Differential Semblance Migration Velocity Analysis via Reverse Time Migration: Gradient Computation

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Introduction

- Differential Semblance Velocity Analysis
  - Smoothness
  - Convexity
- Reverse Time Migration
  - Dip limitation
- DSMVA-RTM
Acoustic wave equation with constant density

\[
\left( \frac{1}{c^2(x)} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) p(x, t; x_s) = f(t) \delta(x - x_s)
\]

\(x\) = position vector
\(x_s\) = position of the point source
\(c(x)\) = velocity
\(p(x, t; x_s)\) = pressure
Source time function \(f(t) = \delta(t)\)
Two-way wave equation operator \(L := \frac{1}{c^2(x)} \frac{\partial^2}{\partial t^2} - \nabla^2\)
Wavefields

Source Wavefields (forward in time):

\[ \mathbf{LS}(x, t; x_s) = \delta(t)\delta(x - x_s) \]

\[ S \equiv 0 \text{ for } t < 0 \]

Receiver Wavefields (backwards in time):

\[ \mathbf{LR}(x, t; x_s) = \int d(x_r, t; x_s)\delta(x - x_r) \, dx_r \]

\[ R \equiv 0 \text{ for } t > t_{max} \]
Write

\[ c(\mathbf{x}) = \nu(\mathbf{x}) + \delta \nu(\mathbf{x}). \]

Forward Born Modeling:

\[ F[\nu] \delta \nu = \delta S|_{surface} \]

Adjoint of Forward Modeling:

\[ F^* d = I \]
RTM formula and DSMVA objective function

Migration formula:

\[ I(x, h) = \int S(x + h, t; x_s)R(x - h, t; x_s) \, dx_s \, dt \]

DSMVA-RTM objective function:

\[ J[\nu] = \frac{1}{2} \| PI \|^2 \]
Gradient Computation

For \( u = P^*P_l, \) \( DI^*u \) gives the gradient.
Introducing \( g_r \) solving

\[
Lg_r(x, t; x_s) = \int R(x - 2h, t; x_s)u(t - h, h) \, dh \\
g_r \equiv 0 \text{ for } t > t_{max}, \text{BC}
\]

Introducing \( g_s \) solving

\[
Lg_s(x, t; x_s) = \int S(x + 2h, t; x_s)u(t + h, h) \, dh \\
g_s \equiv 0 \text{ for } t < 0, \text{BC}
\]

The gradient of DSMVA-RTM is

\[
\nabla_v J = \int \frac{2}{v^3} \left( \frac{\partial^2 S}{\partial t^2} g_r + \frac{\partial^2 R}{\partial t^2} g_s \right) \, dx_s \, dt
\]
Inversion procedure

Input initial velocity $v$

Fwd – S, Bwd – R
CrossCorrelate – I

Compute objective function $J = |P|^2$

Fwd in time – gs, Bwd in time – gr

Compute gradient

Update $dv$ by gradient-based iterative method

$v = v + dv$
Comparison between one-way and two-way WEMVA

One-way wave equation migration:
- Pros: low computational cost
- Cons: dip limitation

Two-way wave equation migration:
- Pros: no dip limitation
- Cons: high computational cost
- Cons: amplitude correction required
Future Work

- Amplitude Correction (see Rami’s work)
- Modification of existing RTM code based on SEAMX, TSOpt to compute objective function and to verify its convexity
- Future modification to create gradient
- Comparison with DS based on downward continuous extrapolation
Perturbation of objective function:

\[ \delta J = \frac{1}{2} \langle \delta (PL), PL \rangle + \frac{1}{2} \langle PL, \delta (PL) \rangle \]

\[ \Rightarrow \delta J = \langle \delta c, \text{Re}((DI)^* P^* PI) \rangle \]

Gradient of objective function:

\[ \nabla_c J = \text{Re}\{(DI)^* (P^* PI)\} \]

For arbitrary \( u(x, h) \), we have

\[ \langle \delta I, u \rangle = \langle DI \delta c, u \rangle = \langle \delta c, DI^* u \rangle \]

When \( u = P^* PI \), \( DI^* u \) gives the gradient.
\langle \delta l, u \rangle = \int \overline{\delta l} u(x, h) \, dx \, dh
\quad = \int \delta S(x + h, x_s, t) \overline{R}(x - h, x_s, t) u(x, h) \, dx \, dx_s \, dt \, dh
\quad + \int S(x + h, x_s, t) \delta \overline{R}(x - h, x_s, t) u(x, h) \, dx \, dx_s \, dt \, dh
\quad = \int \delta S(x, x_s, t) \{ \int \overline{R}(x - 2h, x_s, t) u(x - h, h) \, dh \} \, dx \, dx_s \, dt
\quad + \int \delta \overline{R}(x, x_s, t) \{ \int S(x + 2h, x_s, t) u(x + h, h) \, dh \} \, dx \, dx_s \, dt
Introducing $g_r$ solving

$$L_{g_r}(x, t; x_s) = \int R(x - 2h, t; x_s)u(t - h, h) \, dh$$

$$\int_\Sigma \nabla S_{g_r} - S\nabla g_r = 0$$

$$\frac{\partial S}{\partial t} g_r - S\frac{\partial g_r}{\partial t} (x, 0; x_s) = 0$$

$$\frac{\partial S}{\partial t} g_r - S\frac{\partial g_r}{\partial t} (x, t_{max}; x_s) = 0$$

Introducing $g_s$ solving

$$L_{g_s}(x, t; x_s) = \int S(x + 2h, t; x_s)u(t + h, h) \, dh$$

$$\int_\Sigma \nabla R_{g_s} - R\nabla g_s = 0$$

$$\frac{\partial R}{\partial t} g_s - R\frac{\partial g_s}{\partial t} (x, 0; x_s) = 0$$

$$\frac{\partial R}{\partial t} g_s - R\frac{\partial g_s}{\partial t} (x, t_{max}; x_s) = 0$$
Then,

\[ \langle \delta l, u \rangle = \int (\delta S L g_r + \delta R L g_s) \, dx \, dx_s \, dt \]

\[ = \int ((L \delta S) g_r + (L \delta R) g_s) \, dx \, dx_s \, dt \]

\[ = \int \frac{2 \delta c}{c^3} \frac{\partial^2 S}{\partial t^2} g_r + \frac{2 \delta c}{c^3} \frac{\partial^2 R}{\partial t^2} g_s \, dx \, dx_s \, dt \]

Thus

\[ \nabla_c J = \int \frac{2}{c^3} \left( \frac{\partial^2 S}{\partial t^2} g_r + \frac{\partial^2 R}{\partial t^2} g_s \right) \, dx_s \, dt \]