Velocity analysis in the presence of uncertainty

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The scale gap

- Primary-based seismic methods can be established theoretically on the basis of the Born approximation (Lailly, 1983):
  - Long scale fluctuations (km for sediments) of the velocity are resolved via **velocity analysis**.
  - Short scale variations (10's m) of the velocity (i.e. the reflectivity) are resolved via **migration**.
- Seismic imaging techniques do **not** appear to estimate the medium scale ($\sim$ 60m - 300m) wavelengths (Claerbout, 1985, Tarantola, 1989).
- The medium scale component is assumed not to influence the seismic response (Lailly and Delprat-Jannaud, 2003).

Proposed work

- Provide a new way to look at this familiar ”fact”.
- Try to understand the influence of the medium scale on the resolution of the long (background velocity) and short (image) scales.
- Take this intermediate scale velocity into account and treat it as a **random process** to model the associated uncertainty (and its consequences).
- **Goal**: Estimate the background velocity by **combining** ideas on time reversal and imaging in randomly inhomogeneous media set forth by Borcea, Papanicolaou et al., and the velocity estimation methods of differential semblance type.

Agenda

- Motivation
- Wave propagation in random media
- Cross-correlation tomography
- Assessing statistical stability
- Conclusions and future work

Three-scale asymptotics

Setting: single scattering approximation, 3-scale asymptotics:

- “Deterministic” reflectors on wavelength scale λ (short-scale component).
- Propagation distance L: scale of the background velocity “macro-model”.
- The medium scale velocity is assumed to randomly fluctuate on the scale a.

Asymptotic assumption: high-frequency regime $\lambda \ll a \ll L$

Wave propagation in random media

- Application to time-reversed acoustics:
  - the refocusing of a time-reversed, backpropagated signal is better in random media than in homogeneous ones.
  - the refocusing property does not depend on the particular realization of the random medium: it is **statistically stable** (in the limit $a/L \to 0$).

- Key to self-averaging: near cancellation of the random phases. Heuristically:

$$G \sim \frac{e^{i(kr+\phi)}}{4\pi r}$$

and since the time-reversed back-propagated field contains $\overline{G}G$, the random phases ϕ nearly cancel (for nearby paths).

Application to seismic imaging

- With the Born approximation, the scattered field measured at a receiver r is:

$$d(s,r,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ (i\omega)^2 e^{-i\omega t}\left[\int d\mathbf{x}\, r(\mathbf{x}) G(s,\mathbf{x},\omega) G(r,\mathbf{x},\omega)\right]$$

- **Idea** (Borcea et al., 2003):

$$d(s,r,t) \star d(s',r',t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\ \overline{D(s,r,\omega)} D(s',r',\omega) e^{i\omega t}$$

  Note that we obtain the terms:

$$\overline{G(s,\mathbf{x},\omega)} G(s',\mathbf{x}',\omega) \quad \text{and} \quad \overline{G(r,\mathbf{x},\omega)} G(r',\mathbf{x}',\omega)$$

- Pre-processing step: start with the fluctuating data $d(s,r,t)$, and obtain a reduced, self-averaging data set .

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Cross-correlation tomography (1/4)

- The convolution model for the linearized forward map is (Symes, 1999):

$$d(t, h) = r(Z(t, h)).$$

  Here $Z(t, h)$ is the inverse Fourier transform of the two-way travel time function.

- To obtain the background velocity, construct an operator which when applied to the data with the **correct** background medium yields a vanishing outcome.

- Denote by v^* the correct background velocity, with corresponding traveltime $T^*(z, h)$ and inverse traveltime $Z^*(t, h)$.

- Assume model-consistent data (i.e. noise-free): $d(t, h) = r^*(Z^*(t, h))$.

Cross-correlation tomography (2/4)

Choose a trial velocity v, compute corresponding Z, and define the weighted cross-correlations of nearby traces:

$$C_1(t,h,h') = \int_{-\infty}^{\infty} d(t+\tau,h)p^2(\tau,h)d(\tau,h')d\tau$$
$$C_2(t,h,h') = \int_{-\infty}^{\infty} d(t+\tau,h)p(\tau,h)\left[\int_{-\infty}^{\tau} d(\cdot,h')\right]d\tau$$
$$C_3(t,h,h') = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\tau} d(t+\cdot,h)\right]\left[\int_{-\infty}^{\tau} d(\cdot,h')\right]d\tau$$

Cross-correlation tomography (3/4)

- Define the functional:

$$I(h) = \left(C_1 + \frac{\partial^2 C_3}{\partial h \partial h'} + 2\frac{\partial C_2}{\partial h'} \right) (t = 0, h, h' = h).$$

- After some algebra, we obtain:

$$I(h) = \int_{-\infty}^{\infty} |d(\tau, h)|^2 \left[p(\tau, h) - p^*(\tau, h) \right]^2 d\tau$$

- $I(t, h)$ vanishes when $p^* = p$. It measures the mismatch of event slowness, weighted by data autocorrelation.

- Velocity analysis algorithm:

$$\min_v J = \tfrac{1}{2} \| I(h) \|^2$$

Use gradient-based optimization methods (assuming J is smooth in v).

Cross-correlation tomography (4/4)

Conjectures:

- Objective just defined has **global** minimums, as has been proved for other DSO variants (e.g. the layered medium case).
- When intermediate scale random fluctuations are allowed, the cross-correlations with (slowly-varying) weights are statistically stable, as is the case without weights.
- The gradient of J is also statistically stable.
- Stationary points of J with cross-correlation weights computed from long-scale velocity component are optimal estimators of background velocity.

Ultimately: Velocity analysis is essentially stable against random fluctuations on the medium scale a!

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Constructing a statistically stable data set

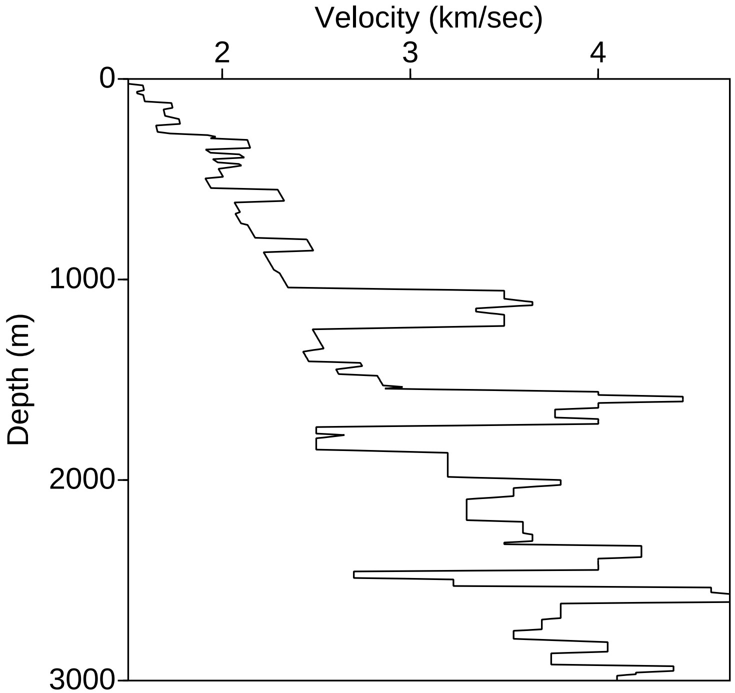
- Task: construct a 3-scale velocity model with the appropriate scaling.
- The medium scale fluctuations δv have the following characteristics:
  - zero mean $\langle \delta v(x) \rangle = 0$
  - specified autocorrelation function of the form:

$$\langle \delta v(\mathbf{x}) \delta v(\mathbf{x}') \rangle = \sigma^2 R(r), \quad r = |\mathbf{x} - \mathbf{x}'|$$

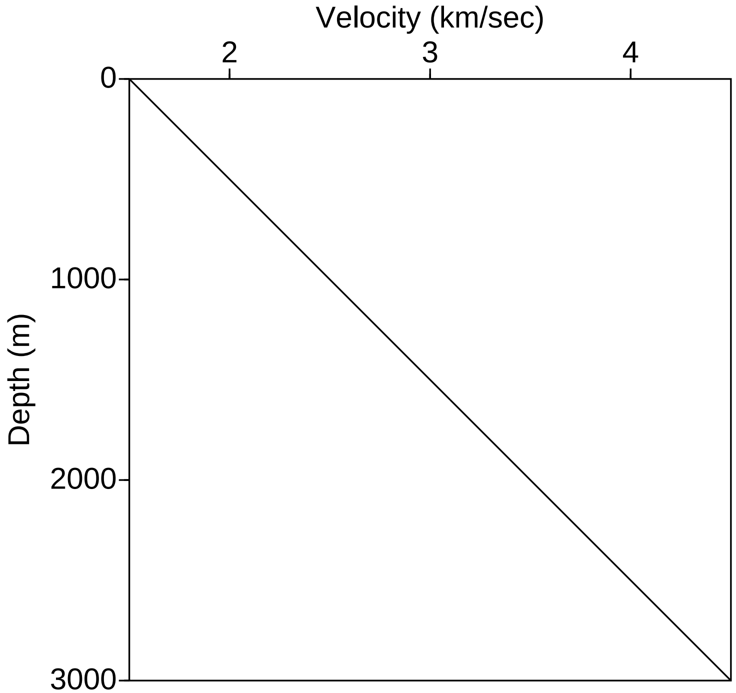
  - For simplicity, we consider a Gaussian autocorrelation function:

$$R(r) = \varepsilon^2 e^{-r^2/a^2}, \quad \varepsilon^2 = R(0) = \frac{1}{\sigma^2} \left\langle \delta v^2(\mathbf{x}) \right\rangle$$

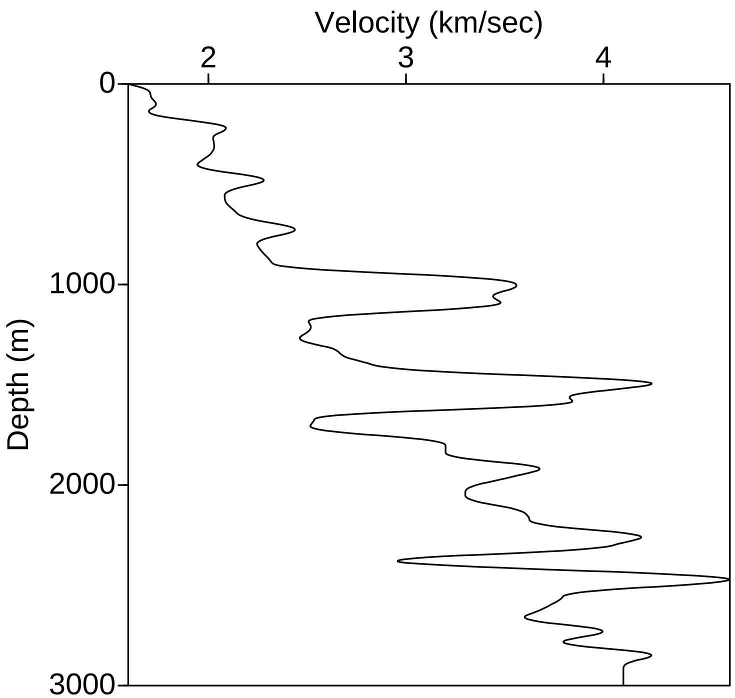
- Conducted 100 linearized simulations. To measure statistical stability, use **sample mean** and **sample variance**.



Velocity profile.

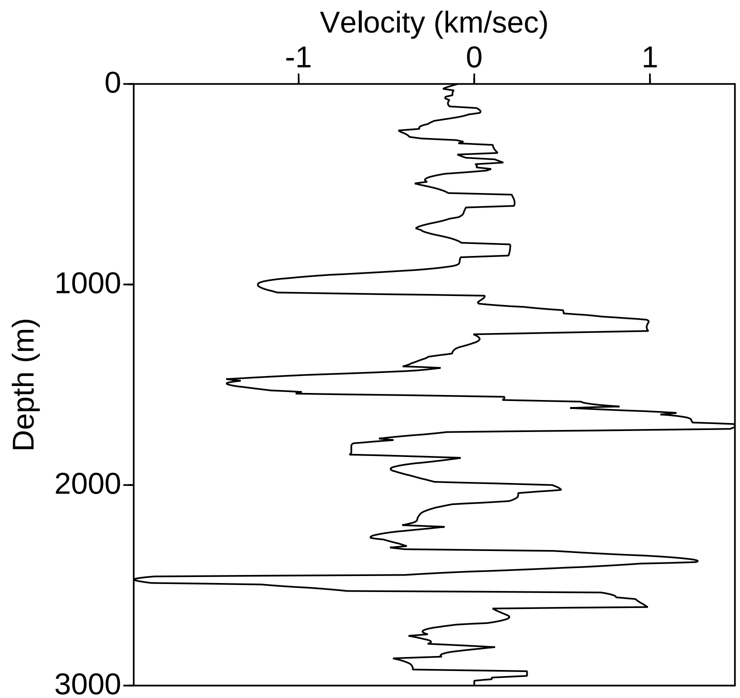


Background velocity profile.

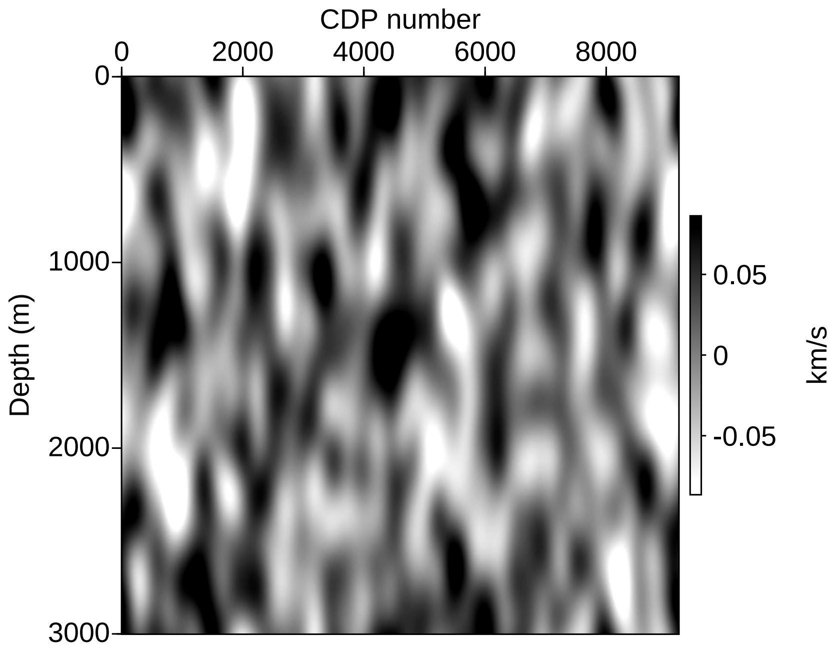


Smoothed velocity profile

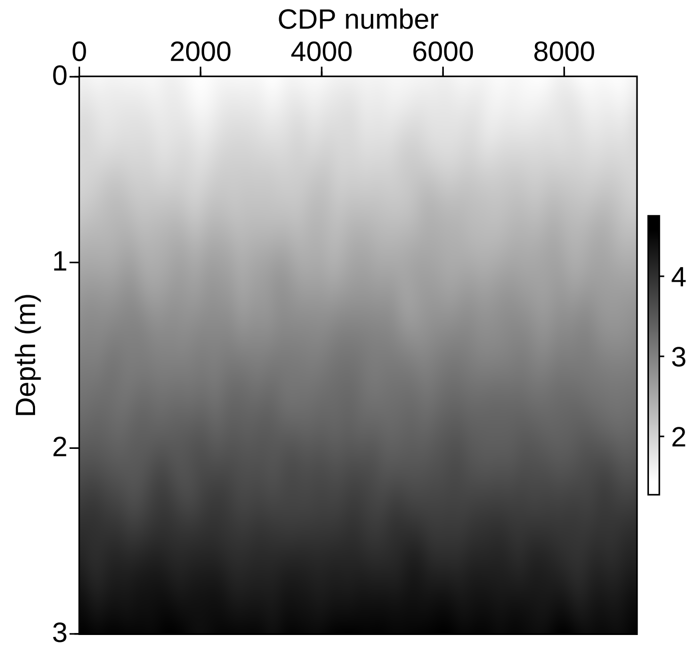
(smoothing width $\sim 100m$)



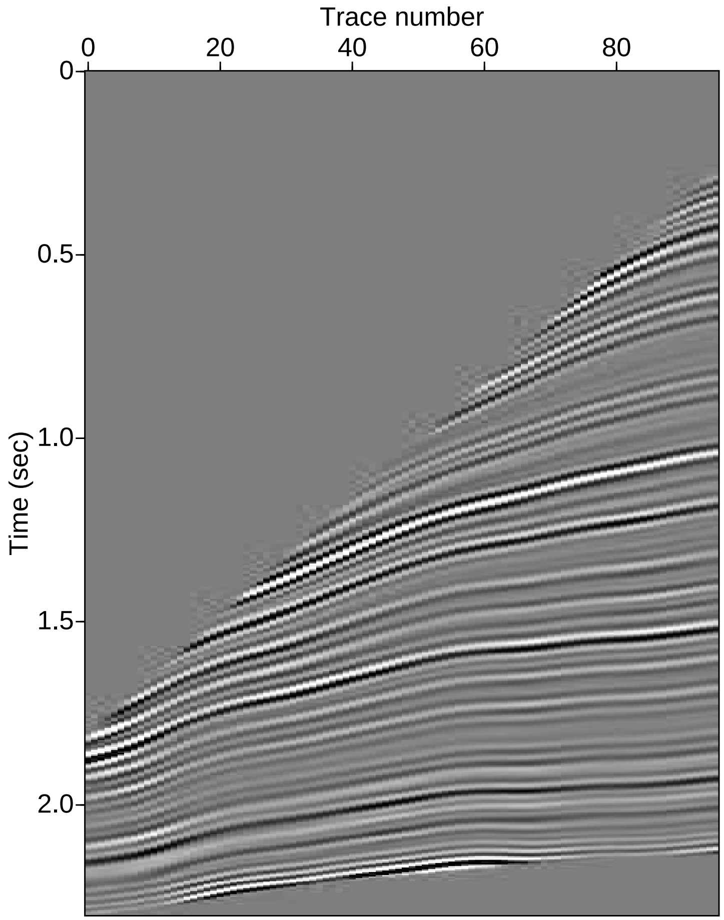
Resulting perturbation velocity



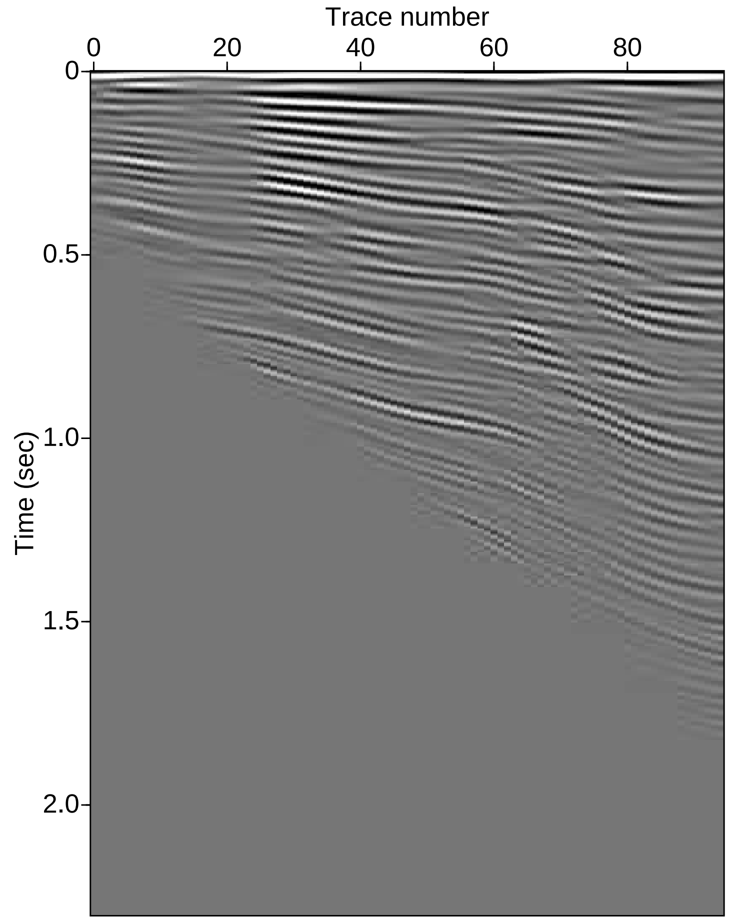
Medium scale fluctuations ($a \sim 300m$).



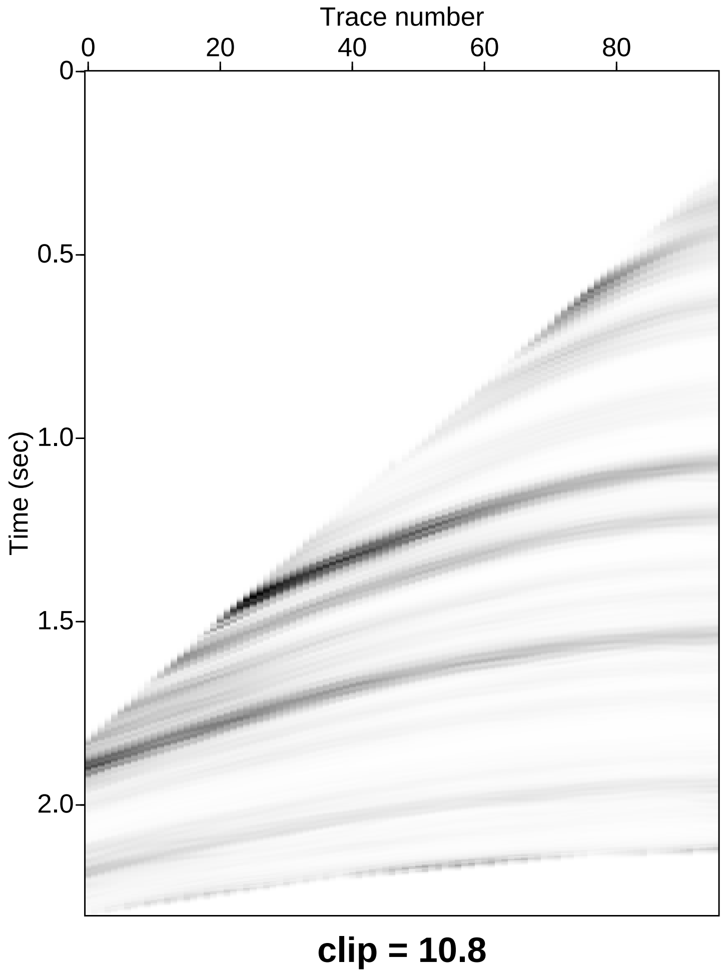
Complete background model.



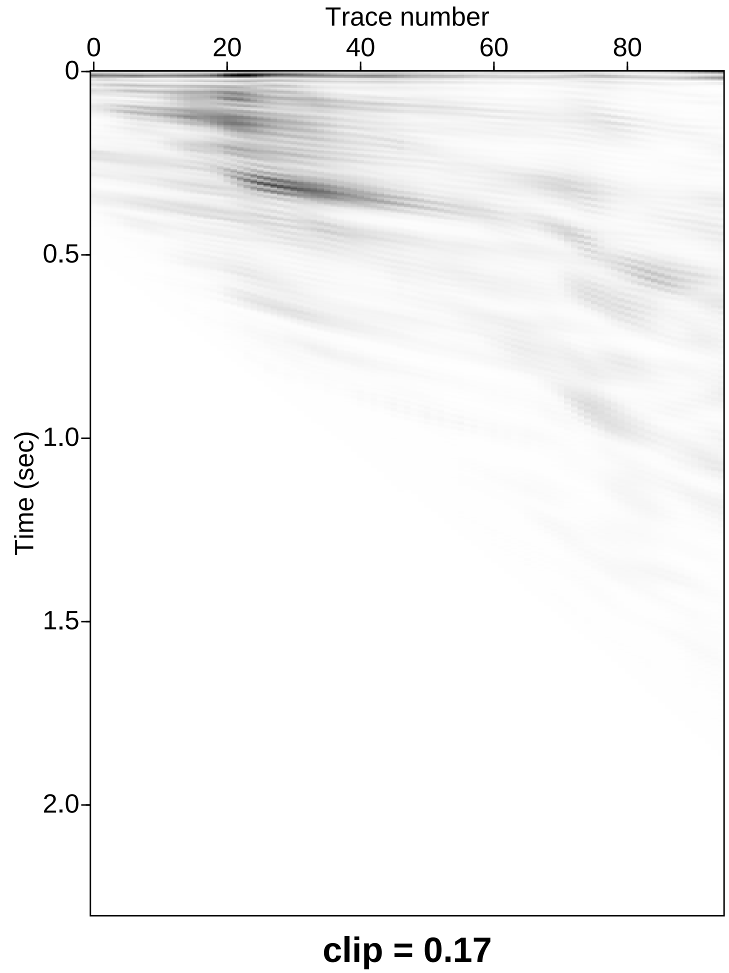
Linearized seismogram.



Cross-correlogram.



Variance of the linearized seismograms.



Variance of the cross-correlograms.

Conclusions

- Next step: velocity analysis
  - using regular differential semblance optimization (c.f. talk by Jintan Li and W.W. Symes) on the raw data.
  - using the new formulation on the cross-correlated data.

- Future work:
  - Extension to more complex models.
  - Investigation of the applicability of the imaging results obtained by Borcea, Papanicolaou and to migration.