Seismic Inversion: Progress and Prospects

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SEG 07

Introduction

Focus: recent developments and near-term prospects in waveform inversion (WI) for *velocity*, and relation to migration velocity analysis (MVA).

- Optimization formulations of MVA using full waveform data
- Merger of WI and MVA

Agenda

1. Waveform Inversion
2. MVA and Semblance
3. MVA via Optimization
4. Extended Modeling: MVA + WI
5. Prospects

Waveform Inversion

The usual set-up:

- $\mathcal{M}$ = a set of *models*;
- $\mathcal{D}$ = a Hilbert space of (potential) data;
- $\mathcal{F} : \mathcal{M} \to \mathcal{D}$: modeling operator or "forward map".

Waveform inversion problem: given $d \in \mathcal{D}$, find $v \in \mathcal{M}$ so that $\mathcal{F}[v] \simeq d$.
$\mathcal{F}$ can incorporate *any physics* - acoustics, elasticity, anisotropy, attenuation,.... (and v may be more than velocity...).

Least squares formulation: given $d \in \mathcal{D}$, find $v \in \mathcal{M}$ to minimize

$$J_{LS}(v,d) = \frac{1}{2}\|d - \mathcal{F}[v]\|^2 \equiv \frac{1}{2}(d - \mathcal{F}[v])^T(d - \mathcal{F}[v])$$

Has long and productive history in geophysics - but not in reflection seismology.

Problem size ⇒ Newton and relatives ⇒ find local minima. BUT....

Waveform Inversion

J_{LS} has lots of useless local minima, for typical length, time, and frequency scales of exploration seismology. $\Rightarrow$ least squares waveform inversion with Newton-like iteration "doesn't work" (Gauthier 86, Kolb 86, Santosa & S. 89, Bunks 95, Shin 01, Shin and Min 06, many others - see Chung SI 2.4).

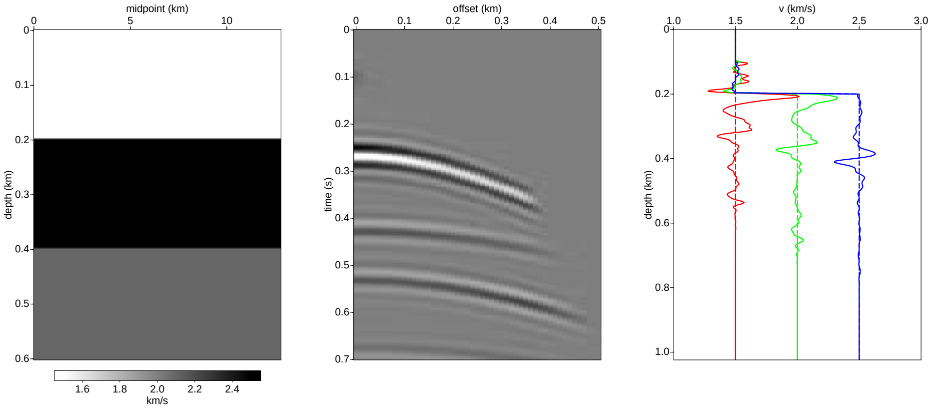
Layered medium, Born approximation, plane wave analysis $\Rightarrow$ incidence angles $\leq 60^\circ$, gradient-based method initialized at ref. vel. $v_0 \neq \langle v \rangle$ cannot update mean velocity over depth interval $[0, z_d]$ if

$$f_{\min} > \frac{v_0}{2z_d}$$

Typical shallow sediment imaging: $v_0 \simeq$3 km/s, $z_d = 5$ km $\Rightarrow$ to recover long-scale structure must have significant energy at $f_{\min} \simeq 0.3 Hz$

If not present (energetics - Ziolkowski 93) and/or filtered from data, then spurious minima must exist (and will be found by gradient-based optimization if $\langle v_{\text{init}} \rangle \neq \langle v \rangle$)!

Waveform Inversion



Left: Layered model. Middle: response to point source in center, 4-10-30-40 Hz bandpass wavelet. Right: LS inversions, dashed=initial, solid=final. Quasi-Newton iteration terminated when gradient reduced by 10^{-2}.

Waveform Inversion

Caveats: least squares WI works

- with synthetic data containing very low frequencies ($<< 1$ Hz): Bunks 95, Shin and Min 06.
- for basin inversion from earthquake data: target of several major efforts. QuakeShow (Ghattas), SpecFEM3D (Tromp, Komatisch), SPICE (Käser, Dumbser). Typical $z_d = 20$ km, $f_{\rm min} = 0.1 Hz$, $\langle v_s \rangle =$ 4 km/s - just OK! Will be done, in 3D, in near future.
- for *transmission* waveform inversion (cf Gauthier 86) with good initial v from traveltime tomography (plus other tweaks) - Pratt 99, Brenders TOM 1.5.

Agenda

Surface-oriented vs. Depth-oriented

MVA based on prestack depth migration - two major variants. Both produce *image volume* $I(\mathbf{x}, \mathbf{h})$ depending on image point $\mathbf{x}$, half-offset $\mathbf{h}$.

(I) Surface oriented: $\mathbf{h} = 0.5$(receiver - source), usually computed by diffraction sum ("Kirchhoff common offset migration"); binwise: offset bin $I(\cdot, \mathbf{h})$ depends only on data traces with offset $\mathbf{h}$.

(II) Depth oriented: $2\mathbf{h}$ = difference between subsurface scattering points, $\mathbf{x}$ = their midpoint. Every point in image volume depends on all data traces. Has diffraction sum rep, but usually computed by one-way (shot profile or DSR) or two-way (RTM) wave extrapolation.

Recent advance: understanding the difference.

Semblance

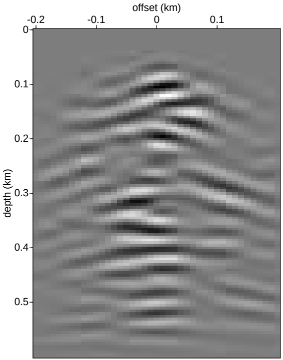
Semblance condition: expresses consistency between data, velocity model in terms of image volume.

(I) Surface oriented: velocity-data consistency when $I(\mathbf{x},\mathbf{h})$ independent of $\mathbf{h}$ (at least in terms of phase), i.e. image gathers are flat.

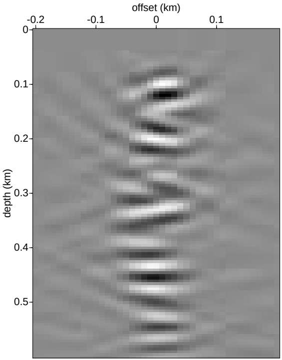
(II) Depth oriented: velocity-data consistency when $I(\mathbf{x},\mathbf{h})$ concentrated near $\mathbf{h}=0$, i.e. image gathers are focused [or flat, when converted to scattering angle].

Main principle of MVA: adjust velocity until image volume satisfies semblance condition.

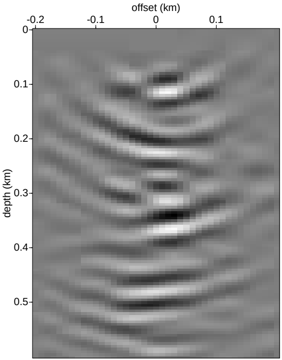
Semblance



OIG, x=1 km: vel 10% low



Offset Image Gather, x=1 km



OIG, x=1 km: vel 10% high

RTM space shift image gathers ($I_D(\mathbf{x},\mathbf{h})$) from velo model $v + \delta v$, $v =$ const., $\delta v =$ randomly distributed point diffractors. Left to Right: migration velocity = 90%, 100%, 110% of true velocity.

Artifacts

Nolan & S. 97, Stolk & S. 04, deHoop & Brandsberg-Dahl 03: multipathing (multiple rays connecting source, receiver, and image points, caustics) leads to kinematic artifacts in surface oriented image volume.

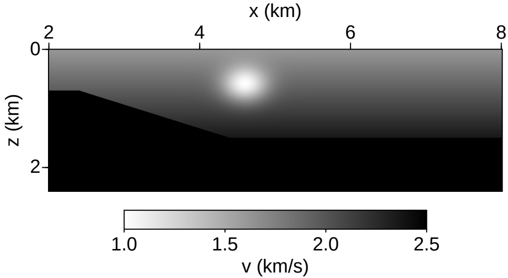
Artifact = coherent event in wrong place, of strength comparable to correct events.

Consequences for velociy analysis: artifacts $\Rightarrow$ semblance condition not satisfied even if velocity is correct!: Nolan and S. 97, Xu TOM 1.4.

Stolk & deHoop 01, S. 02, Stolk 05: depth-oriented image volume generally free of artifacts, even with strong multipathing.

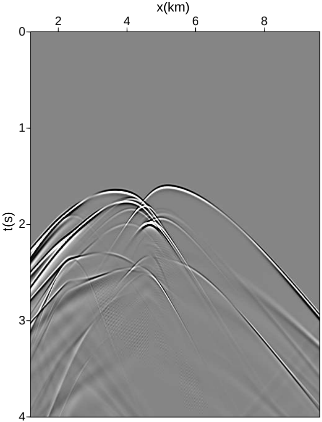
So the two types of image volume are not even kinematically equivalent! Accounts for perceived superiority of "wave equation migration".

Artifacts



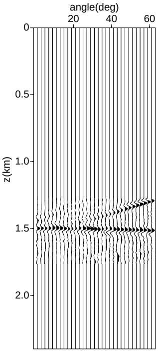
Velocity model after Valhall field, North Sea. Note sloping reflector at left, large low-velocity lens (modeling gas accumulation) in center. Both tend to produce multipathing. (Thanks: M. de Hoop, A. Malcolm)

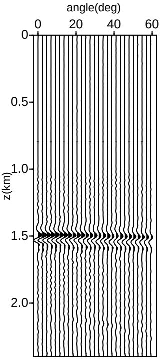
Artifacts



Typical shot gather over center of model, exhibiting extensive multipathing.

Artifacts





Angle common image gathers at same horizontal position from surface-oriented (Kirchhoff) and depth-oriented (DSR) migrated image volumes. Left: ADCIG from Kirchhoff migration: kinematic artifacts clearly visible. Right: ADCIG from DSR migration: no artifacts!

Agenda

Traveltime vs. Waveform Tomography

Semblance condition leads to two methods for velocity updating:

(I) Depth domain reflection traveltime tomography:

- (auto)pick events in migrated image volume
- backproject inconsistency (eg. residual moveout of angle gather events) to construct velocity update as in standard traveltime tomography.

Used with both surface oriented and depth oriented image volume formation.

Drawback: uses only small fraction of events in typical image volume.

Traveltime vs. Waveform Tomography

(II) Depth domain reflection waveform tomography ("differential semblance"):

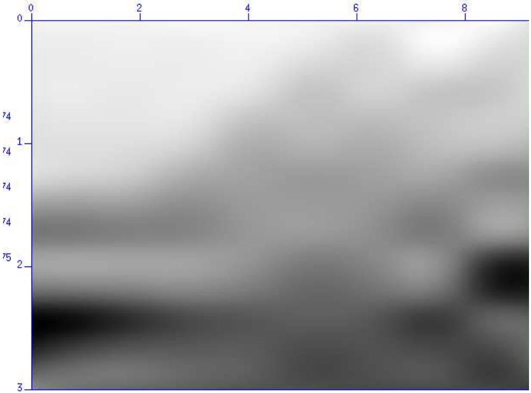
- form measure of deviation of image volume from semblance condition - function of velocity model; all energy not conforming to semblance condition contributes.
- optimize numerically: gradient = backprojection of semblance-inconsistent energy into velocity update.

Also used with both surface and depth oriented image volumes. Recent contributions: Shen 03, 05, Li & S. 05, Foss 06, Albertin 06, Khoury 06, Verm 06, Kabir SVIP 2.3.

Inherently uses all events in data, weighted by strength.

Example: for depth-oriented, minimize $J[v] = \sum |\mathbf{h}I(\mathbf{x},\mathbf{h})|^2$ - penalizes energy at $\mathbf{h} \neq 0$. Apparently: no local mins!

Synthetic Example (Shen SEG 05)



Starting velocity model for waveform tomography. Data: Born version of Marmousi, fixed receiver spread across surface.

Synthetic Example (Shen SEG 05)

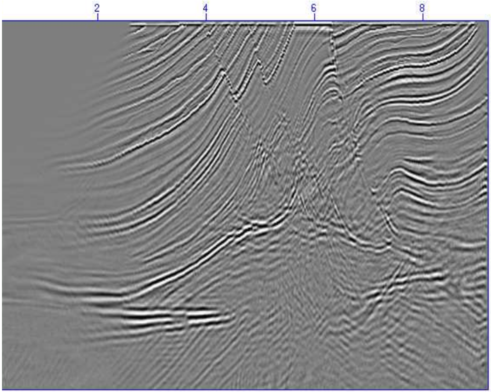
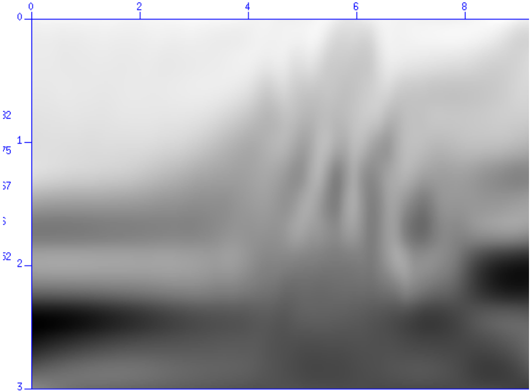


Image ($I_D(\mathbf{x}, \mathbf{H} = 0)$) at initial velocity.

Synthetic Example (Shen SEG 05)



Final velocity (47 iterations of descent method). Note appearance of high velocity fault blocks.

Synthetic Example (Shen SEG 05)

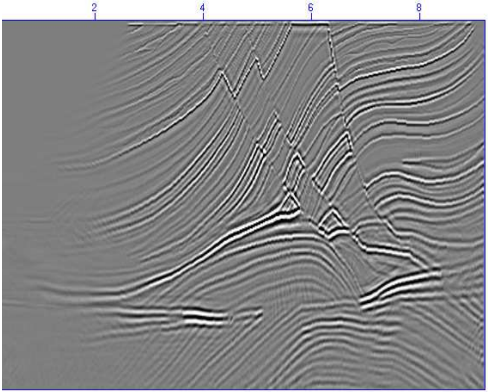


Image ($I_D(\mathbf{x}, \mathbf{H} = 0)$) at final velocity.

Field Example - Trinidad (Kabir SVIP 2.3)

[see extended abstract]

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Extended Modeling

Extended model $\bar{\mathcal{F}} : \bar{\mathcal{M}} \rightarrow \mathcal{D}$, where $\bar{\mathcal{M}}$ is a *bigger model space*= models depending on $\mathbf{x}$ *and* $\mathbf{h}$, i.e. $\bar{v}(\mathbf{x}, \mathbf{h})$.

Physical (normal) model identified with extended model - for depth oriented modeling, $v(\mathbf{x}) \mapsto v(\mathbf{x})\delta(\mathbf{h}) = \bar{v}(\mathbf{x}, \mathbf{h})$ (satisfies semblance condition!). Extension property: $\mathcal{F}[v] = \bar{\mathcal{F}}[\bar{v}]$.

Lailly, Tarantola, Claerbout (80's): migration operator (producing image) is *adjoint* or transpose $D\mathcal{F}[v]^T$. *True amplitude* migration is (pseudo)inverse $D\mathcal{F}[v]^{-1}$. Same for extended modeling $\bar{\mathcal{F}}[\bar{v}]$:

$$D\bar{\mathcal{F}}[\chi[v]]^T d(\mathbf{x}, \mathbf{h}) = I(\mathbf{x}, \mathbf{h}), \; D\bar{\mathcal{F}}[\chi[v]]^{-1} d(\mathbf{x}, \mathbf{h}) = \delta\bar{v}(\mathbf{x}, \mathbf{h}).$$

MVA and WI

(1) MVA (with true amplitude) solves “partially linearized” problem: find reference velocity v and perturbation δv so that $D\mathcal{F}[v]\delta v \simeq d - \mathcal{F}[v]$.

Proof: successful true amplitude MVA produces image volume satisfying imaging condition and fitting data, i.e. $I(\mathbf{x},\mathbf{h}) \simeq \delta v(\mathbf{x})\delta(\mathbf{h})$...

(2) Nonlinear MVA, or WI based on semblance:

$$\min_{\bar{v}\in\bar{M}} \|h\bar{v}\|^2 \text{ subj } \bar{\mathcal{F}}[\bar{v}] \simeq d$$

- $\mathcal{F}$ can be *any modeling* operator - acoustic, elastic, ... - So: MVA extended to elastic modeling with multiples, for instance.
- For depth-oriented extension, $\bar{\mathcal{F}}$ expresses action at a distance: elastic moduli are nonlocal, stress at $\mathbf{x}+\mathbf{h}$ results from strain at $\mathbf{x}-\mathbf{h}$. So Claerbout's semblance principle is actually Cauchy's no-action-at-a-distance hypothesis! [Thanks: Scott Morton]

Agenda

Conclusions

Takeaway messages of this talk:

- Least squares WI prone to get trapped in useless local minima - avoidance requires either initial velocity estimates good to 0.5 (shortest) wavelength, or longest wavelength exceeding the survey depth.
- MVA: "Kirchhoff" and "Wave Equation" prestack migrations have different kinematic properties.
- MVA via waveform tomography ("differential semblance"), based on semblance condition and numerical optimization, uses all events to constrain velocity updates, much less tendency towards local minima than least squares WI.
- MVA solves a "partially linearized" WI problem based on *extended modeling* - nonphysical degrees of freedom.
- Nonlinear extended scattering = framework for uniting MVA and waveform inversion.

Prospects

- Waveform MVA via RTM and differential semblance - kinematic accuracy, fast linear inversion (SI 2.2, Moghaddam SPMI 3.2).
- Alternatives to reflection waveform tomography / differential semblance: Biondi-Sava 2004, Muharramov TOM 1.2, van Leeuwen SI 2.8.
- Nonlinear inversion via model extension ("nonlinear MVA") including multiple scattering.
- integration of source estimation into inversion (Minkoff & S. 97, Lailly & Delprat 06).

Thanks to

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- All of you, for listening

Extended version of slides : `www.trip.caam.rice.edu`