Agenda, Morning

0845 Welcome and logistical announcements
0900 J.-L. Qian, UCLA: Recent developments in level set methods for traveltime and related computations
0945 C. C. Stolk, U. Twente: Aspects of wave equation imaging
1030 break
1040 E. Dussaud, Explicit extrapolators and common azimuth migration
1110 W. W. Symes and F.-C. Gao, Rice U: HOCIGs and VOCIGs via two-way reverse time migration
1130 E. Dussaud, Rice U: A sparse, bound-respecting parametrization of velocities
1140 W. W. Symes and J. Li, Rice U: NMO-based DSO: implementation and initial noise studies
Agenda, Afternoon

1200 Lunch, Cohen House
1300 E. Dussaud, Rice U: Velocity analysis in the presence of uncertainty
1320 P. Shen, Rice U and Total: Wave equation velocity analysis
1350 W. W. Symes, Rice U: Velocity analysis and nonlinear inverse scattering
1420 Discussion: immediate plans, future directions
1500 Adjournment
NMO-Based DSO

Objectives:

- automatic velocity analysis accounting for mild lateral heterogeneity
- accommodate both 2D and 3D data in standard input format (SEGY)
- produce velocity models in depth with controlled resolution, using PIGrid data structure

Working version: uses hyperbolic traveltimes, estimates isotropic P-wave velocity
NMO-Based DSO - Fundamentals

\( d(t, h, m) = \text{CMP gathers}, h, m = (3D) \text{ half-offset, midpoint}, v = v(z, m) \text{ midpoint dependent interval velocity. NMO = layered medium approximation to migration:} \)

\[
d_{\text{NMO}}[v](t_0, h, m) = d(t[v](t_0, h), h, m)
\]

Differential semblance measures flatness of nmo-corrected CMP:

\[
s[v](t_0, h, m) = \frac{\partial}{\partial h} d_{\text{NMO}}[v](t_0, h, m)
\]

Differential semblance optimization:

\[
\min_v \left\{ J_{\text{DSO}}[v, d] \equiv \sum_{t_0,h,m} |s[v](t_0, h, m)|^2 \right\}
\]
NMO-Based DSO - Implementation

- change of variables $t \rightarrow t_0$ by local cubic interpolation - smooth enough (barely) for differentiation w.r.t. $v$.

- use Fortran for basic numerical kernels. Motivation: availability of automatic differentiation (TAMC) to produce derivatives and adjoints required for optimization.

- kernels wrapped in C++ to produce Standard Vector Library Operator subclasses

- SU and SEP data structures implemented as SVL Space, DataContainer subclasses

- linked to SVL implementation of limited-memory quasi-Newton optimization algorithm to produce final NMOOpt.x driver.

- SU-style self-doc provided.
NMO-Based DSO - Limitations

- Accounts only for isotropic P-wave (or single velocity) moveout
- Accounts only for primary reflection data from (near-)layered structure
- Sensitive to coherent noise: multiple reflections, mode conversions, etc. (see WWS and Gockenbach, SEG 99)

Jintan Li MA project: assess accuracy, ease of use, influence of various types of noise using synthetic and field data
NMO-Based DSO - Future

• will remain a tool for inversion of *primaries only* data - dependent on multiple suppression technology

• anisotropy accommodated through (a) approximate high-order corrections to hyperbolic TT, (b) ray trace TT (also interesting for isotropic case) via eikonal solvers

• multiple modes handled *without mode separation* through *concatenated annihilators* (see TRIP annual report 2000).

• for multiple reflections, we will pursue another path...
HOCIGs and VOCIGs


This talk:

- Fuchun Gao: how to produce offset image gathers using frequency domain two-way migration, and their focussing property when DSR condition holds;
- in order to avoid imaging ambiguity when rays turn, image volume must include nonhorizontal offsets;
- midpoint dip filtering produces artifact-free horizontal and vertical offset CIGs - reduce cost by decimating midpoints, avoid midpoint dip filtering, and still eliminate artifacts;
- Details: paper *Reverse time shot-geophone migration* (“RTSGM”)
**Kinematics**

Phase space description: reflector has *location* \((y_r, y_s)\) and *dip* \((k_r, k_s)\).

Similarly, reflection event in data at location \((x_r, t; x_s)\) and dip \(\omega(p_r, 1; p_s)\). Event slownesses \(p_r, p_s\) determined by data for ”true 3D”, otherwise many data-compatible slownesses (eg. for idealized streamer geometry).

**Kinematic Relation** of S-G modeling/migration: reflection event \((x_r, t; x_s), \omega(p_r, 1; p_s)\) occurs ⇔ reflector exists at \(y_r, y_s, k_r, k_s\) and

- a ray begins at \(x_s\) with takeoff slowness \(p_s\) and reaches \(y_s\) with arrival slowness \(k_s/\omega\), in time \(t_s\);
- a ray begins at \(x_r\) with takeoff slowness \(p_r\) and reaches \(y_r\) with arrival slowness \(k_r/\omega\), in time \(t_r\);
- \(t_s + t_r = t\)
Kinematics

Kinematic relation of S-G modeling/migration
Too many image points!

Note: for any given reflection event in data, many corresponding (double) reflectors: all points on rays from source, receiver with correct total time.

⇒ gross imaging ambiguity

The ”traditional” fix: (1) DSR assumption, i.e. no turning rays; (2) ”sunken offset” vector horizontal
DSR, good \( v \Rightarrow \text{focus at } h = 0 \)

\[
X_s(t_s) = X_r(t_r)
\]

Kinematic relation of S-G modeling/migration + DSR + horizontal offset: NO IMAGING AMBIGUITY (Stolk-deHoop 2001)
Q. Why drop DSR?

A. Because in complex structure, rays turn.

Q. Why drop horizontal offsets? A. Because reflectors structures may be vertical or near-vertical, and then horizontal offset images will be smeared (i.e. ambiguous reflector locations!)

Nonvertical reflector \(\Rightarrow\) total traveltime determines reflection point uniquely when velocity is correct and horizontal offset assumed.

Vertical reflector \(\Rightarrow\) many different (double) reflectors correspond to single physical reflector, all having same traveltimes and horizontal offset.
Nonvertical reflector: $t_r + t_s = t'_r + t'_s$, but depths can only be the same at one point (which must be the physical reflection point, if velocity is correct, by S-deH).
(Near) vertical reflector: $t_r + t_s = t'_r + t'_s$, and depths can be the same at a continuum of points, besides the physical reflection point $\Rightarrow$ reflector is smeared, location ambiguous.
Horizontal and vertical offsets via filtering

Suggested approach (differs from Biondi-Symes 2004): create HO and VO image volumes, then filter in midpoint dip (i.e. in $x, z$, not in $h$): remove near-vertical reflector components from HO volume, near-horizontal reflector components from VO volume.

See paper RTSGM for details.

Difficulty: computation of (HO) image volume

$$I(x, z, h) = \int dt \int dx_s u(x_s, x - h, z, t)v(x_s, x + h, z, t)$$

requires $N_tN_sN_xN_hN_z$ flops - and this can overwhelm the cost of solving the wave equation if all axes are sampled densely!

Reasonable cost requires decimation in midpoint, i.e. compute only a relatively small number of HOCIGs, VOCIGs.
Horizontal and vertical offsets via filtering

Decimated midpoints ⇒ can’t filter in midpoint dip.

Alternate process: *high-cut filter*

- HOCIGs in $z$
- VOCIGs in $x$

Also removes horizontal dips from HOCIGs, vertical dips from VOCIGs, but carried out *per midpoint*, i.e. fixed $x$ for HOCIGs, fixed $z$ for VOCIGs - compatible with decimated midpoints.
Velocity model with velocity increasing with depth, generating turning rays, and vertical reflector.
Example

VOCIGs ($z = 30 \text{ m}, 35 \text{ m}$) are artifact-free - no imaging ambiguity
Example

HOCIG at reflector midpoint has substantial low freq component - *smearing*
Filtered HOCIG at reflector midpoint has horizontal dip / LF components removed.
Focussing property of HO/VO image volume

Regard prestack image as

- filtered HOCIGs + VOCIGs

Then: at correct velocity, energy is focussed at zero offset in both HOCIGs and VOCIGs within an offset “corridor” of width $h_{\text{min}}$ - depends on amount of ray bending, qualitative version of TIC assumption.

Proof: see RTSGM.

Note that apparently image artifacts may exist at large enough offsets, in contrast to DSR case. Future project: illustrate the existence, extent of such artifacts, explore implications for VA.
Velocity Analysis and Nonlinear Inverse Scattering

Overview of past, present, planned TRIP efforts on velocities

- A common framework for VA
- Differential semblance
- Nonlinear inverse scattering via an analogue of standard MVA
- A nonlinear version of S-G MVA
A common framework for VA
Constant Density Acoustic Model

*acoustic potential* \( u(x, t) \), *sound velocity* \( c(x) \) related to pressure \( p \) and particle velocity \( v \) by

\[
p = \frac{\partial u}{\partial t}, \quad v = \frac{1}{\rho} \nabla u
\]

Second order wave equation for potential

\[
\left( \frac{1}{c(x)^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) u(x, t) = w(t) \delta(x - x_s)
\]

plus initial, boundary conditions.

*Forward map*: \( \mathcal{F}[c] \equiv p|_Y \), where \( Y = \{(t, x_r, x_s) : 0 \leq t \leq T, \ldots\} \) is acquisition manifold.
(Partly) linearized inverse scattering

Formally, $\mathcal{F}[v(1+r)] \simeq \mathcal{F}[v] + F[v]r$ where $F[\cdot]$ is linearized forward map defined by

$$
\left( \frac{1}{v(x)^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \delta G(x_s, x, t) = 2 \frac{r(x)}{v^2(x)} \frac{\partial^2 G}{\partial t^2}(x_s, x, t)
$$

$$
F[v]r = \frac{\partial \delta G}{\partial t} \bigg|_Y
$$

- basis of most practical data processing procedures.
- $v$ is no more known than $r$, inverse problem for $[v, r]$ still nonlinear!
- linearization error contains many effects observable in field data, notably multiple reflections, which can be quite strong, or even dominant - so major open issue in this subject is how to go beyond linearization!!!
Extended models

*Extension of $F[v]$ (aka extended model):* manifold $\bar{X}$ and maps $\chi : \mathcal{E}'(X) \to \mathcal{E}'(\bar{X})$, $\bar{F}[v] : \mathcal{E}'(\bar{X}) \to \mathcal{D}'(Y)$ so that

\[
\begin{array}{ccc}
\mathcal{E}'(\bar{X}) & \rightarrow & \mathcal{D}'(Y) \\
\chi & \uparrow & \uparrow \text{id} \\
\mathcal{E}'(X) & \rightarrow & \mathcal{D}'(Y) \\
F[v] & & \\
\end{array}
\]

commutes, i.e.

\[\bar{F}[v] \chi r = F[v] r\]

Extension is “invertible” iff $\bar{F}[v]$ has a right parametrix $\bar{G}[v]$, i.e. $I - \bar{F}[v] \bar{G}[v]$ is smoothing, or more generally if $\bar{F}[v] \bar{G}[v]$ is pseudodifferential (“inverse except for wrong amplitudes”). Also require existence of a left inverse $\eta$ for $\chi$: $\eta \chi = \text{id}$.

**NB:** The trivial extension - $\bar{X} = X$, $\bar{F} = F$ - is virtually never invertible.
Grand Example

The Standard Extended Model: $\tilde{X} = X \times H$, $H =$ offset range.

$$\chi r(x, h) = r(x), \quad \eta \tilde{r}(x) = \frac{1}{|H|} \int_H dh \, \tilde{r}(x, h) \text{ ("stack")}. $$

$\tilde{r} \in \text{range of } \chi \iff \text{plots of } \tilde{r}(\cdot, \cdot, z, h) \text{ ("(prestack) image gathers") appear flat.}$

$$\bar{F}[v]\tilde{r}(x_r, x_s, t) = \frac{\partial^2}{\partial t^2} \int dx \int d\tau \, G(x, x_r, t - \tau)G(x, x_s, \tau) \frac{2\tilde{r}(x, h)}{v^2(x)}$$

(recall $h = (x_r - x_s)/2$)

**NB:** $\bar{F}$ is “block diagonal” - family of operators (FIOs) parametrized by $h$. 
Reformulation of inverse problem

Given $d$, find $v$ so that $\tilde{G}[v]d \in$ the range of $\chi$.

Claim: if $v$ is so chosen, then $[v, r]$ solves partially linearized inverse problem with $r = \eta \tilde{G}[v]d$.

Proof: Hypothesis means

$$\tilde{G}[v]d = \chi r$$

for some $r$ (whence necessarily $r = \eta \tilde{G}[v]d$), so

$$d \simeq \tilde{F}[v] \tilde{G}[v]d = \tilde{F}[v] \chi r = F[v]r$$

Q. E. D.
Application: Migration Velocity Analysis

Membership in range of $\chi$ is *visually evident*

$\Rightarrow$ industrial practice: adjust parameters of $v$ *by hand* (!) until visual characteristics of $\mathcal{R}(\chi)$ satisfied - “flatten the image gathers”.

For the Standard Extended Model, this means: until $\bar{G}[v]d$ is independent of $h$.

Practically: insist only that $\bar{F}[v]\bar{G}[v]$ be pseudodifferential, so adjust $v$ until $\bar{G}[v]d$ is “smooth” in $h$. 
Differential semblance
Automating the reformulation

Suppose $W : \mathcal{E}'(\bar{X}) \to \mathcal{D}'(Z)$ annihilates range of $\chi$:

\[
\begin{array}{c}
\chi & W \\
\mathcal{E}'(X) & \to \mathcal{E}'(\bar{X}) & \to \mathcal{D}'(Z) & \to 0
\end{array}
\]

and moreover $W$ is bounded on $L^2(\bar{X})$. Then

\[
J[v; d] = \frac{1}{2} \|W \bar{G}[v] d\|^2
\]

minimized when $[v, \eta \bar{G}[v] d]$ solves partially linearized inverse problem.

Construction of annihilator of $\mathcal{R}(F[v])$ (Guillemin, 1985):

\[
d \in \mathcal{R}(F[v]) \iff \bar{G}[v] d \in \mathcal{R}(\chi) \iff W \bar{G}[v] d = 0
\]
Annihilators, annihilators everywhere...

For Standard Extended Model, several popular choices:

•

\[ W = (I - \Delta)^{-\frac{1}{2}} \nabla_h \]

(“differential semblance” - WWS, 1986)

•

\[ W = I - \frac{1}{|H|} \int dh \]

(“stack power” - Toldi, 1985)

•

\[ W = I - \chi F[v]^\dagger \bar{F}[v] \]

\[ \Rightarrow \text{minimizing } J[v, d] \text{ equivalent to reduced least squares.} \]
But not many are good for much...

Since problem is huge and data is noisy, only $W$ giving rise to differentiable $v, d \mapsto J[v, d]$ are useful - must be able to use Newton!!! Once again, idealize $w(t) = \delta(t)$.

**Theorem** (Stolk & WWS, 2003): $v, d \mapsto J[v, d]$ smooth $\iff W$ pseudodifferential.

i.e. only *differential semblance* gives rise to smooth optimization problem even with noisy data.

Nonlinear inverse scattering via an analogue of standard MVA
A nonlinear common-shot extension

Simply replace \( F \) by an extension of \( \bar{F} \):

\[
\left( \frac{1}{\bar{c}(x, x_s)^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \right) u(x, t) = w(t)\delta(x - x_s)
\]

plus initial, boundary conditions.

Extended Forward map: \( \bar{F}[\bar{c}] \equiv p|_Y \), where \( Y = \{(t, x_r, x_s) : 0 \leq t \leq T, \ldots\} \) is acquisition manifold.

Extension map: same as for partially linearized common shot extension, i.e. \( \chi[c](x, x_s) = c(x) \).

Q: What replaces the right inverse of the linear extended operator?
Nonlinear common-shot DS

A: Inverse scattering, what else.

A em feasible model \( \bar{c} \) at noise level \( \epsilon \) satisfies

\[
\| \mathcal{F}[\bar{c}] - d \| \leq \epsilon \|d\|
\]

Feasible points are easy to find, for extended models!!!

The natural common-shot differential semblance operator is \( W = \partial / \partial x_s \).

Nonlinear differential semblance, common shot version:

\[
\min_{\bar{c}} \|W\bar{c}\| \quad \text{subj} \quad \|\mathcal{F}[\bar{c}] - d\| \leq \epsilon \|d\|
\]
Nonlinear common-shot DS - implementation

\[
\min_{\bar{c}} \|W\bar{c}\| \quad \text{subj} \quad \|\bar{F}[\bar{c}] - d\| \leq \epsilon \|d\|
\]

Inequality constrained optimization problem, (relatively) easy access to feasible points ⇒ interior point method.

Classic IPM = log-barrier method (Fiacco & McCormack 1967): (1) initialize penalty parameter \(\mu\); (2) while (not satisfied) (i) minimize log-barrier function

\[
\|W\bar{c}\|^2 - \mu \log(\epsilon \|d\|^2 - \|\bar{F}[\bar{c}] - d\|^2)
\]

(ii) when gradient of log-barrier function small enough, reduce \(\mu\) and do it again.

Status: log-barrier method implemented, being tested. Next: couple to already-implemented operator, gradient computations.
A nonlinear version of S-G MVA
Invertible Extensions

Beylkin (1985), Rakesh (1988): if $\|\nabla^2 v\|_{C^0}$ “not too big” (no caustics appear), then the Standard Extension is invertible.

Nolan & WWS 1997, Stolk & WWS 2004: if $\|\nabla^2 v\|_{C^0}$ is too big (caustics, multi-pathing), Standard Extension is not invertible! Not in any version - common offset, common source, common scattering angle,...

Brings the whole program to a screeching halt, unless there are other, inequivalent extensions.
Claerbout’s extension

\[ \chi r(x, h) = r(x)\delta(h), \eta \bar{r}(x) \rightleftharpoons \bar{r}(x, 0) \]  
(Claerbout’s zero-offset imaging condition)

\[ \bar{r} \in \text{range of } \chi \rightleftharpoons \text{plots of } \bar{r}(\cdot, \cdot, z, h) \text{ (i.e. image gathers) appear focused at } h = 0 \]

\[
\bar{F}[v]\bar{r}(x_r, x_s, t) = \frac{\partial^2}{\partial t^2} \int dx \int dh \int d\tau G(x+h, x_r, t-\tau)G(x-h, x_s, \tau) \frac{2\bar{r}(x, h)}{v^2(x)}
\]

This extension is invertible, assuming (i) \( \bar{r}(x, h) = \hat{r}(x, h_1, h_2)\delta(h_3) \) (horizontal offset only) and (ii) ”DSR hypothesis”: waves propagate up and down, not side-ways (”rays do not turn”) [Stolk-DeHoop 2001] and sometimes under more general conditions [RTSGM].
Differential Semblance for Claerbout’s Extension

\[ W \bar{r}(x, h) = h \bar{r}(x, h), \quad J[v, d] = \frac{1}{2} ||W \bar{G}[v]d||^2 \]

Same smoothness properties as DS for Standard Extension.

P. Shen (2004): implementation, optimization via quasi-Newton algorithm, synthetic and field data.

Conclusion: successfully estimates \( v \) in settings (strong refraction) in which Standard Extension based DS fails.
Claerbout’s Extension as a linearization

Write differential equation for $\bar{F}[v]$, by applying wave operator to both sides of integral representation: $\bar{F}[v]r = \delta\bar{u}|_Y$ where

$$\left(v^{-2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\delta\bar{u}(x, x_s, t) = \int_H dh \, 2\bar{r}(x - h, h)v^{-2}(x - h)\frac{\partial^2 G}{\partial t^2}(x - 2h, x_s, t)$$

Observe that this equation describes the linearization of the system

$$V^{-2} \left[\frac{\partial^2 u}{\partial t^2}\right] - \nabla^2 u(x, x_s, t) = w(t)\delta(x - x_s),$$

in which the “velocity” $V$ is an operator: formally,

$$Vw(x) = \int_H dh \, K_V(x - h, h)w(x - 2h)$$

and the linearization takes place at $V$ with $K_V(x, h) = v(x)\delta(h) = \chi v(x, h)$. 

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The Nonlinear Claerbout Extension

That is, you can view Claerbout’s extension of the linearized scattering problem as the linearization of an extension of the original scattering problem:

\[ v^{-2} \left[ \frac{\partial^2 u}{\partial t^2} \right] - \nabla^2 u(x, x_s, t) = w(t)\delta(x - x_s), \]

where \( v \) is the operator of multiplication by the positive function \( v \), versus

\[ V^{-2} \left[ \frac{\partial^2 u}{\partial t^2} \right] - \nabla^2 u(x, x_s, t) = w(t)\delta(x - x_s), \]

with self-adjoint positive \( V \).

This generalized nonlinear scattering problem makes sense: J.-L. Lions showed in the late ’60s how to demonstrate the well-posedness of the initial value problem for operators like the above, with self-adjoint positive operator coefficients [also Stolk 2000].
Extended Inverse Scattering

The extended inverse scattering problem takes the place of the right inverse map $\tilde{G}$ of the linear Claerbout extension: define the extended forward map $\tilde{F}$ by $\tilde{F}[V] = u|_Y$, where $u$ solves

$$V^{-2} \left[ \frac{\partial^2 u}{\partial t^2} \right] - \nabla^2 u(x, x_s, t) = w(t)\delta(x - x_s),$$

plus appropriate initial and boundary conditions. Given a nominal noise level $\epsilon$, an $\epsilon$-solution of the extended inverse scattering problem is a positive self-adjoint $V$ so that

$$\|\tilde{F}[V] - d\| \leq \epsilon\|d\|$$

(1)

In itself, this problem is grossly underdetermined - so use it as a constraint!
Nonlinear Differential Semblance

Natural differential semblance op for Claerbout extension: \( W = \text{multiply by} \ h. \)

The *nonlinear differential semblance* problem is: given \( d, \epsilon \), find \( V \) to minimize

\[
\min_V \| WK_V \|^2 \quad \text{subj} \| \mathcal{F}[V] - d \| \leq \epsilon \|d\|
\]

where \( K_V \) is the distribution kernel of \( V \).

Many open questions to be studied in near future, for instance:

- What is a good class of operators? Must have well-behaved kernels!
- How to sensibly define the norm on \( WK_V \).
- Economical implementation?