max}} K_{j,k,o} D \cdot \mathbf{v}^{n+\frac{1}{2}}_{j-o,k}$$

BIG problem: involves dense matrix multiply *at every space-time gridpoint*

Solution, motivated by typical imaging practice (how adjoint is used): retain *sparse* (sunken) receiver grid for $o \neq 0$, spacing $\Delta r >> \Delta x$:

$$K_{j,k,o} = \frac{1}{\Delta o} \delta_{o0} \kappa_{j,k} \text{ if } (j - j_{\min})\%[\Delta r/\Delta x] \neq 0$$

Implementation in IWAVE++

Store $K_{j,k,o}$ using *sparse matrix* compression: for each sunken receiver position (j, k) ("row"),

- number $n(j, k)$ of nonzero values of $K(j, k, o)$,
- offset index $m(j, k)$ of first nonzero
- array K of nonzeros
- `size_t` workspace ia for array position of first nonzero

(band matrix variant of compressed row storage). Both n and p are `int` arrays even for 3D analog.

Then (linear indexing) constitutive law term in pressure update is

$$\sum_{i=0}^{n(j,k)-1} K(ia + i) * \nabla \cdot \mathbf{v}(j - i + m(j, k) + n_x * k), \;\; ia += n(j, k)$$

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Implementation in IWAVE++

Similarly, (per source, per time) imaging condition is: for $i = 0$ to $n(j,k) - 1$,

$$I(i + ia) += \nabla \cdot \mathbf{v}(j - i + m(j,k) + n_x * k) * q(j + n_x * k)$$

Gather sparsity: $n(j,k) > 1$ for (very) few j (x index)

Overlapping code: initialization of n, m arrays flags diag op timestep rules, much faster as no innermost loop over offset.

Needs: modification of IWAVE internals, i/o, timestepping functions fwd/adj, extraction utilities for visualization

Status: implementation in progress - stay tuned!

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