Approximate Inverse Scattering Using Pseudodifferential Scaling

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The Rice Inversion Project

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Outline

1. Linearization of the inverse problem
2. Properties of the normal operator
3. One-parameter: Pseudodifferential scaling
   - Formulation
   - Experiments and Results
4. Multi-parameters: Operator Krylov
   - Formulation
   - Experiments and Results

Let:

- $m(x)$: model (consists of p-parameters: impedance, density,...)
- $p(x,t)$: state (the solution of the system: pressure)

Then, if S is the Forward Map:

- The Forward Problem:

$$S[m] = p|_{surface}$$

- The Inverse Problem:

$$S[m] \approx S^{obs}$$

Given S^{obs}, get $m(x)$

Nonlinear and Large Scale !

Linearization

Solution depends nonlinearly on coefficients; if we have an approximation m_0 to the model, Linearization is advantageous:

- Write $m = m_0 + \delta m$
  m_0: Given reference model
  δm: First order perturbation about m_0

- Define Linearized Forward Map $F[m_0]$ (Born Modeling):

$$F[m_0]\delta m = \delta p$$

- Reduce to the Linear Subproblem

$$F[m_0]\delta m \approx S^{obs} - S[m_0] := d$$

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Normal Operator

Interpreted as a least squares problem, linear subproblem yields the normal equations

$$F^*[m_0]F[m_0]\delta m = F^*[m_0]d$$

$F^*[m_0]F[m_0]$ is the Normal Operator (Modeling + Migration)

The problem is still *Large Scale*,order of Pflops/Pbytes $\Rightarrow$ cannot use direct methods to invert F^*F.

Properties of the normal operator have been extensively studied (Beylkin,1985; Rakesh,1988) for *smooth* m_0

- Pseudodifferential Operator for one parameter, nearly diagonal in a basis of localized monochromatic pulses
- $p \times p$ matrix of pseudodifferential operators for p parameters in polarization preserving scattering

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Normal Operator

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Migration Vs Inversion

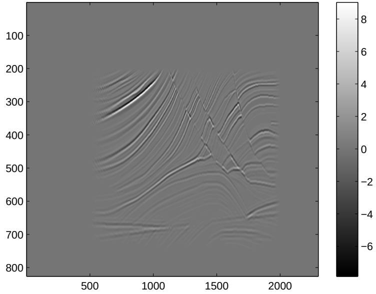

Figure: $m_{mig} = F^* d$

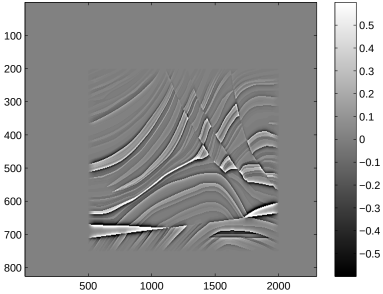

Figure: $m_{true} = (F^* F)^{\dagger} F^* d$

Lessons Learned

- Discontinuities preserved
- Amplitudes distorted

Solution: amplitude correction

- Idea: use near diagonality of the normal operator in the right basis to derive an approximation to $(F^*F)^\dagger$ that scales the amplitudes of the migrated image to the true image.

Scaling Methods

- Spatial delta function approximation of eigenvectors $\Rightarrow$ Hessian $\approx$ multiplication by a smooth function (Claerbout and Nichols, 1994; Rickett, 2003)
- Near Diagonal Approximation of Hessian (Guitton, 2004)
- Special case (well defined dip): normal operator $\approx$ multiplication by smooth function after composition with power of Laplacian (correction to Claerbout-Nichols - Symes, 2008)
- Herrmann et al. (2007) derive a scaling method using *curvelets* to approximate eigenvectors

Pseudodifferential Scaling

Want to solve:

$$Nx = b,$$

where $N = F^*F$ and $b = F^*d \in \mathit{Range}(N)$.

Given b and Nb, compute a scaling factor c_1:

$$c_1 = \underset{c_1 \in \Psi DO}{\mathit{argmin}} \|b - c_1\, Nb\|^2.$$

Then,

$$x = N^{\dagger}b \approx N^{\dagger}c_1\, Nb \approx c_1 b := x_{inv}$$

The Scaling Factor

c_1 scales the amplitudes of the migrated image to those of the true image $\Rightarrow c_1 \approx N^\dagger$.

Require c_1 to be:

- Like $N^\dagger$
  - c_1 is pseudodifferential
  - c_1 is dip-dependent
- Unlike $N^\dagger$
  - Efficient to compute
  - Efficient to apply to data

How to represent c_1 *?*

Approximation of ΨDO

- The action of the ΨDO (Bao and Symes, 1996):

$$Q_m u(x,z) \approx \int\int q_m(x,z,\xi,\eta)\hat{u}(\xi,\eta)e^{i(x\xi+z\eta)}\,d\xi\,d\eta$$

  q_m is the principal symbol, homogeneous of degree m.
  $\hat{u} = \mathcal{F}[u]$.
- Direct Algorithm $O(N^4 \log(N))$ complexity ($N = \mathcal{O}(10^3)$)!
- Writing $\xi = \omega\cos(\theta)$, $\eta = \omega\sin(\theta)$. Then,
  $q_m(x,z,\xi,\eta) = \omega^m \tilde{q}_m(x,z,\theta)$
-

$$\tilde{q}_m \approx \sum_{l=-K/2}^{l=K/2} a_l(x,z)e^{il\theta} = \sum_{l=-K/2}^{l=K/2} \omega^{-l} a_l(x,z)(\xi+i\eta)^l$$

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Algorithm

$$Q_m u \approx \sum_{l=-K/2}^{l=K/2} a_l(x,z)\mathcal{F}^{-1}[\omega^{m-l}(\xi + i\eta)^l \hat{u}(\xi,\eta)]$$

1. Calculate $\hat{u} = \mathcal{F}[u]$
2. Calculate $\mathcal{F}^{-1}[\omega^{m-l}(\xi + i\eta)^l \hat{u}(\xi,\eta)]$
3. Calculate $a_l(x,z) \approx \mathcal{F}[\tilde{q}_m]$
4. Estimate $Q_m u$

Use FFT $\Rightarrow O(KN^2[\log(N) + \log(K)])$ complexity vs $O(N^4 \log(N))$ complexity for the direct algorithm. K independent of N.

Algorithm

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Recap

To solve

$$Nx = b,$$

where $N = F^*F$, $b = F^*d$.

Given, $b = F^*d = m_{mig}$ and $Nb = F^*Fm_{mig} = m_{remig}$

- Represent $c_1 = Q_m[q_m]$
- Compute $c_1 =$ *argmin* $\|b - c_1\, Nb\|^2$
- Approximate $x_{inv} := c_1 b \approx N^{\dagger} b = x$

Pseudodifferential scaling method that resolves multiple dip events for $K > 1$.
Reduces to optimal scaling (Symes, 2008) for $K = 1$.

Results I

On Marmousi 2D data:

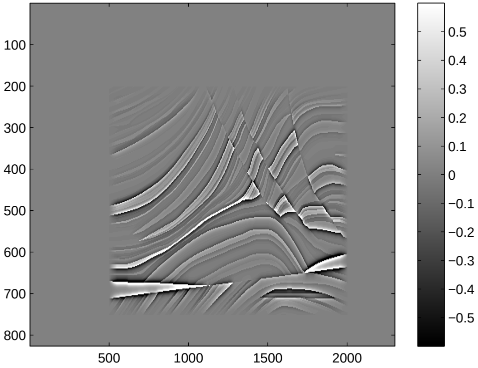

Figure: m_{true}

Migration - Remigration

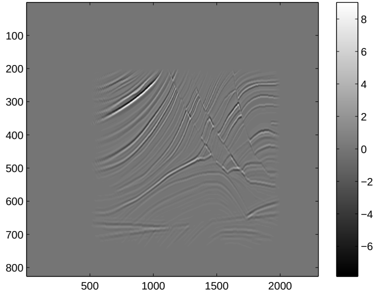

Figure: $m_{mig} = F^{*}d$

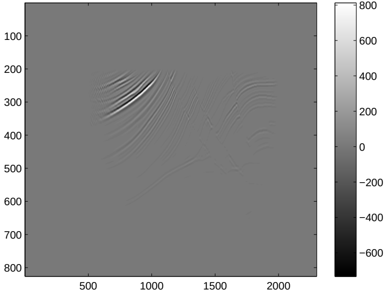

Figure: $m_{remig} = F^{*}Fm_{mig}$

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Scaling $K = 1$

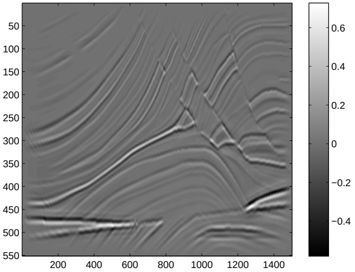

Figure: m_{inv} with $K = 1$

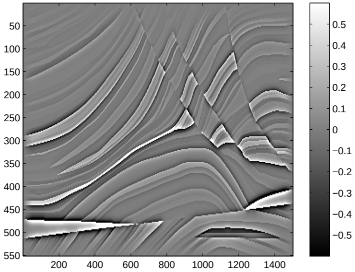

Figure: m_{true}

Scaling $K = 5$

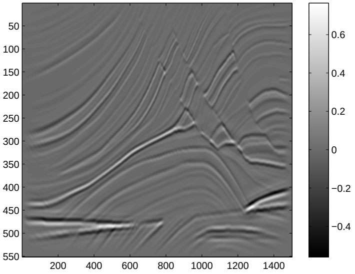

Figure: m_{inv} with $K = 5$

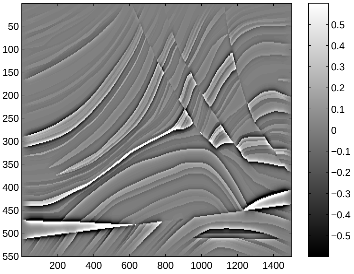

Figure: m_{true}

Difference between $K = 1$ and $K = 5$

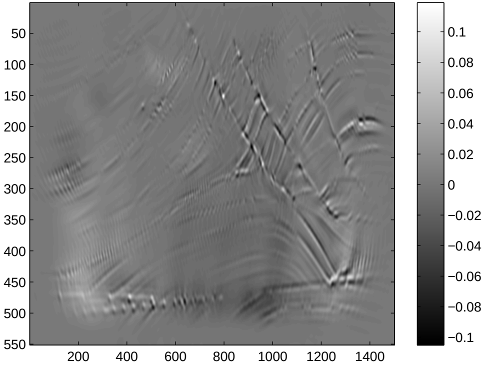

Figure: Difference between $K = 1$ and $K = 5$

Plaid Model I

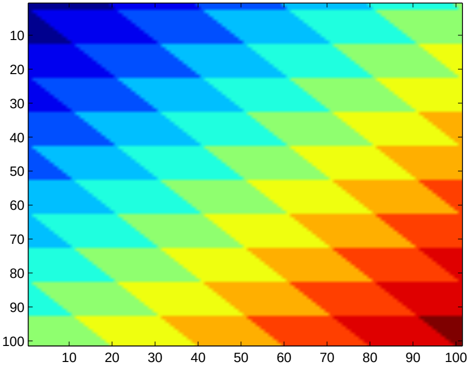

Figure: Plaid Data = b

Plaid Model II

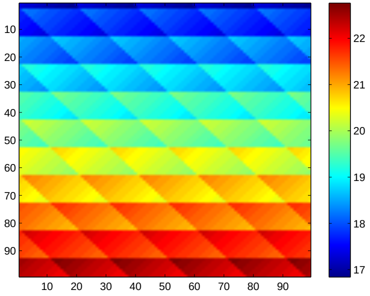

Figure: $Ab = Q[\cos^2(\theta)]b$

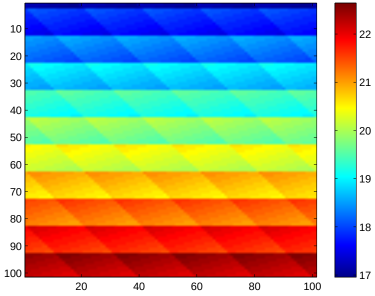

Figure: $A^2b = Q[\cos^2(\theta)]Ab$

Results I

$$c_1 = \underset{c_1 \in \Psi DO}{\textit{argmin}} \|c_1 A b - A^2 b\|^2 \Rightarrow im_{inv} = c_1 b \approx Ab$$

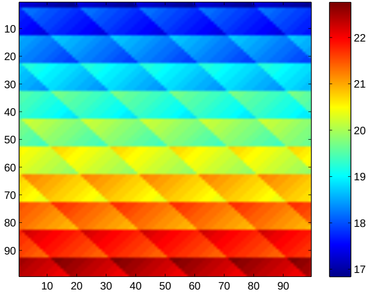

Figure: True Image: Ab

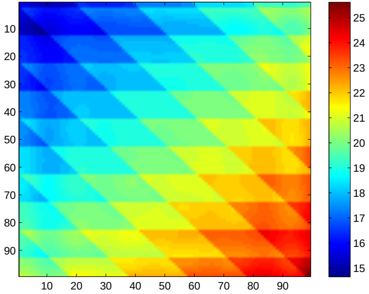

Figure: Inverted Image $K = 1$: $c_1 b$

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Results II

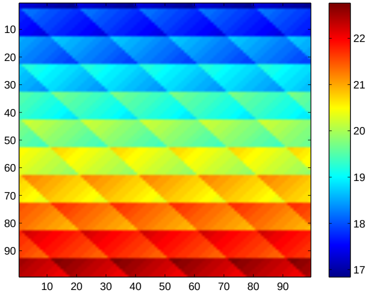


Figure: True Image: Ab

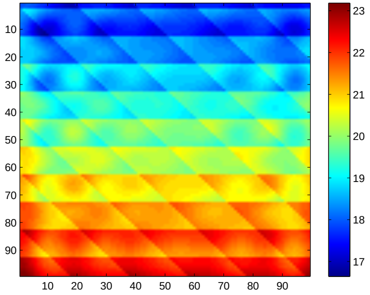


Figure: Inverted Image $K = 5$: $c_1 b$

Error

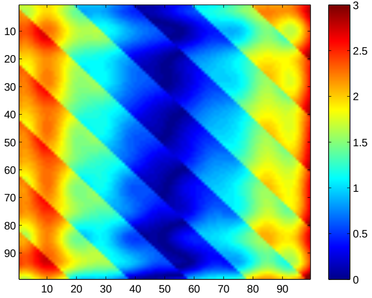


Figure: Error for $K = 1$

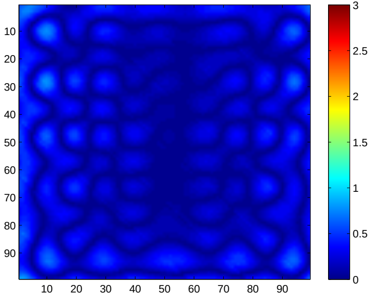


Figure: Error for $K = 5$

Applications

- When m_0 is a good approximation:
  - Fast solution of the Linear Inverse Problem
  - Constant density acoustics
- When m_0 is not a good approximation:
  - View the linear problem as a Newton step
  - Preconditioning of iterative methods (Herrmann et al. 2008)
- Future Work:
  - Extend the method to 3D (spherical harmonics)

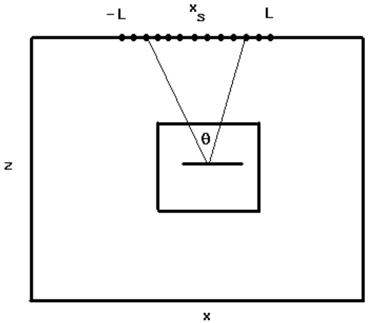
Multi-parameter Case

Example: Variable density acoustics, impedance and density. Formally the same, solve

$$Nx = b$$

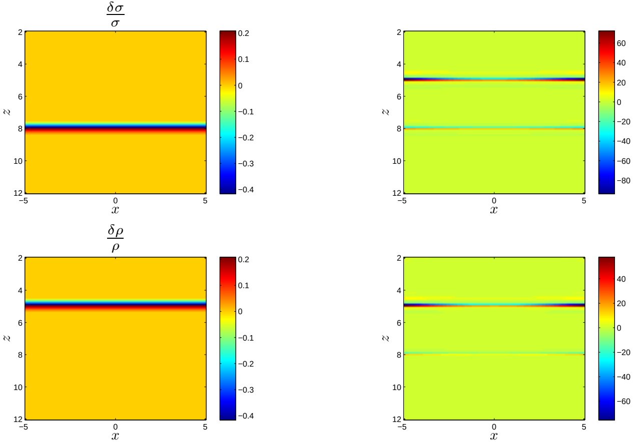
- N is a 2×2 matrix of pseudodifferential operators
- $b = F^* d$ consists of two images, one for each parameter

Geometry



N may be calculated analytically in the case of variable density acoustics.

The Challenge: Separation



True model: x

Mig images: b

Operator Krylov

Generalization of one parameter case, to solve

$$Nx = b,$$

where $N = F^*F$ and $b = F^*d \in \textit{Range}(N)$.

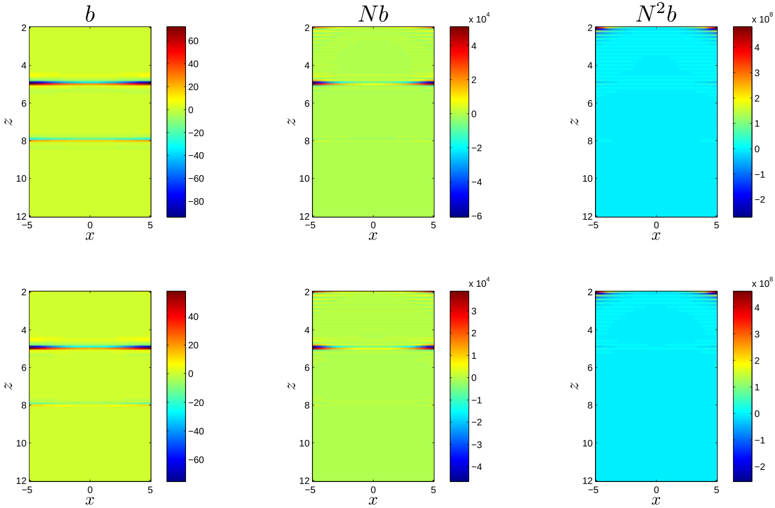
Given b, Nb and N^2b. Compute c_1, c_2:

$$\{c_1, c_2\} = \underset{c_1, c_2 \in \Psi DO}{\textit{argmin}} \|b - c_1\, Nb - c_2\, N^2 b\|^2.$$

Then,

$$x = N^\dagger b \approx N^\dagger(c_1\, Nb + c_2\, N^2 b) \approx c_1 b + c_2\, Nb := x_{inv}$$

What to expect from N



Conditioning of N

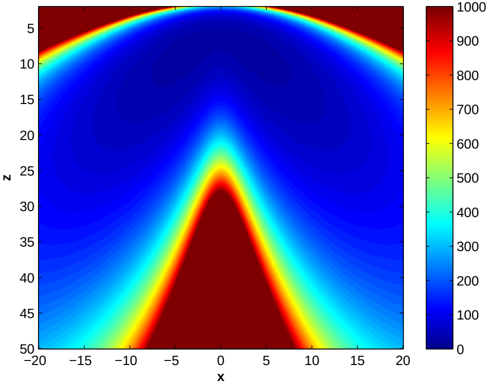


Figure: spatial variation of the condition number of N

Preconditioning

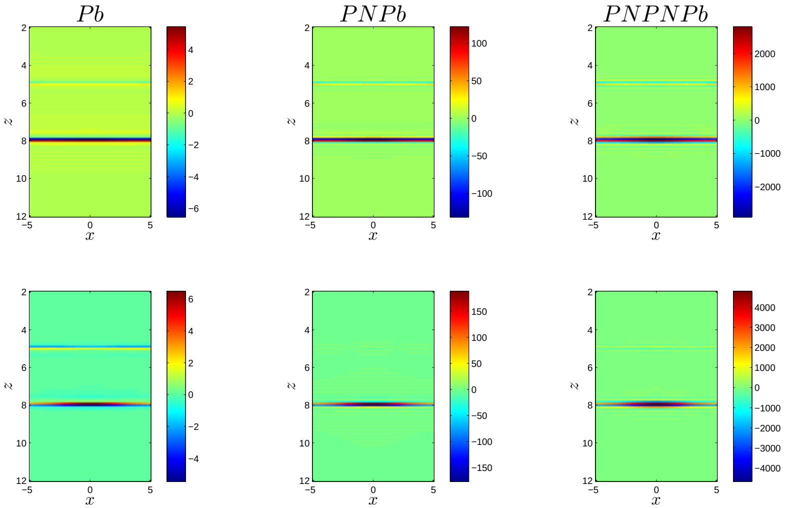
Compute a preconditioner $P \approx N^\dagger$ analytically. Then,

- $b \to Pb$
- $Nb \to PNPb$
- $N^2 b \to PNPNPb$

Compute operator krylov:

- $\{c_1, c_2\} = \underset{c_1, c_2 \in \Psi DO}{\text{argmin}} \|Pb - c_1\, PNPb - c_2\, PNPNPb\|^2$
- $x = N^\dagger b = (PN)^\dagger Pb \approx c_1\, Pb + c_2\, PNPb := x_{inv}$

Preconditioned Images



Results

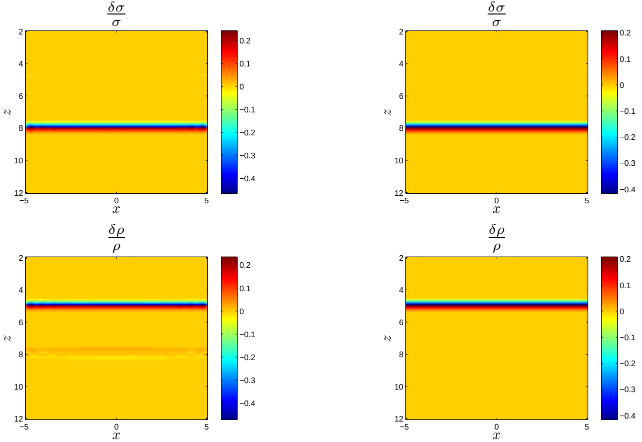


Figure: Comparison between inverted and true image

Conditioning of NP

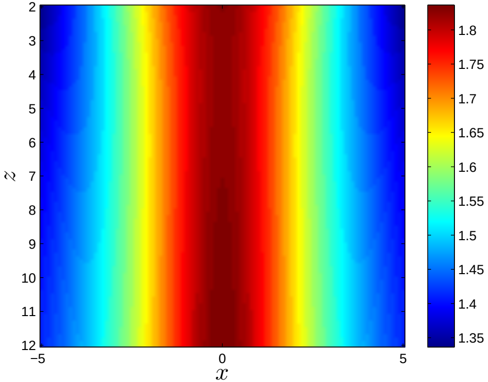


Figure: spatial variation of the condition number of NP

Future Work

- Derive a class of preconditioners for different geometries
- Precondition a RTM code for variable density acoustics
- Apply for variable density acoustics
- Generalize for linear elasticity

Summary

- one-parameter case: Pseudodifferential Scaling
  - Fast and reliable solution if m_0 is a good reference model
  - Preconditioning iterative methods when m_0 is not a good reference model
- multi-parameter case: Operator Krylov
  - Necessity of preconditioning for success
  - Apply to variable density acoustics
  - Linear elasticity ...

THANK YOU !

References

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