2006-Present:

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Interval velocity estimation via NMO-based Differential Semblance

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Classical Semblance is equivalent to least squares data fitting and has local maxima.

All stationary points of Differential Semblance are global minimizers.

A recent approach to Differential Semblance has some numerical problems. I proposed an alternative approach to overcome these difficulties.
The simplest acoustic model

The acoustic wave equation where density is considered constant and equal to one, with a point source:

\[
\frac{1}{c^2(x)} \frac{\partial^2 p}{\partial t^2}(x, t; x_s) - \nabla^2 p(x, t; x_s) = f(t)\delta(x - x_s)
\]

\(x\) is the position vector
\(x_s\) is the position of the point source
\(f(t)\) is the source time function
\(c(x)\) is the particle velocity
\(p(x, t; x_s)\) is the pressure
Forward map: \( S[c] = p|_{Y=(x_r,t;x_s)} \) (predicted seismic data)
\( x_r \) is the receiver position and \( x_s \) is the source position.

Inverse problem: given observed seismic data \( d \), find velocity field \( c \) so that

\[
S[c] \approx d
\]

The inverse problem is large scale and nonlinear.
Write $c = v(1 + r)$, then $\delta p(x, t; x_s)$ satisfies

$$\frac{1}{v^2(x)} \frac{\partial^2 \delta p}{\partial t^2}(x, t, x_s) - \nabla^2 \delta p(x, t, x_s) = \frac{2r(x)}{v^2(x)} \frac{\partial^2 p}{\partial t^2}(x, t, x_s)$$

Linearized forward map: $F[v] r = \delta p|_{Y=(x_r, t; x_s)}$

- $v$ smooth, $r$ oscillatory $\Rightarrow F[v] r$ approximates primary reflections
- Error consists of multiple reflections.
- No mathematical results are known which justify these observations in any rigorous way.
Convolutional model for layered media

(Theoretical derivation by Winslow 2000, based on linearization and high frequency approximation)

\[ F[v]r(t, h) = f(t) \ast r(T_0(t, h)) \]

\( h \) is the half offset
\( t_0 \) is the traveltime at zero offset
\( f(t) \) is the source time function
\( r(t_0) = \frac{\delta v(t_0)}{v(t_0)} \)
\( T_0(t, h) \) is a change of variables function. It is the inverse function of \( T(t_0, h) \) (Hyperbolic approximation to two-way traveltime)

Ideal case: \( f(t) = \delta(t) \). Then

\[ F[v]r(t, h) = r(T_0(t, h)) \]
Classical Semblance is equivalent to least squares data fitting

Turn the linearized inverse problem into a least squares problem: given CMP data \(d\), find \(v\), \(r\) so that

\[
\min J[v, r] = \|F[v]r - d\|^2
\]

\[
= \int \int dt \, dh \, (r(T_0(t, h)) - d(t, h))^2
\]

\[
= \|d\|^2 + \int \int dt_0 \, dh \, \frac{\partial T}{\partial t_0}(t_0, h) \times (r(t_0)^2 - 2r(t_0)d(T(t_0, h), h))
\]

\[
= \|d\|^2 + \int dt_0 \, j(t_0) r(t_0)^2 - 2 \int dt_0 \, r(t_0) \int dh \, \frac{\partial T}{\partial t_0}(t_0, h) \times d(T(t_0, h), h)
\]
Then
\[ J[v, r] = \| d \|^2 + < jr, r > - 2 < r, Sd >, \]
where \( Sd \) is the weighted stacking
\[ Sd[v](t_0) = \int dh \frac{\partial T}{\partial t_0}(t_0, h)d(T(t_0, h), h), \]
and
\[ j[v](t_0) = \int dh \frac{\partial T}{\partial t_0}(t_0, h). \]

Since \( Sd, j \) only depend on \( v \), then if \( v \) is fixed, we can get the optimal \( r = \frac{1}{j} Sd \)

\[ \min J[v, r] = \| d \|^2 - \left< \frac{1}{j} Sd, Sd \right> \]
\[ \iff \max J_S[v] = \left< \frac{1}{j} Sd, Sd \right> \]

Then the classical semblance turns out to be equivalent to the least squares data fitting.
Introduce nonphysical model \( r(t_0, h) \). Physical model satisfies constraint \( \frac{\partial r}{\partial h} = 0 \).

\[
\min J[v, r] = \int \int dt \; dh \; (r(T_0(t, h), h) - d(t, h))^2
\]

\[
= \int \int dt_0 \; dh \; \frac{\partial T}{\partial t_0}(t_0, h)(r(t_0, h) - d(T(t_0, h), h))^2
\]

The objective function is very easy to minimize without constraint: \( r(t_0, h) = d(T(t_0, h), h) \). Then the model is infeasible since

\[
\left\| \frac{\partial r}{\partial h} \right\|^2 > 0.
\]

To reduce the infeasibility: \( \min_v \left\| \frac{\partial r}{\partial h} \right\|^2 \)

Differential Semblance objective function is

\[
J_{DS}[v] = \left\| \frac{\partial}{\partial h} d(T(t_0, h), h) \right\|^2
\]
Comparison between Classical Semblance and Differential Semblance

(a) Classical Semblance (Chauris, 2001):

Figure: Classical Semblance cost function
(b) Differential Semblance (Chauris, 2001):

Figure: Differential Semblance cost function
Motivation for DS

- **DS**
  - All stationary points of DS are global minimizers. (Symes, TR99-09)
  - DS uses gradient method to solve the optimization problem.

- **OLS**
  - Output Least squares objective function has local minima and these local minimizers are far from any global minimizer.
  - Gradient methods are unreliable.
  - Computational cost for global optimization methods is high.
A recent approach to DS (Jintan Li, 2007)

- **Objective function:**

\[ J[v] = \left\| \frac{\partial}{\partial h} d(T(t_0, h), h) \right\|^2 \]

- \( J[v] \) and \( \nabla J[v] \) have to be computed numerically. But grid points in the \( t_0 \) axis are not mapped to grid points in the \( t \) axis.

\[ d(t, h) \rightarrow d(T(t_0, h), h) \]

- Local cubic interpolation is needed to compute the oscillatory data \( d \) which will cause error.
Discretization

\( t_{0j} = j \Delta t_0, \ h_i = i \Delta h, \)

\[ d( T(t_{0j}, h_i), h_i ) \simeq d^{\text{int}}( T(t_{0j}, h_i), h_i ) \]

\[ \frac{\partial}{\partial h} d( T(t_{0j}, h_i), h_i ) \simeq \frac{1}{\Delta h} ( d^{\text{int}}( T(t_{0j}, h_{i+1}), h_{i+1}) - d^{\text{int}}( T(t_{0j}, h_i), h_i ) ) \]

Define the discrete moveout derivative operator:

\[ M[v] d(t_{0j}, h_i) = \frac{1}{\Delta h} ( d^{\text{int}}( T(t_{0j}, h_{i+1}), h_{i+1}) - d^{\text{int}}( T(t_{0j}, h_i), h_i ) ) \]

Thus the discrete objective function

\[ J[v] = \sum_{ij} | M[v] d(t_{0j}, h_i) |^2 \]
Figure: Original CDP

Figure: Corrected CDP
**Figure:** Instability of DSVA velocity estimates
Alternative approach to DS

\[ J[\nu] = \left\| \left( p \frac{\partial d}{\partial t} + \frac{\partial d}{\partial h} \right)(t, h) \right\|^2 \]

where slowness

\[ p(t, h) = \frac{\partial T}{\partial h}(T_0(t, h), h) \]

This approach involves interpolation of smooth function \( p(t, h) \) instead of oscillatory data \( d(t, h) \), then the interpolation error in \( p \) is smaller than the previous approach. Then the noises in \( J \) and \( \nabla J \) are smaller. Thus this optimization is more stable.
Recall

\[ J[\nu] = \left\| (p \frac{\partial d}{\partial t} + \frac{\partial d}{\partial h})(t, h) \right\|^2 \]

Since \( t \) is oversampled, we don’t have problem in computing \( \frac{\partial d}{\partial t} \). Since offset \( h \) is often undersampled, \( \frac{\partial d}{\partial h} \) has to be calculated carefully.

How to deal with \( \frac{\partial d}{\partial h} \)?
If $v_0 - \Delta v \leq v \leq v_0 + \Delta v$, then

$$p \frac{\partial d}{\partial t} - \frac{\partial d}{\partial h} \simeq N[v_0]d + (p - p_0) \frac{\partial d}{\partial t}$$

where operator $N$ has been defined by

$$N[v]d(t_j, h_i) = M[v]d(T_0(t_j, h_i), h_i)$$

Then

$$(p \frac{\partial d}{\partial t} - \frac{\partial d}{\partial h})(t_j, h_i) \simeq N[v_0]d(t_j, h_i) + (p - p_0) \frac{\partial d}{\partial t}(t_j, h_i)$$

This will be accurate if $f_{max} \leq f(\Delta v)$
Summary:

- CS vs. DS
- Recent approach vs. my approach

Future works:

- Justify the proposed strategy.
- Implement the algorithm
Thank you