Do your worst: optimal selection of synthetic sources for waveform inversion

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Agenda

Source Encoding - why

Deterministic source synthesis

A few initial experiments

Conclusions and Plans

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Source Encoding - why

Several motivations for source encoding:

- Cheap acquisition - shoot from several sources simultaneously, or with smaller time delay than necessary to separate recorded data in time (Womack 1990, Beasley et al. 1998, 2008, Berkhout 2008, Blaquiere et al. 2009,...)
- Cheap modeling and migration - multiplex multishot data into encoded data with fewer effective sources (eg. Romero et al. 2000, Neelamani et al. 2008)
  - random filtering, incoherence important
  - accurate recovery of shot gathers $\Rightarrow$ limited compression, reduction in modeling workload

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Source Encoding - why

- Cheap inversion - use fewer sources (ideally, one for entire data set) in each iterative inversion step
  - length-1 encoding (Krebs et al. 2009)
  - inversion using source blending, simultaneous shooting (Ayeni et al. 2009)
  - random filtering, incoherency *presumed* important

Explicit recovery of individual shots not primary goal - instead, choose sources to drive model towards optimal inversion solution

= model which best fits *any* data (so shots are *implicitly* recovered...)

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Deterministic source synthesis

based on *distinguishability* concept - introduced into biomedical Electrical Impedance Tomography (EIT) by Isaacson (1986)

David Isaacson

EIT: image anomalies interior to body by measuring voltage response to applied current on boundary.

Deterministic source synthesis

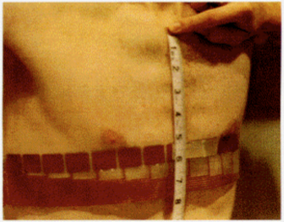
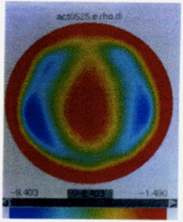


Photo 5. The chest of the normal subject shown in Photos 6 and 7. The electrode arrays are in place on the chest.



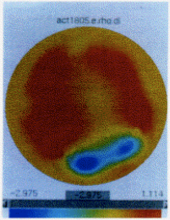


Photo 6 (left). Change in permittivity (displayed as $\omega\epsilon$) with inspiration in a normal human subject. The scale is from -9.4 mS/m (blue) to -1.5 mS/m (red). Inspiration causes a decrease in permittivity in bilateral lung fields. View is as if the subject were facing the viewer seen from above. Dorsal is at top of image.

Photo 7 (right). Change in permittivity (displayed as $\omega\epsilon$) with cardiac systole in a normal human subject. The scale is from -3.0 mS/m (blue) to 1.1 mS/m (red). In systole, permittivity of the heart region decreases, while that in the lungs increases.

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Deterministic source synthesis

current pattern f = current density as function of electrode location - can be controlled experimentally

Measured response $= \Lambda^d f$ - linear in f (electrostatics - f determines boundary flux $\partial u/\partial n$ of voltage potential u

Predicted response for model m of body conductivity $= \Lambda[m]f$, computed by FE or FD or...

Many possible current patterns - *for most, response nearly same regardless of interior structure!*

Q: can you find *any* current pattern f for which the response distinguishes the model-predicted voltage response $\Lambda[m]f$ from the measured one $\Lambda^d f$?

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Deterministic source synthesis

Isaacson: seek normalized f so that RMS difference is largest: given estimated model m,

$$\mathrm{maximize}_f\,(\Lambda^d f - \Lambda[m]f)^T(\Lambda^d f - \Lambda[m]f)\,\mathrm{subj}\,f^T f = 1$$

max value $\lambda[m]$ = largest eigenvalue (operator norm) of

$$A[m] = (\Lambda^d f - \Lambda[m])^T(\Lambda^d f - \Lambda[m])$$

= *largest discrepancy* in response for *any* (normalized) current pattern.

Isaacson's algorithm: Estimate $\lambda[m]$ by power method, apply a gradient descent method to minimize it over m.

Deterministic source synthesis

Translation to seismic source synthesis:

(1) choose search space for sources $f(\mathbf{x}_s, t)$

- length-1 filters: assuming uniform point source model with wavelet $w(t)$,
$$f(\mathbf{x}_s, t) = a_f(\mathbf{x}_s) w(t)$$
(Krebs et al. 09 - $a_f(\mathbf{x}_s) = \pm 1$)
- arbitrary length filters
$$f(\mathbf{x}_s, t) = \int d\tau \, a_f(\mathbf{x}_s, t - \tau) w(\tau)$$
(Romero et al. 00) - first is special case of second with $a_f(\mathbf{x}_s, t) = a_f(\mathbf{x}_s)\delta(t)$

Deterministic source synthesis

(2) given conventional fixed spread shot-order data $\{d(\mathbf{x}_r, t; \mathbf{x}_s)\}$, calculate measured response to f ("encoded" or "blended" or "simultaneous" data)

$$\Lambda^d f(\mathbf{x}_r, t) = \sum_{\mathbf{x}_s} \int d\tau \, a_f(\mathbf{x}_s, t-\tau) d(\mathbf{x}_r, \tau; \mathbf{x}_s)$$

(3) for a model (acoustic, elastic,...) of earth response m, compute $\Lambda[m]f$ by solving one wave equation with source term $f(\mathbf{x}_s, t)$, then sampling the resulting fields as appropriate

(4) transpose operator $\Lambda[m]^T = R\Lambda[m]R$, $R =$ time-reversal op

Deterministic source synthesis

Analog of Isaacson's alternating algorithm:

- initialize m, f
- while (not satisfied),
  - fixed m, update f: perform several power method steps: $f \leftarrow A[m]f$, $f \leftarrow (1/\sqrt{f^t f})f$
  - fixed f, update m: perform several quasi-Newton steps with objective function $\lambda_{\rm est}[m] = f^t A[m] f$ (standard OLS)

Each step of both types involves 2 or 3 forward and/or reverse time loops, *for single (array) source*

What can be proved: under some circumstances, $m \rightarrow m_{\rm opt}, \lambda_{\rm est}[m] \rightarrow \lambda[m_{\rm opt}]$.

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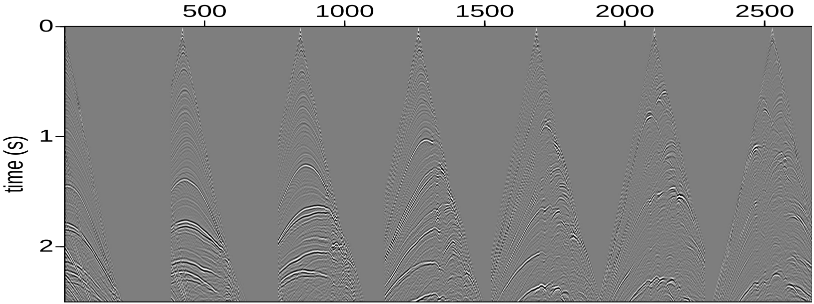
Explore first half of algorithm (power method for optimal source synthesis)

Based on Marmousi constant-density acoustic model, fixed spread simulation

- 96 shots, $\Delta x_s = 80$ m, $x_{s,\min} = 800$ m, $z_s = 4$ m
- 381 receivers (fixed), $\Delta x_r = 20$ m, $x_{r,\min} = 200$ m, $z_r = 8$ m
- nt = 626, dt = 4 ms
- source wavelet $w(t) =$ Ricker wavelet, peak frequency 25 Hz
- IWAVE-based implementation of $\Lambda[m]$ - array source, RVL `LinearOp` interface

A few initial experiments



Shot gathers for sx = 800, 1600, 2400,...8000 m

A few initial experiments

Question to be explored: what is the optimal synthesized source to start the inversion from constant background $v = 1.5$ km/s?

"Optimal" means: has largest Rayleigh quotient, i.e produces largest predicted difference in recorded data for a given "size" of input source

Search space: length-1 filters, that is, weighted stacks over sx - what is the optimal weight choice? Some possibilities:

- a randomly placed single point source ($a(x_s) = 1$ if $x_s = x_s^*$, $= 0$ else)
- a plane wave ($a(x_s) =$ const.)
- random signs ($a(x_s) = \pm 1$) (per Krebs et al. 2009)

A few initial experiments

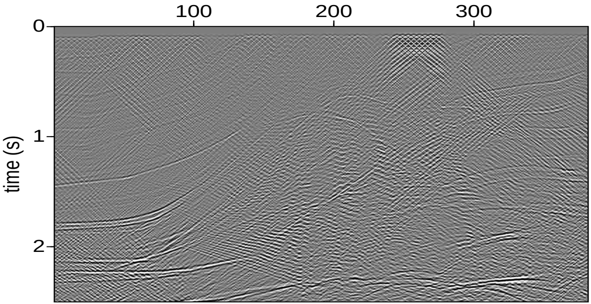
Comparison: plane wave vs random ± 1 - after 10 iterations of power method

- plane wave weights: initial RQ $= 823$, final RQ $= 1413$
- random ± 1 weights: initial RQ $= 802$, final RQ $= 1375$

$\Rightarrow$ source generated by power method considerably more sensitive to difference between Marmousi and const velo modes ("per unit source energy") than either plane wave or random ± 1 synthetics.

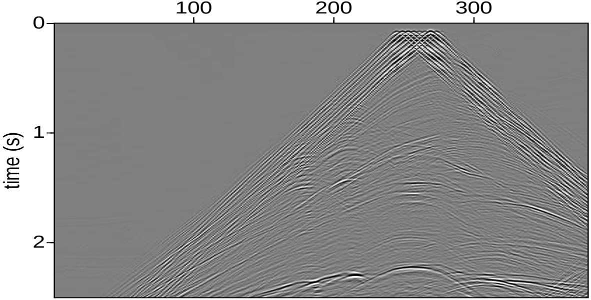
Observation: most energetic part of data is reflection from near-surface fault blocks near 5500 m, so optimal sources concentrate energy there

A few initial experiments



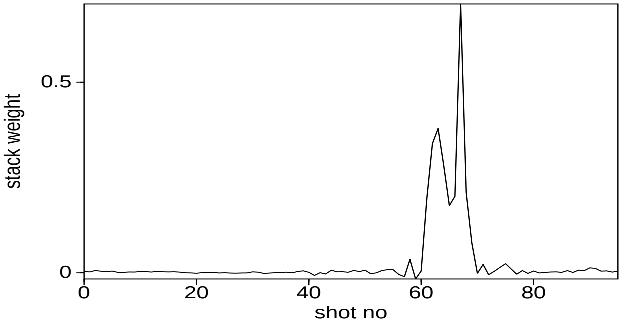
Normal incidence plane wave stack

A few initial experiments



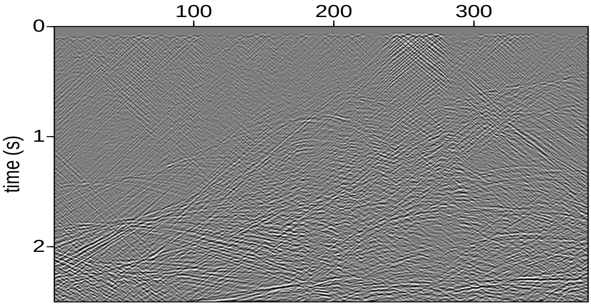
Power method - stack starting with normal incidence plane wave

A few initial experiments



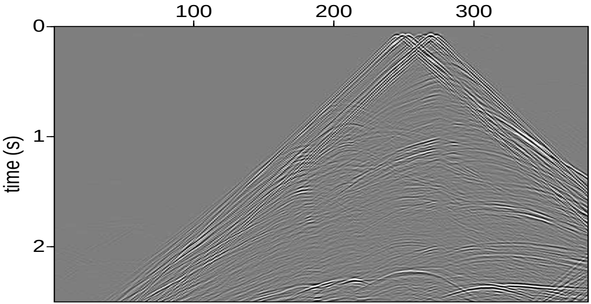
Power method - weights starting with normal incidence plane wave

A few initial experiments



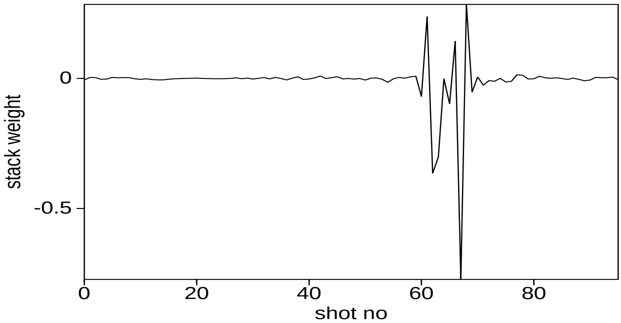
Random ±1 weights - stack

A few initial experiments



Power method - stack starting with random ± 1 weights

A few initial experiments



Power method - weights starting with random ± 1 weights

A few initial experiments

To avoid domination by the fault block reflections, mute data above 0.8 s.

$\Rightarrow$ can create optimal source to illuminate data zones

Limitation: muting must be shot-independent, so that the same data is created for each shot

Show only result with plane wave initial weights - random ± 1 *very* similar. Initial RQ $=$ 652, final RQ $=$ 852 (so plane wave either does not have components with large singular values, or is already near-optimal)

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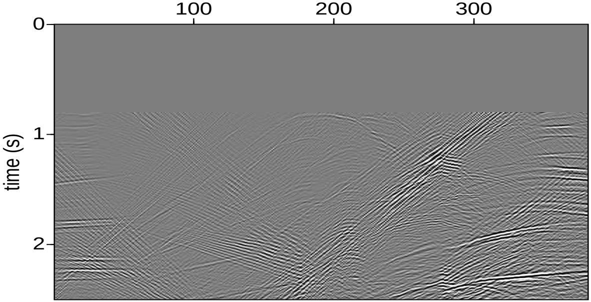
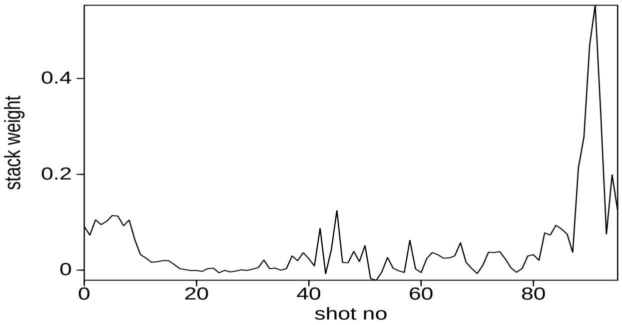
A few initial experiments



Power method - stack starting with normal incidence plane wave, mute to 0.8 s

A few initial experiments



Power method - weights starting with normal incidence plane wave, mute to 0.8 s

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RICE

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Appears to be eminently possible to design *deterministic* source at each gradient step of WI to optimally drive *all possible residuals* to minima.

Amounts to measuring error in terms of operator norm of $\Lambda[m] - \Lambda^d$, rather that conventional OLS = Frobenius norm

Next up: try alternating direction algorithm - combine power method for source update with quasi-Newton medium update (IWAVE implementation - Dong) - also explore larger source search space

Much related work in array ultrasonics (Fink & Prada 2004), ocean acoustics (Roux & Kuperman 2005), and eigenvalue design literature - other algorithms may outperform simple alternating direction alg. Borcea & Papanicolaou (2007): other eigenvalues (other than max) may also be useful