Differential Semblance Velocity Analysis using Kirchhoff Common Offset PSDM

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Basic Kirchhoff Imaging

$$I(\mathbf{x}, h) = \sum_{(r,s) \in B(h)} A(\mathbf{x}_r, \mathbf{x}_s, \mathbf{x}) d(r, s, T(\mathbf{x}_r, \mathbf{x}) + T(\mathbf{x}_s, \mathbf{x}))$$

- $I(\mathbf{x}, h)$ = prestack image volume. $\sum_h I(\mathbf{x}, h)$ = image.
- $B(h) = \{(r, s) : h \leq |\mathbf{x}_r - \mathbf{x}_s| < h + \Delta h\}$ = source-receiver index pairs for offset bin $[h, h + \Delta h)$.
- $d(r, s, t)$ = traces
- $T(\mathbf{x}, \mathbf{y})$ = (oneway) traveltime from $\mathbf{x}$ to $\mathbf{y}$.
- $A(\mathbf{x}_r, \mathbf{x}_s, \mathbf{x})$ = amplitude field of asymptotic inverse theory or approximation.
- To avoid kinematic artifacts in prestack image volume $I(\mathbf{x}, h)$, must rule out multipathing (Nolan & S 97; Stolk & S 04).

Differential Semblance

Simple formulation of objective: minimize over v

$$J[v;d] = \frac{1}{2}\sum_{\mathbf{x}}\sum_{h}|D_h I(\mathbf{x},h)|^2$$

where $D_h f(h) = (f(h+\Delta h) - f(h))/\Delta h$ is forward h-difference operator.

Other formulations:

- Chauris & Noble (2001) - scale by total energy in section. Compute traveltimes by ray tracing.
- Mulder & Plessix (2001) - following S., use fwd modeling to make DS operator "near unitary", apply Laplace power to make it bounded. Also use ray tracing.

Based on experience with NMO-based DS, we don't bother with these refinements.

Gradient

Ignoring dependence of amplitude on velocity,

$$\nabla J[v,d] \simeq \sum_h \sum_{(r,s)\in B(h)} (DT[v,\mathbf{x}_r]^T + DT[v,\mathbf{x}_s]^T) A(\mathbf{x}_r,\mathbf{x}_s,\cdot)$$

$$\times [D_h^T D_h I(\cdot,h)] \frac{\partial d}{\partial t}(r,s,T(\mathbf{x}_r,\cdot)+T(\mathbf{x}_s,\cdot))$$

Like an imaging computation with three major differences:

- input is time derivative of trace;
- each trace is spread over isochron then scaled by image 2nd h deriv for its offset bin;
- migrated, scaled trace processed by adjoint traveltime Jacobians for source and receiver points, added.

Tomography → Velocity Analysis

Q. Where have you see adjoint traveltime Jacobians before?

A. Traveltime tomography!

If R_{TT} is (transmission) traveltime error (residual), and $J_T T[v]$ is its mean square, then

$$\nabla J_{TT}[v] = \sum_s DT[v, \mathbf{x}_s]^T R_{TT}(\cdot, \mathbf{x}_s)$$

A classic syllogism:

- Since only models without multipathing can be handled by COM-based VA, might as well used Eikonal Solver (first arrival = all arrivals!);
- JQ's Eulerian tomography package provides adjoint Jacobians;
- therefore Kirchhoff PSDM + Eulerian tomography ⇒ consistant VA package.

Status and Prospects

General plan: construct Kirchhoff framework for general imaging and corresponding gradient, that will accomodate

- amplitude-neglecting acoustic imaging ($A \equiv 1$);
- elastic multiparameter inversion with P-P or 3/4C reflections;
- teleseismic P-S imaging from forward scattering and surface multiples (joint CMG project with Alan Levander);
- standardized interface to Eikonal solver package;
- coupling with RVL optimization;

Current status: framework built, connected with Qian's package. Accuracy tests underway, efficiency improvements planned (don't compute every $T(\cdot, \mathbf{x}_s)$!).