Interval velocity estimation via Kirchhoff-based differential semblance

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Introduction

Velocity analysis updates velocity parameters to flatten primary reflections in image gathers.

Differential semblance velocity analysis (DSVA) optimizes a differential measure of gather flatness:

- mean square of all differences between neighboring traces in prestack image volume (DS). (so tries to flatten everything!)

Theory:

- DS is unique velocity-dependent quadratic form in prestack image volume, which is smooth in data and velocity (Kim & WWS 98, Stolk & WWS 03)
- For version based on NMO, all stationary points are global minima for noise-free primaries only data (WWS 01)
Versions of DSVA

- Hyperbolic or ray-trace NMO correction in CMP and plane wave domains (Carazzone & S. 91, Minkoff & WWS. 97, S. 98)
- Two way reverse time (Versteeg & WWS. 93, Kern & WWS. 94)
- Prestack Kirchhoff migration (Chauris & Noble 01, Mulder & ten Kroode 01, Brandsberg-Dahl & deHoop 03)
- Shot profile and DSR migration (Shen et al. 03, Foss et al. 04)

For reliable results, all variants require

- suppression of coherent noise (multiples, unmodeled converted waves,...)
- velocity models compatible with imaging engine (usually: nonreflecting)
Part I : NMO-based DSVA

When:

- lateral heterogeneity is weak
- coherent noise (e.g. multiple reflection) can be suppressed

NMO-based DSVA interval velocity estimation is:

- kinematically accurate
- robust
- fast (100+ traces /s on modest workstation)
NMO-based DSVA

Simplest possible prestack imaging engine ⇒ \textit{fastest} but accurate only for slow lateral variation of velocity and reflector structure.

Our implementation:

- standard data structure (SEGY) for data input and diagnostic output
- standard preprocessing: filter, mute, various noise suppression methods
- flexible velocity modeling accommodating 1D, 2D, and 3D variation, but enforcing smoothness and upper and lower velocity envelopes (use ”PIGrid”, see TRIP 2005 report)
- effective numerical optimization (limited memory BFGS)
NMO and Derivatives

NMO-based DSVA requires:

• NMO transformation - can get this anywhere (eg. SU)

• Derivative and adjoint derivative of the NMO transformation with respect to velocity

• Implemented hyperbolic moveout using local polynomial interpolation, degree $\geq 3$ to assure sufficiently smooth response to velocity perturbation
Adaptive Choice of Velocity Model Dimension

- First pass: find optimal 1D model. If the gathers are flattened adequately, end; otherwise go to second pass

- Second pass: find optimal 2D model. If the gathers are flattened adequately, end; otherwise go to third pass.

- Third pass: find optimal 3D model, end.
Example

38 CMPs extracted from the Viking Graben data set. Preprocessing: *hyperbolic radon transform multiple suppression*, zero phase band pass filter, mute. (Keys and Foster, 1996)
NMO corrected gathers after DSVA, $v(z)$:
NMO corrected gathers after DSVA, $v(x, z)$:
Detail: gathers from $v(z)$ DSVA
Detail: gathers from $v(x, z)$ DSVA
DSVA : $v(z)$ Interval velocities
DSVA: $v(x, z)$ Interval velocities
Conclusion for Part I

- For regions of mild lateral heterogeneity and data from which coherent noise can be suppressed, NMO-based DSVA may give useful, fast, and robust initial velocity estimates.

- Coherent noise (e.g., multiple reflections) degrades accuracy - for better automatic VA, such noise must be either suppressed from data or included in VA wave propagation model.
Kirchhoff migration:

- A high-frequency approximation of the wave equation

- Image grid can be arbitrarily specified

- Can yield good and reasonable results except when the multipathing occurs

- Low computational cost
Part II: Kirchhoff migration

The image is computed by:

\[ r_{[v]}(x, h) = \int dx_s d_{[v]}(x_s, x_r, T_{[v]}(x_s, x) + T_{[v]}(x, x_r)) \ast A_{[v]}(x_s, h...) \]  

(1)

\( T_{[v]}(x_s, x) \): travel time from source to the reflection point;
\( T_{[v]}(x, x_r) \): travel time from reflection point to receiver

\( A_{[v]}(x_s, h...) \): the specified amplitude, which can vary under different circumstances
Differential semblance

The DS objective is:

\[
J[v, d] = \frac{1}{2} \sum \left| \frac{\partial r}{\partial h} \right|^2 = \frac{1}{2} \sum_{i=0}^{Nh-1} \left\langle \frac{r_{i+1} - r_i}{\Delta h}, \frac{r_{i+1} - r_i}{\Delta h} \right\rangle
\]  

(2)

(image: \( r_i = r(h_i, \tilde{x}) \), offset: \( h_i = h_0 + i\Delta h \))

hence

\[
\delta J = \sum_{i=0}^{Nh-1} \left\langle \frac{r_{i+1} - r_i}{\Delta h}, \frac{\delta r_{i+1} - \delta r_i}{\Delta h} \right\rangle
\]  

(3)

let

\[
f_i = \frac{r_{i+1} - r_i}{\Delta h}
\]  

(4)
\[ \delta J = \sum_{i=0}^{N_h-1} \langle f_i, \frac{\delta r_{i+1}}{\Delta h} \rangle - \sum_{i=0}^{N_h-1} \langle f_i, \frac{\delta r_i}{\Delta h} \rangle \]

\[ = \langle f_{N_h-1}, \delta r_{N_h} \rangle - \langle f_0, \delta r_0 \rangle + \sum_{i=1}^{N_h-1} \langle \frac{f_{i-1} - f_i}{\Delta h}, \delta r_i \rangle \]

\[ = \sum_{i=1}^{N_h-1} \langle \frac{r_i - r_{i-1} - r_{i+1} - r_i}{\Delta h}, \delta r_i \rangle + \langle f_{N_h-1}, \delta r_{N_h} \rangle - \langle f_0, \delta r_0 \rangle \]

\[ = \sum_{i=1}^{N_h-1} \langle \frac{2r_i - r_{i+1} - r_{i-1}}{\Delta h^2}, \delta r_i \rangle + \langle f_{N_h-1}, \delta r_{N_h} \rangle - \langle f_0, \delta r_0 \rangle \]

here

\[ q(\tilde{x}) = \begin{cases} 
  f_0 & i = 0 \\
  \frac{f_{i-1} - f_i}{\Delta h} & i = 1, \ldots, N_h - 1 \\
  f_{N_h-1} & i = N_n 
\end{cases} \]
The derivative of the image

The derivative of the image is:

\[
\delta r_{[v]}(\bar{x}, h) = \int dx_s \frac{\partial d}{\partial t}(x_r, x_s, T_{[v]}(x_s, \bar{x}) + T_{[v]}(x_r, \bar{x})) \ast (\delta T_{[v]}(x_s, \bar{x}) + \delta T_{[v]}(x_r, \bar{x}))
\]

(7)

Bring \( \delta r \) into \( \delta J \), we get:

\[
\delta J = - \sum_{i=0}^{N_h-1} \left( \sum_{\bar{x}} \sum_{x_s} q_i(\bar{x}) \frac{\partial d}{\partial t}(x_r, x_s, T_s + T_r)(\delta T(x, x_s) + \delta T(x_s + 2h, x)) \right)
\]

\[
= - \sum_{\bar{x}} g(\bar{x})(\delta T(x, x_s) + \delta T(x_s + 2h, x)) + \text{boundary conditions}
\]

(8)
The gradient of the objective function

Find an operator $A(y)$ such that

$$\langle A(y)g, \delta v \rangle = \langle g(\tilde{x}), \delta T(y, \tilde{x}) \rangle$$

which means

$$\sum_{\tilde{x}} (A(y)g)(\tilde{x})\delta v(\tilde{x}) = \sum_{\tilde{x}} g(\tilde{x})\delta T(y, \tilde{x})$$

do this for $y = x_s$, $y = x_s + 2h$, then the gradient objective is:

$$\nabla J[v, d] = \sum_{\tilde{x}, h} (A(x_s)g)(x_s, \tilde{x}, h) + (A(x_s + 2h)g)(\tilde{x}, x_s + 2h, h)$$
Velocity representation

Possibilities:

- constant velocity (test first)

- piecewise linear

- B-splines

- PIGrid (“Partially Irregular Grid”) (see 2005 TRIP report)
Travel time and amplitude calculation

Travel time calculation:

Fast sweeping method for both time and adjoint calculation (see Jianliang Qian’s talk)

Amplitude calculation: (later...)
Data preparation

• Coherent noise and direct waves should be sufficiently suppressed or muted

• The data is presorted by increasing 3D offset - will work for both 2D and 3D data with the header words critically defined (sx,gx,gelev,selev,offset....) (use suazimuth & susort)
Implementation

- Define a global image grid \((NX, NY, NZ, OX, OY, OZ, dx, dy, dz)\)
- Loop over each trace in each common offset gather
  - Compute the travel time table and the amplitude on the global image grid
  - Choose the local grid within which all the traveltime \(\leq\) Max data time \((nt \times ns + delrt)\)
  - Calculate the image on this local grid
- Update the common offset image gather
- Compute the derivative of the image gather
- Compute the objective and the adjoint derivative of the objective
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