Explicit extrapolators and common azimuth migration

Eric Dussaud

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Agenda

- Downward continuation via explicit extrapolation.
- Design of explicit extrapolators.
  - Design of 1-D convolution operators.
  - Transformation filters.
- Extension to unequal sampling - Li correction.
- Application to common-azimuth migration.
- Conclusions and future work.

Explicit extrapolation (1/3)

- Downward continuation of zero-offset data:

$$\frac{\partial P}{\partial z}(k_x, k_y, z, \omega) = ik_z P(k_x, k_y, z, \omega), \quad k_z = \sqrt{\frac{4\omega^2}{v^2} - k_x^2 - k_y^2}.$$

- The analytical solution is given by:

$$P(k_x, k_y, z + \Delta z, \omega) = e^{ik_z \Delta z} P(k_x, k_y, z, \omega) \equiv W(k_x, k_y, \Delta z, \omega) P(\omega, k_x, k_y, z).$$

- Inverse Fourier transform over k_x and k_y yields:

$$\begin{aligned} p(x, y, z + \Delta z, \omega) &= w(x, y, \omega) * p(x, y, z, \omega) \\ &= \iint dx' dy' \; w(x', y', \omega) p(x - x', y - y', z + \Delta z, \omega) \end{aligned}$$

Explicit extrapolation (2/3)

- Lateral changes in v are accommodated by "allowing" w to vary laterally, i.e. write $w(x', y', \omega) \equiv w(x, y, x', y', \omega)$ in the previous expression.

- In practice, substitute w with convolution operators f that are designed to fit the exact operator for a range of ω/v ratios.

- The operators f are optimized in such a way that their Fourier transform F over k_x and k_y approximate the exact phase-shift operator.

Explicit extrapolation (3/3)

- Major drawback of the method: computationally **very** expensive.

- Hale (1991): break the design procedure into **two** parts,
  - computation of the coefficients of 1-D extrapolation filters that would be used for 2-D migration.
  - computation of a small transformation filter that can be used to transform 1-D extrapolation operators into 2-D ones.

Upshot: computational cost is greatly reduced, and the design is simplified.

Design of 1-D convolution operators

- Compute 1-D short convolution operators $f(x, \Delta z, \omega)$ with complex coefficients $f_n \equiv f_n(\Delta z, \omega, v)$, and with a wavenumber spectrum:

$$F(k_x, \Delta z, \omega) \sim W(k_x, \Delta z, \omega) \equiv \exp\left[i\Delta z\sqrt{\frac{\omega^2}{v^2} - k_x^2}\right], \quad |k_x| \in [0, k_c]$$

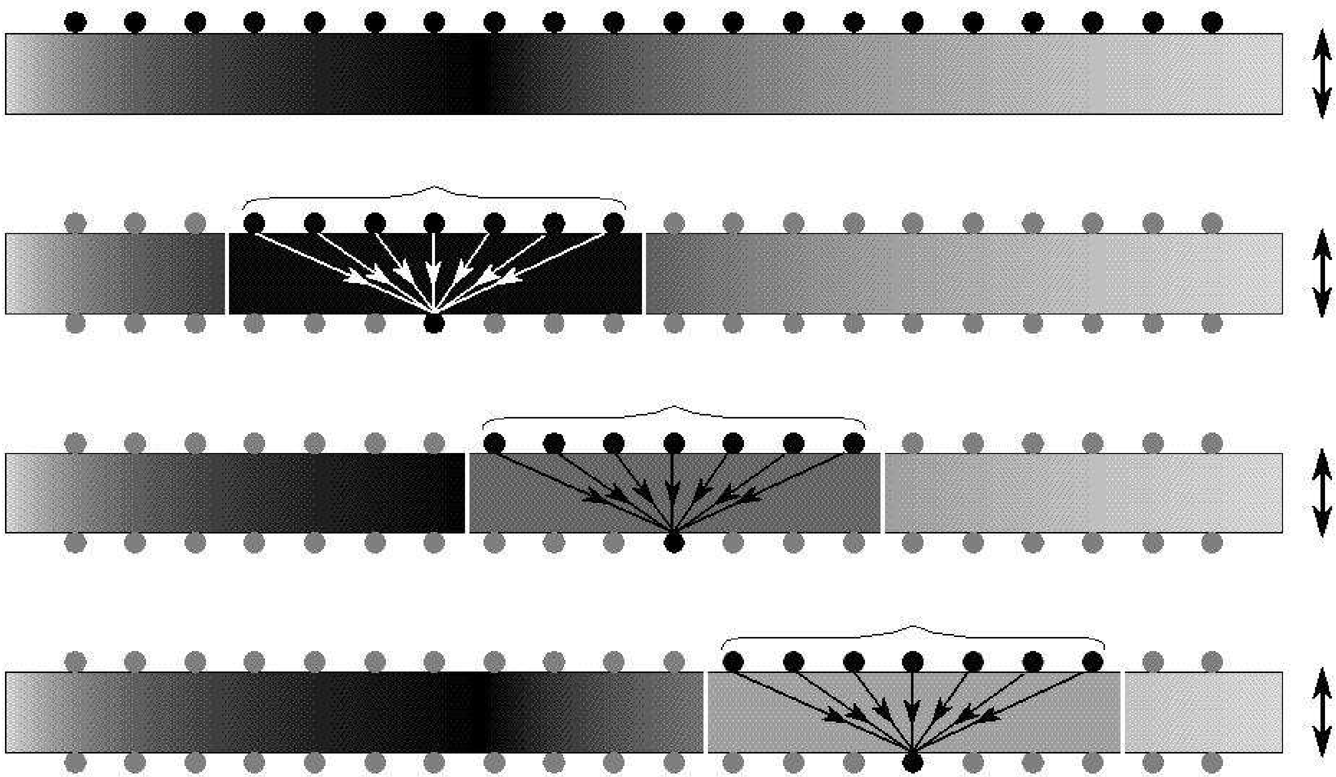
- The (discrete) Fourier transform of $f(x, \Delta z, \omega)$ is:

$$F(k_x, \Delta z, \omega) \approx \sum_{n=(-N+1)/2}^{(N-1)/2} f_n e^{-in\Delta x k_x} = f_0 + 2\sum_{n=1}^{(N-1)/2} f_n \cos\left(n\Delta x k_x\right).$$

- Optimization problem: find the coefficients f_n so as to minimize $\|F - W\|$.

- Do it for a range of ω/v ratios and store the coefficients in a file.

Principle of recursive extrapolation



Courtesy: Jan Thorbecke.

Transformation filters (1/3)

- To transform 1-D filters into 2-D filters, use the fact that:

$$W(k_x, k_y, \Delta z, \omega) = W(|\mathbf{k}|, \Delta z, \omega), \quad |\mathbf{k}| = \sqrt{k_x^2 + k_y^2}$$

Therefore:

$$F(k_x, k_y, \Delta z, \omega) \approx f_0 + 2 \sum_{n=1}^{(N-1)/2} f_n \cos\left(n \Delta x |\mathbf{k}|\right) \tag{1}$$

- Use the Chebyshev recursion formula

$$\cos(n\Delta x|\mathbf{k}|) = 2\cos(\Delta x|\mathbf{k}|)\cos\left[(n-1)\Delta x|\mathbf{k}|\right] - \cos\left[(n-2)\Delta x|\mathbf{k}|\right]$$

to write:

$$F(k_x, k_y, \Delta z, \omega) \approx \sum_{n=0}^{(N-1)/2} \hat{f}_n \cos^n(\Delta x|\mathbf{k}|)$$

Transformation filters (2/3)

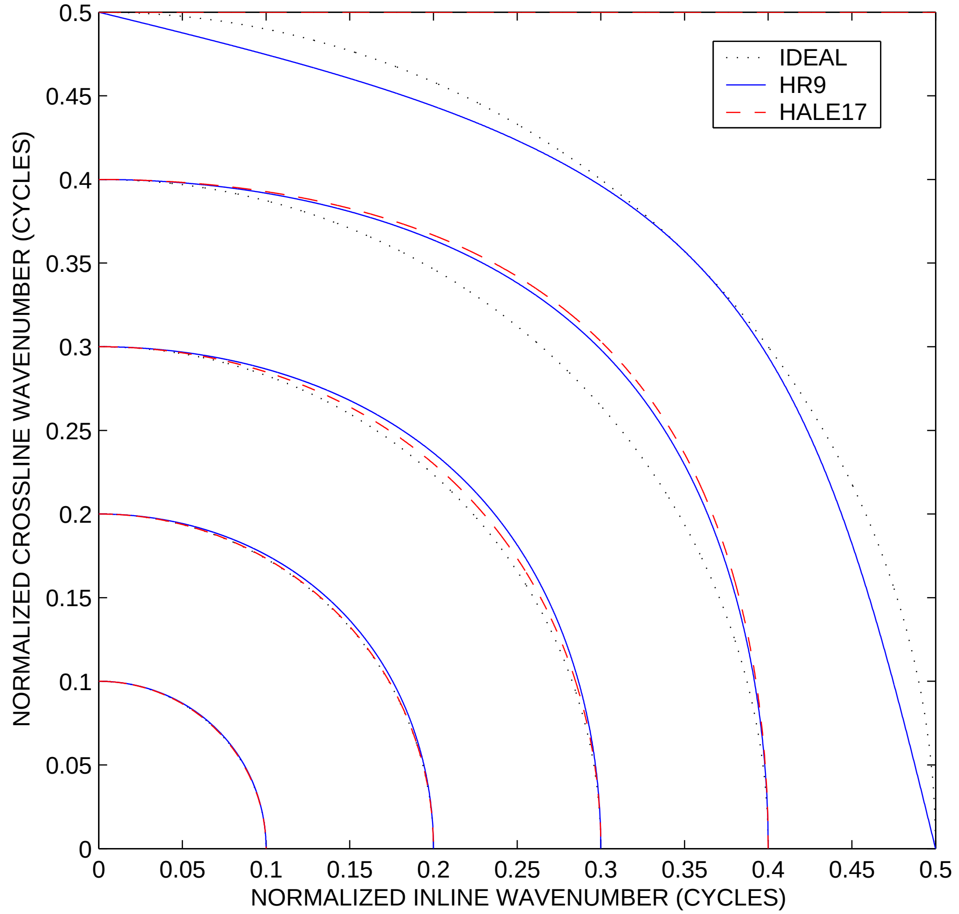
- To approximate $\cos(\Delta x|\mathbf{k}|)$, Hale (1991) suggested the following transform:

$$\cos\left(\Delta x\sqrt{k_x^2+k_y^2}\right) \approx G(k_x,k_y) \equiv -1+\frac{1}{2}\left[1+\cos(\Delta x k_x)\right]\left[1+\cos(\Delta x k_y)\right] - \frac{c}{2}\left[1-\cos(2\Delta x k_x)\right]\left[1-\cos(2\Delta x k_y)\right]$$

with $c = 0.0255$.

- Denote by $g(x,y)$ the corresponding 5x5 spatial stencil (obtained by IFT):

$$g(x,y) \equiv \begin{bmatrix} -c/8 & 0 & c/4 & 0 & -c/8 \\ 0 & 1/8 & 1/4 & 1/8 & 0 \\ c/4 & 1/4 & -(1+c)/2 & 1/4 & c/4 \\ 0 & 1/8 & 1/4 & 1/8 & 0 \\ -c/8 & 0 & c/4 & 0 & -c/8 \end{bmatrix}$$



Contours of constant amplitude and phase (constant $|\mathbf{k}|$) for the (scaled and shifted) Hazra & Reddy transformation compared to the improved McClellan transformation.

Transformation filters (3/3)

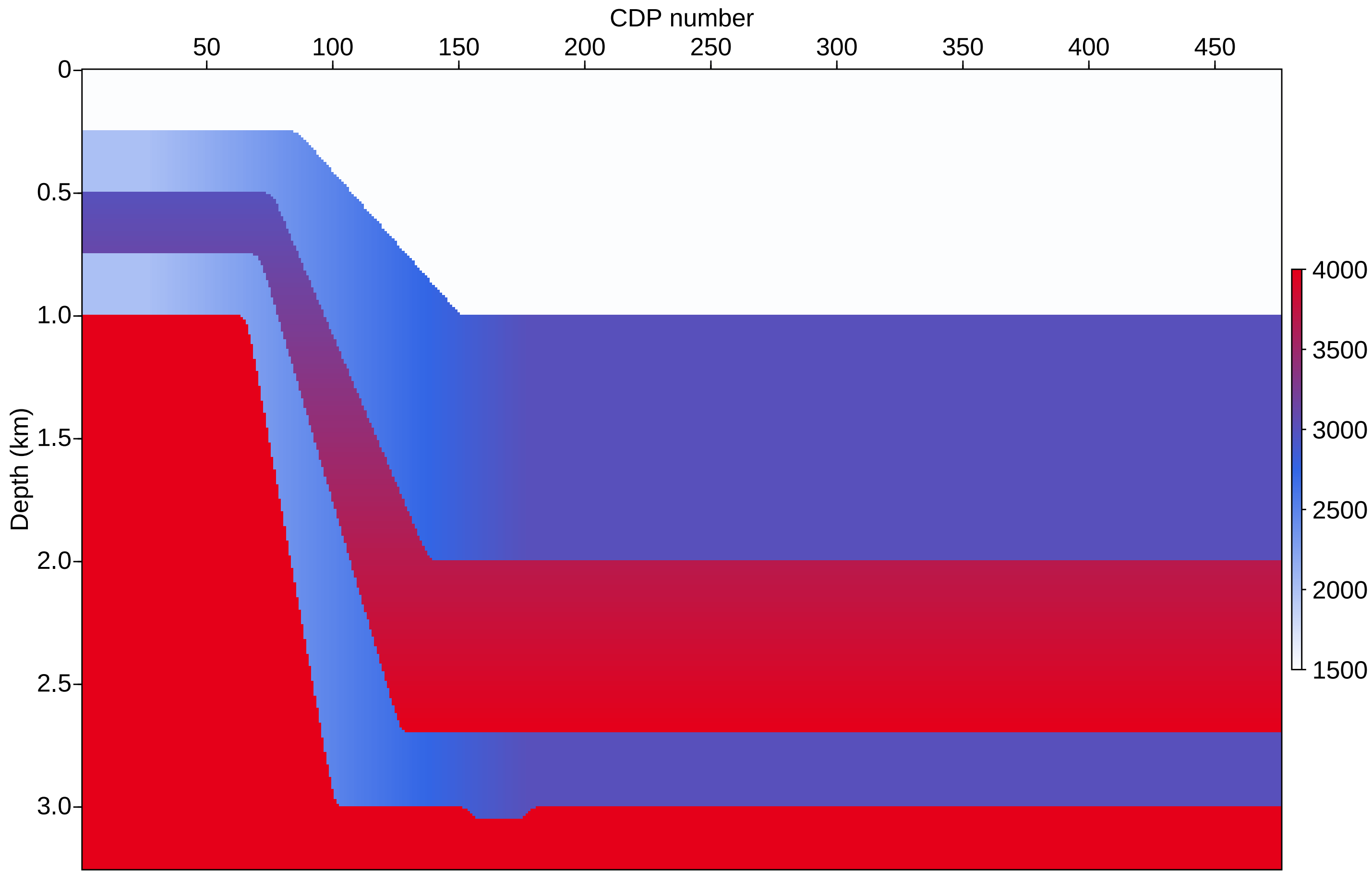
- The 2-D inverse Fourier transform of (1) is given by:

$$f(x,y,\Delta z,\omega) = f_0\,\delta(x,y) + 2\sum_{n=1}^{N} f_n\, g_n(x,y),$$

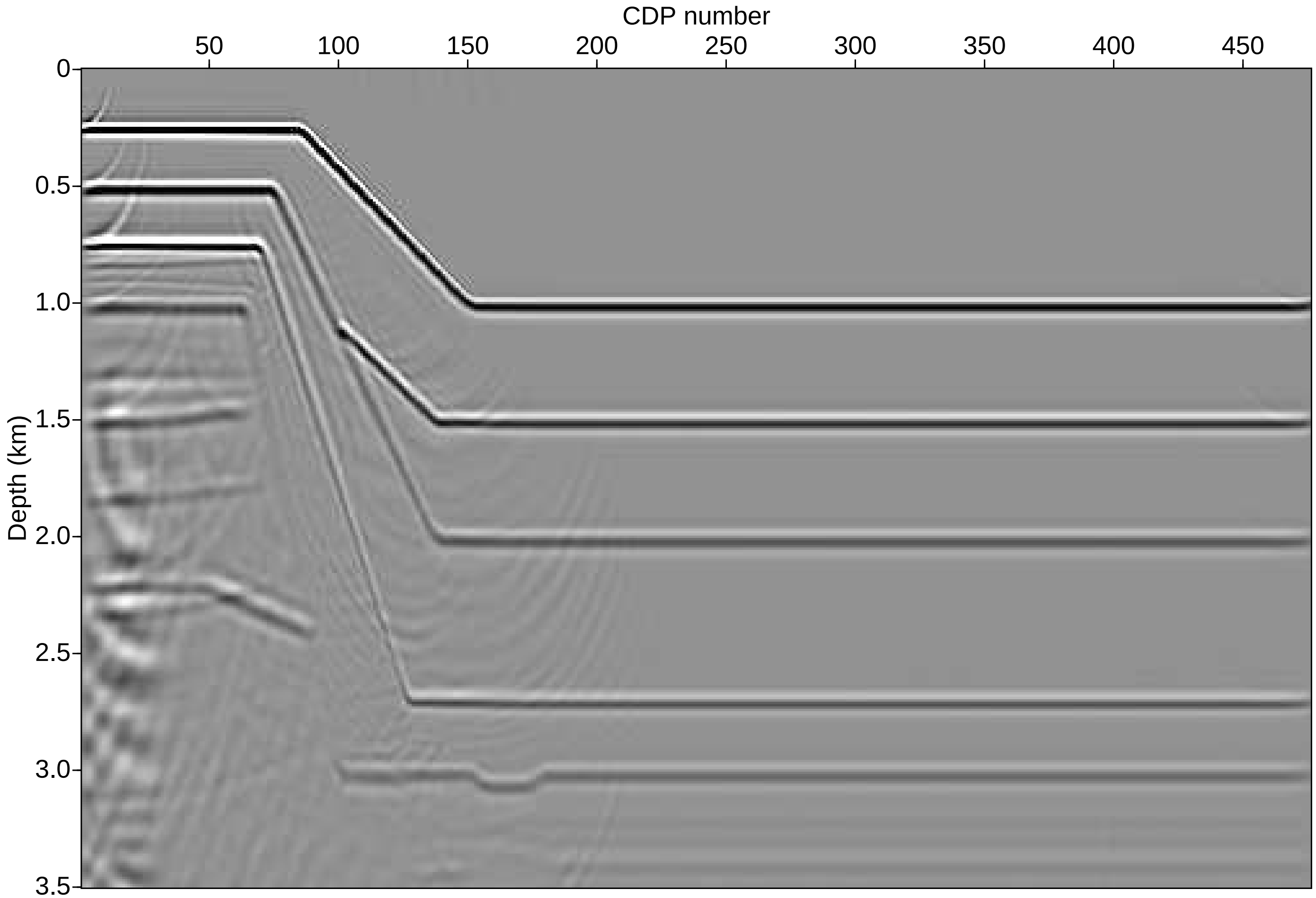
where $g_0(x,y) \equiv \delta(x,y)$, $g_1(x,y) \equiv g(x,y)$ and $g_n = 2g_{n-1} * g - g_{n-2}$, $n > 1$.

- The extrapolation step consists of convolving the wavefield with $f(x,y,\Delta z,\omega)$:

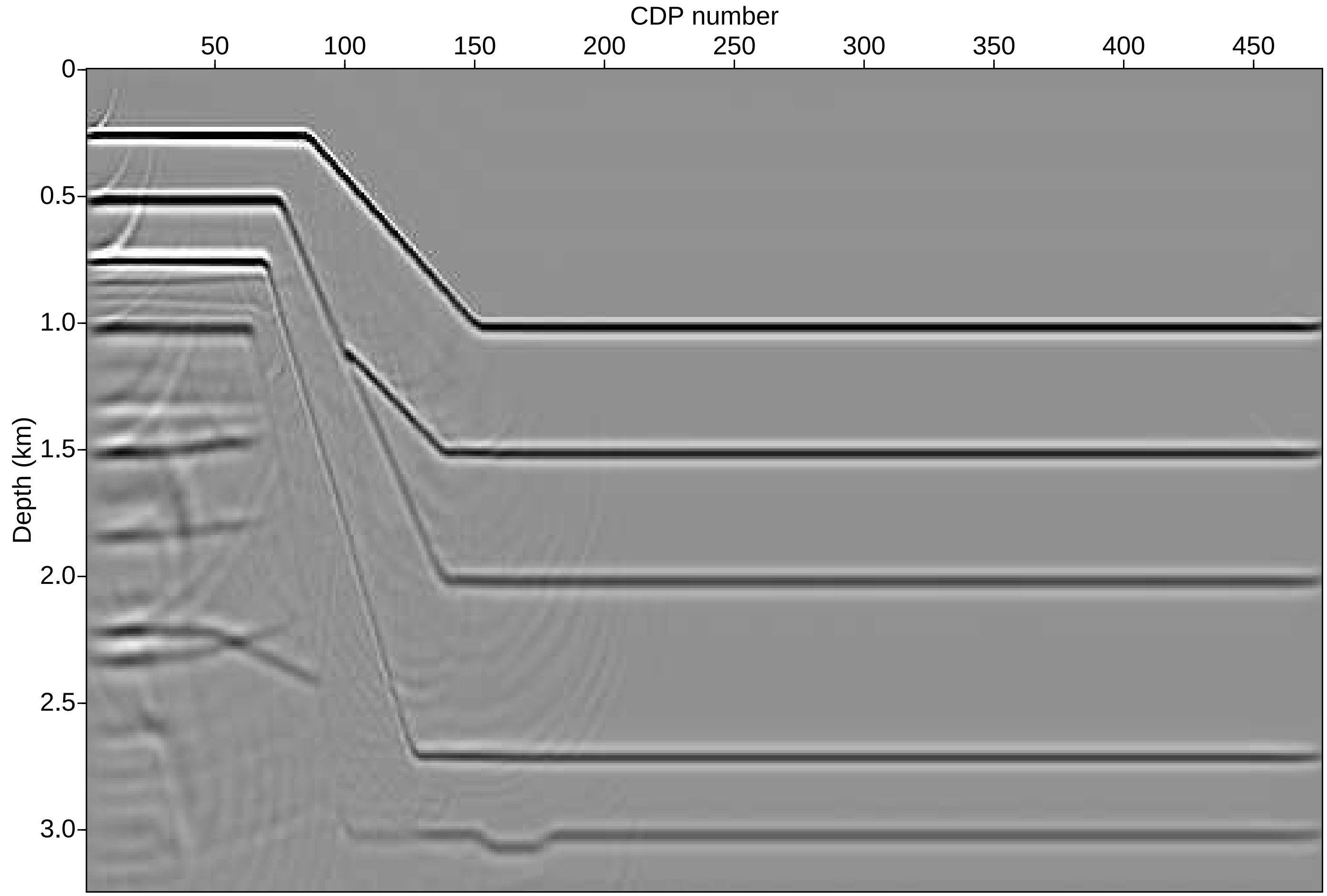
$$p(x,y,z+\Delta z,\omega) = f_0\, p(x,y,z,\omega) + 2\sum_{n=1}^{N} f_n\,[g_n * p(x,y,z,\omega)]$$



IFP model: in-line section at constant cross-line coordinate $y = 0$m.



IFP model: migrated section using an explicit scheme.

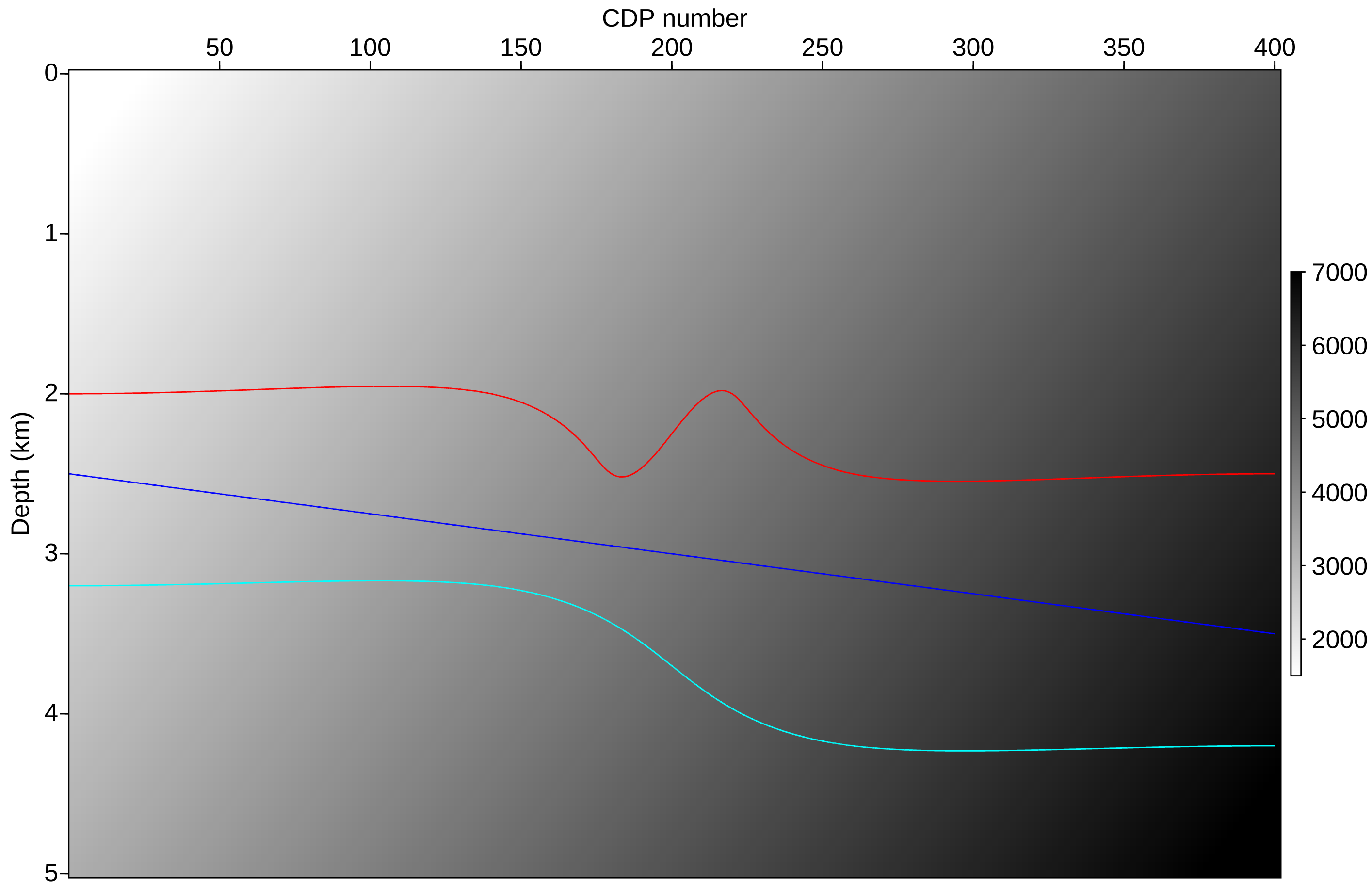


IFP model: migrated section using GSP.

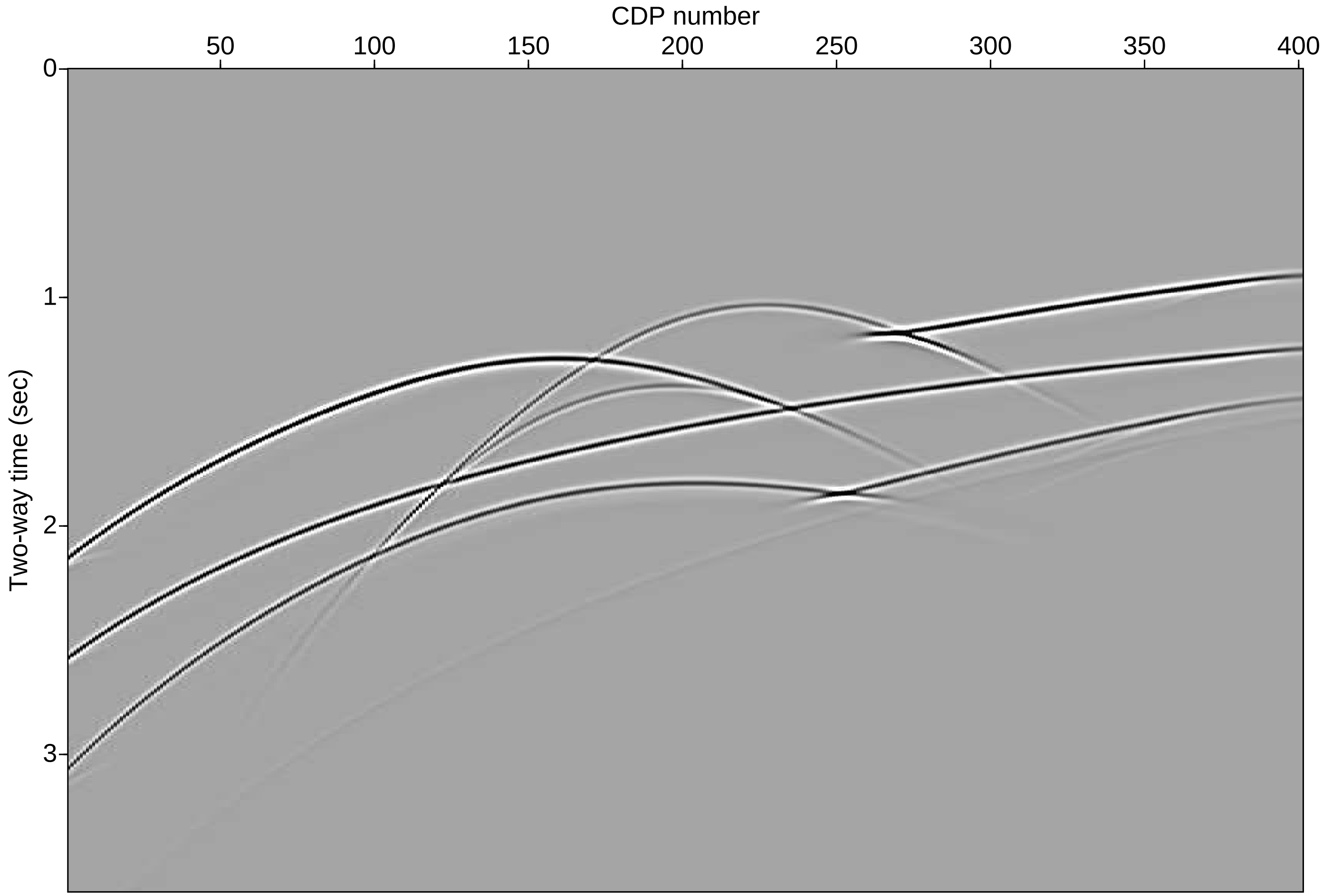
Extensions

- Accomodate unequal in-line and cross-line sampling intervals, following the idea set forth by Levin (1999, 2004).

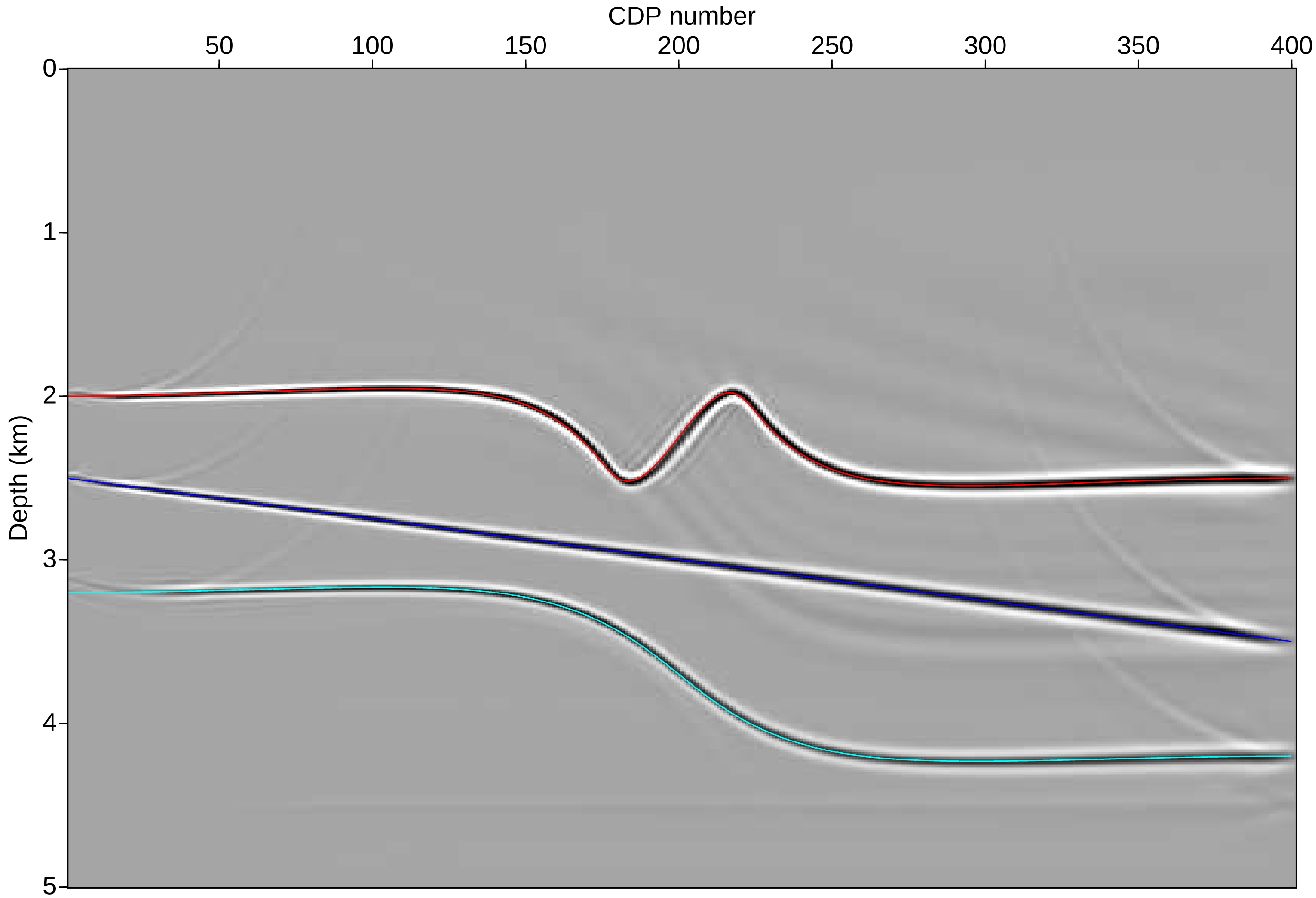
- Application of a correction filter (Li correction) to remove some of the phase error introduced by the Hale-McClellan tranformation filter (Etgen and Nichols, 1999).



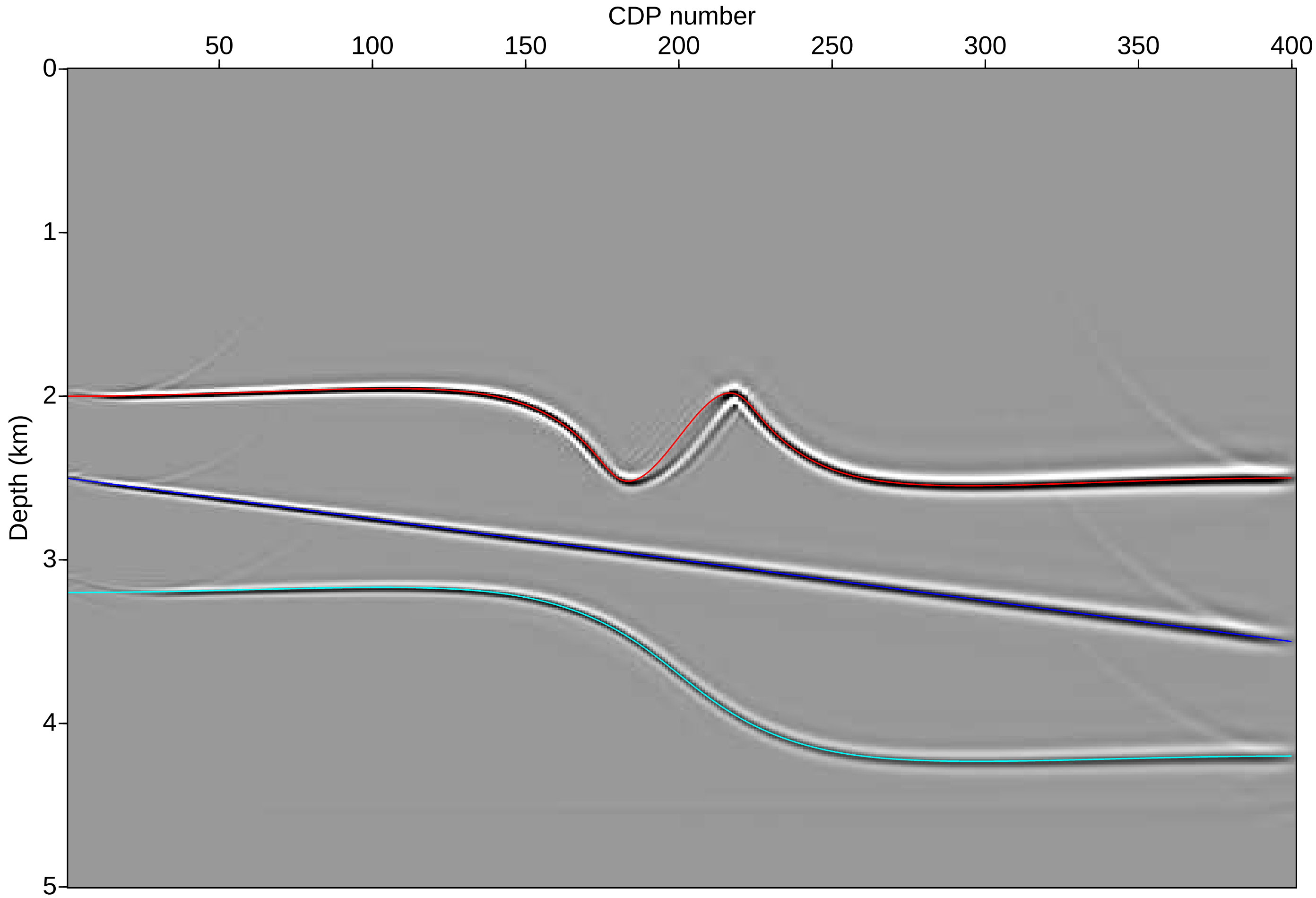
Curve model: in-line section at constant cross-line coordinate $y = 0$m.



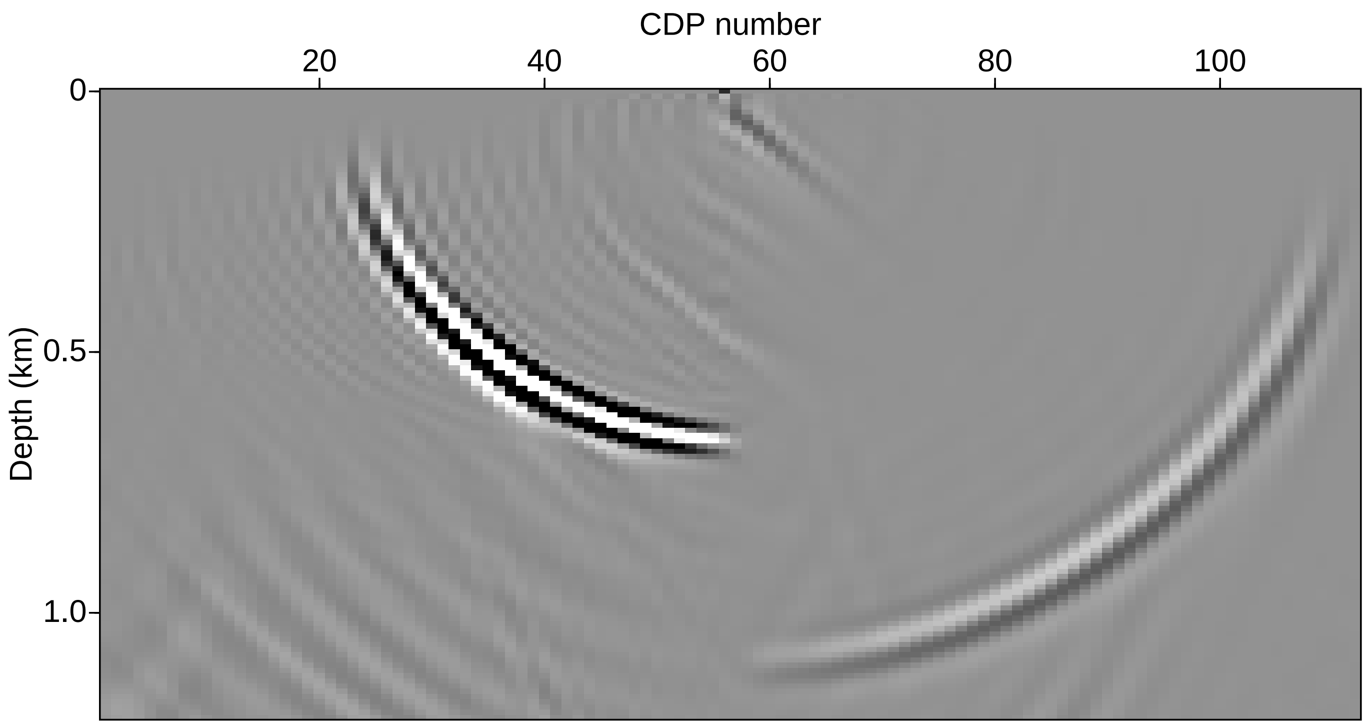
Synthetic data created with 101 cross-lines spaced 40m apart, and 401 in-lines spaced 25m apart.



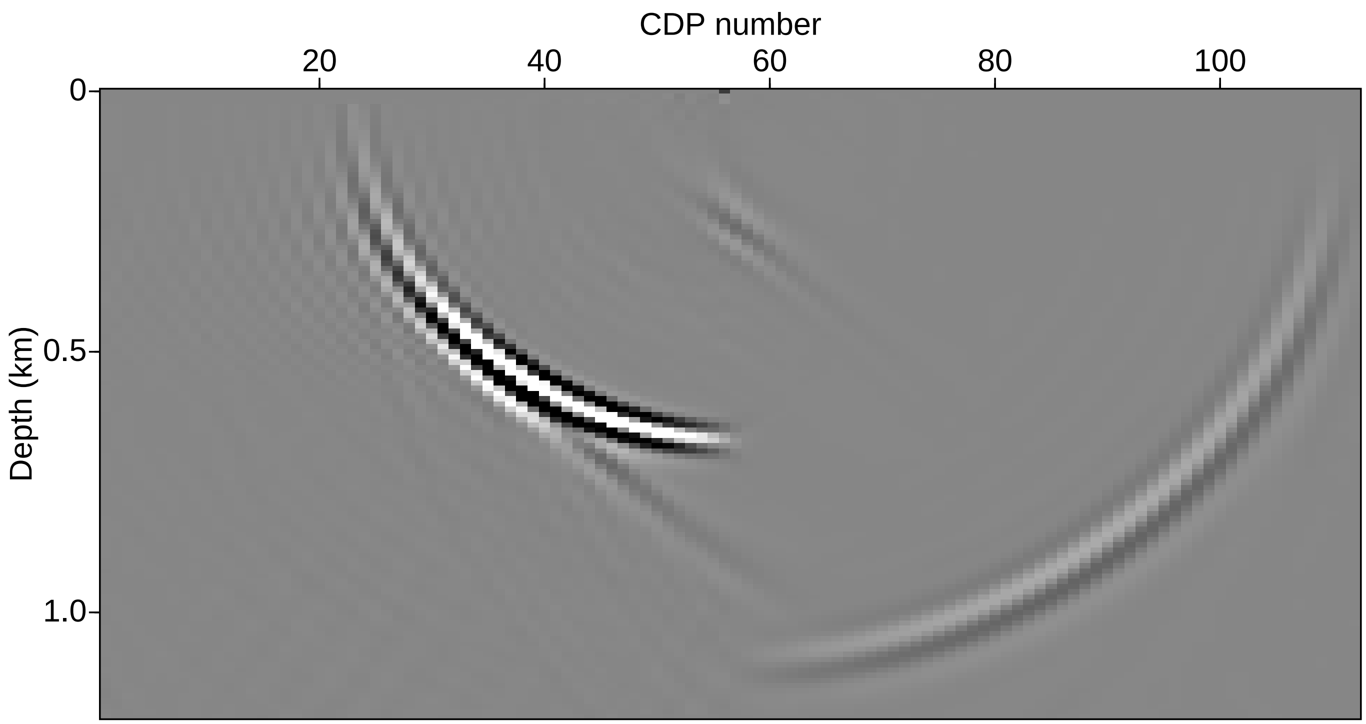
Migrated section obtained using the explicit scheme.



Migrated section obtained using GSP.



Vertical slice through the migrated cube of an impulse response in a two-velocity medium. The operator used is a 25-point WLSQ operator, along with the Hale-McClellan transform.



Same result with the Li correction applied every 10 steps.

Application to common-azimuth migration

- Convenient restriction of the DSR operator for downward-continuing data that share a single azimuth (Biondi & Palacharla, 1996).

- After approximation with splitting, the dispersion relation associated with common-azimuth downward continuation is given by:

$$\widehat{k}_z = \underbrace{\sqrt{\frac{\omega^2}{v_s^2} - \frac{1}{4}\left(k_{m_x} - k_{h_x}\right)^2} + \sqrt{\frac{\omega^2}{v_g^2} - \frac{1}{4}\left(k_{m_x} + k_{h_x}\right)^2}}_{\text{Convolution in } x} + \underbrace{\sqrt{\frac{4\omega^2}{v_m^2} - k_{m_y}^2}}_{\text{Convolution in } y} - \frac{2\omega}{v_m}$$

Common-azimuth migration

- Downward-continuation step:

$$P(k_{m_x}, k_{m_y}, k_{h_x}, z+\Delta z, \omega) = D_1(\ldots)D_2(\ldots)D_3(\ldots)P(k_{m_x}, k_{m_y}, k_{h_x}, z, \omega),$$

with:

$$D_1(k_{m_x}, k_{h_x}, \Delta z, \omega) = \exp\left[i\Delta z \sqrt{\frac{\omega^2}{v_s^2} - \frac{1}{4}\left(k_{m_x} - k_{h_x}\right)^2} \right]$$

$$D_2(k_{m_x}, k_{h_x}, \Delta z, \omega) = \exp\left[i\Delta z \sqrt{\frac{\omega^2}{v_g^2} - \frac{1}{4}\left(k_{m_x} + k_{h_x}\right)^2} \right]$$

$$D_3(k_{m_y}, \Delta z, \omega) = \exp\left[i\Delta z \sqrt{\frac{4\omega^2}{v_m^2} - k_{m_y}^2} - \frac{2\omega}{v_m} \right]$$

Implementation (1/2)

- The operator D_3 is designed and implemented exactly as in the zero-offset case.
- Consider D_1. Setting $k_s = k_{m_x} - k_{h_x}$, we have:

$$D_1(k_{m_x}, k_{h_x}, \Delta z, \omega) = \exp\left[i\Delta z\sqrt{\frac{\omega^2}{v_s^2} - \frac{1}{4}\left(k_{m_x} - k_{h_x}\right)^2}\right] = \exp\left[i\frac{\Delta z}{2}\sqrt{\frac{4\omega^2}{v_s^2} - k_s^2}\right].$$

It can be approximated by the finite-length summation:

$$D_1(k_{m_x}, k_{h_x}, \Delta z, \omega) \approx d_0 + 2\sum_{n=1}^{N_h-1} d_n \cos(n\Delta x k_s), \qquad (2)$$

where Δx represents the CMP in-line sampling interval.

Implementation (2/2)

- The Chebyshev recursion can be used to write $\cos(n\Delta x k_s)$ in terms of an n-th order polynomial of $\cos(\Delta x k_s)$.

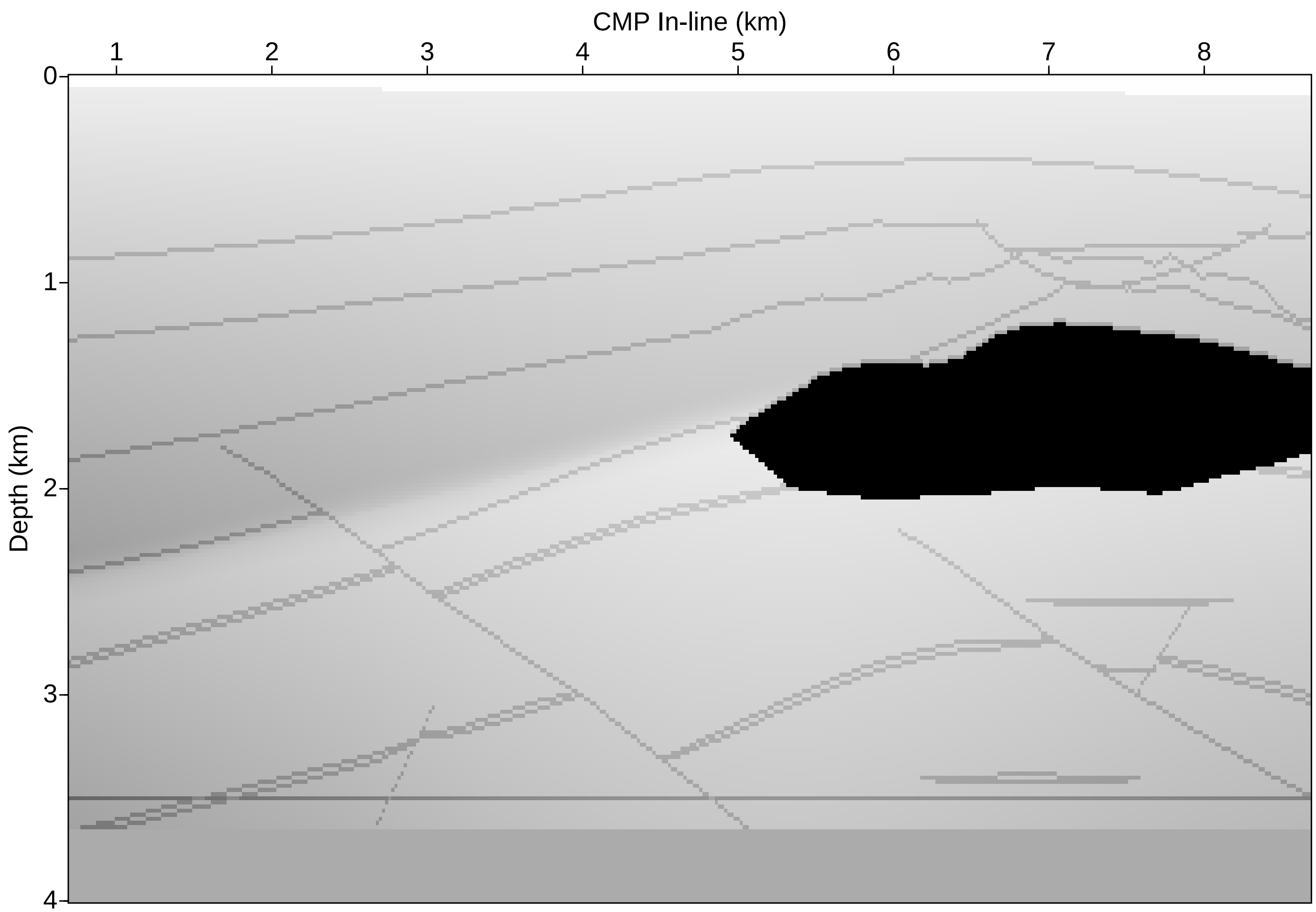
- Need to find spatial stencil corresponding to $\cos(\Delta x k_s)$. We have:

$$\begin{aligned} G(k_{m_x}, k_{h_x}) &\equiv \cos\left[\Delta x \left(k_{m_x} - k_{h_x}\right)\right] \\ &= \cos\left(\Delta x k_{m_x}\right)\cos\left(\Delta x k_{h_x}\right) + \sin\left(\Delta x k_{m_x}\right)\sin\left(\Delta x k_{h_x}\right) \end{aligned}$$

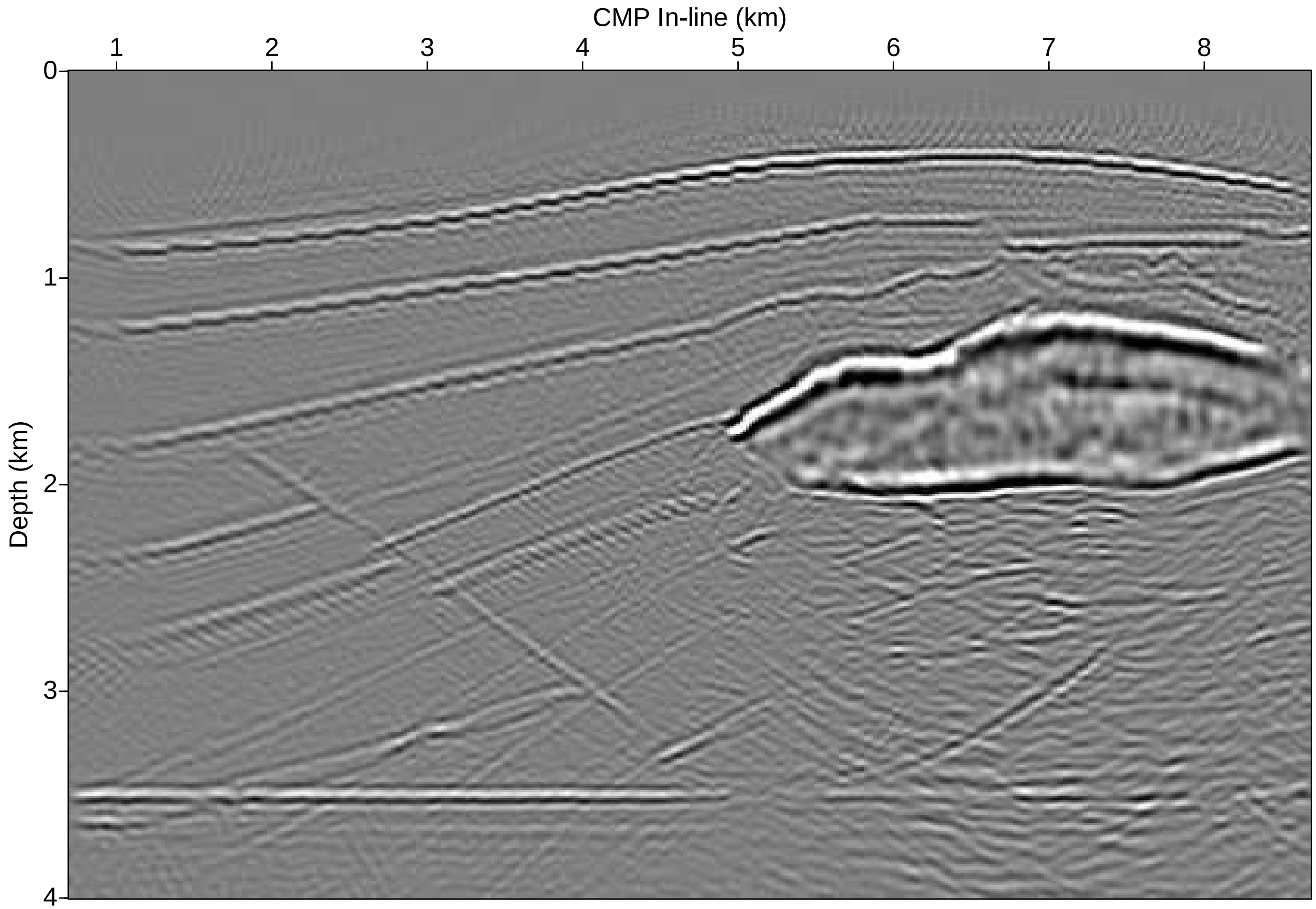
  By inverse Fourier transform, we obtain:

$$g(m_x, h_x) = \begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}$$

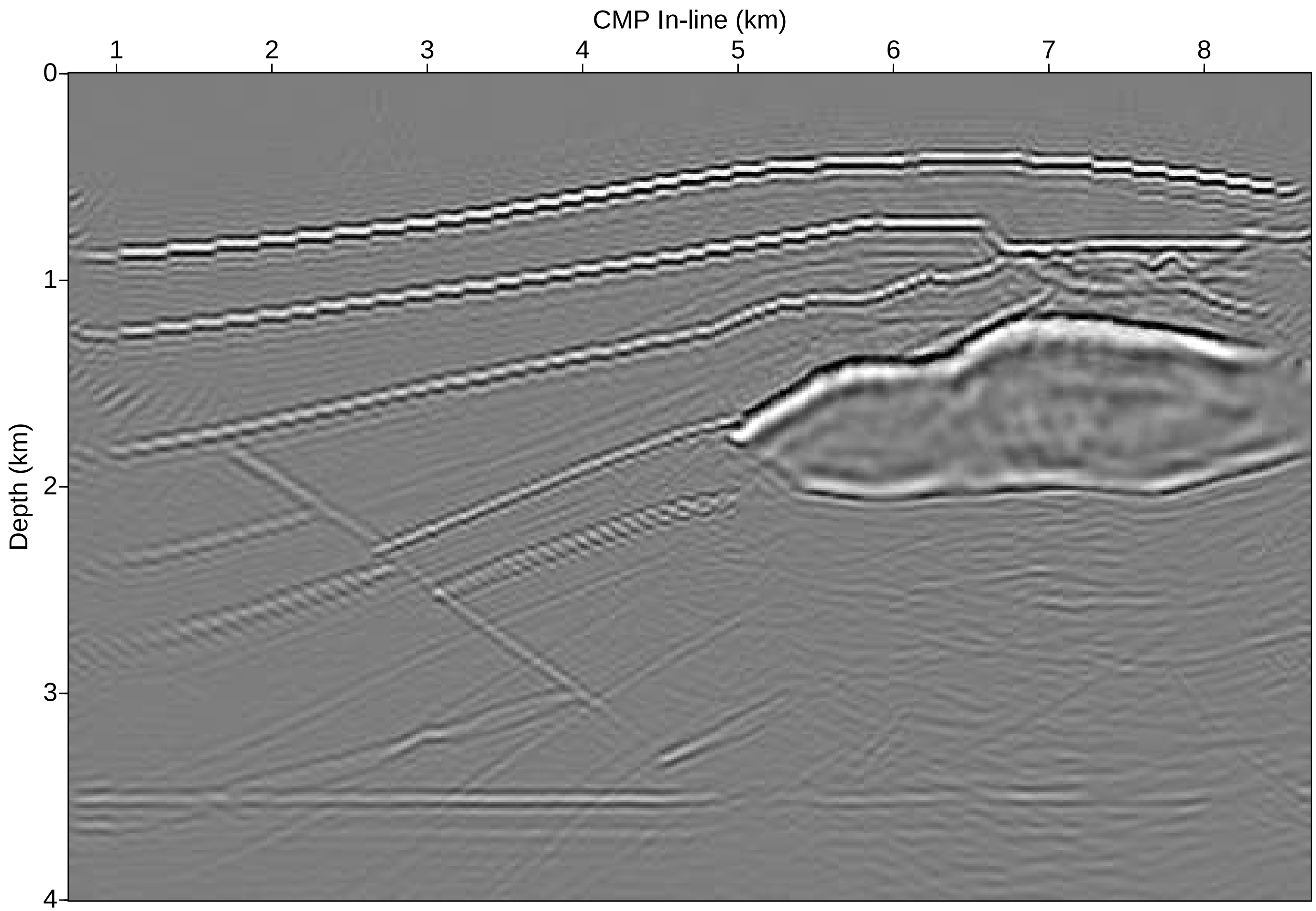
- Note: the main anti-diagonal of the filter represents the impulse response of the $\cos k_s$ filter, that is, the convolution is done in effect along the shot direction.



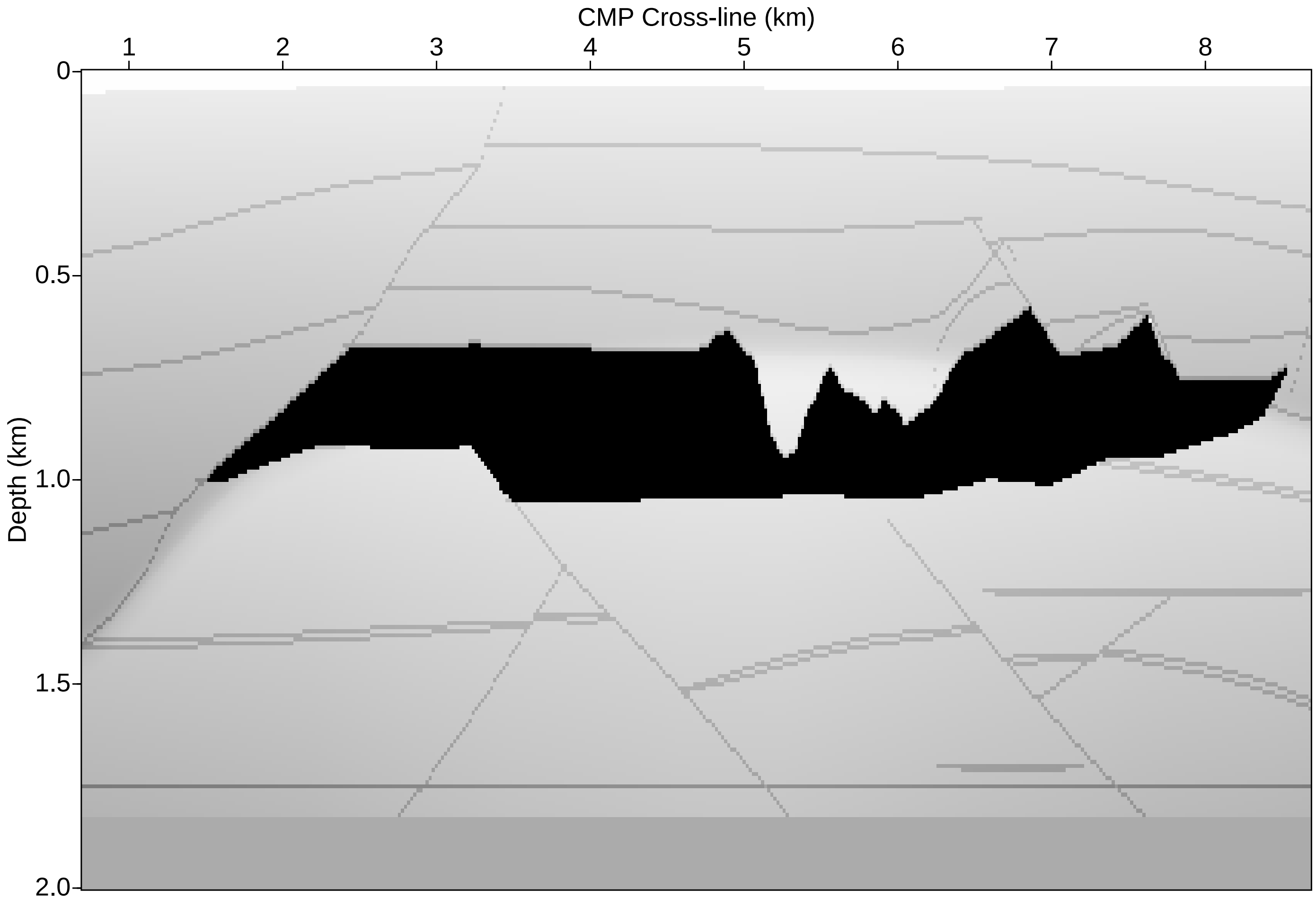
SEG-EAGE salt model: in-line section at constant cross-line coordinate $y = 9,820$m.



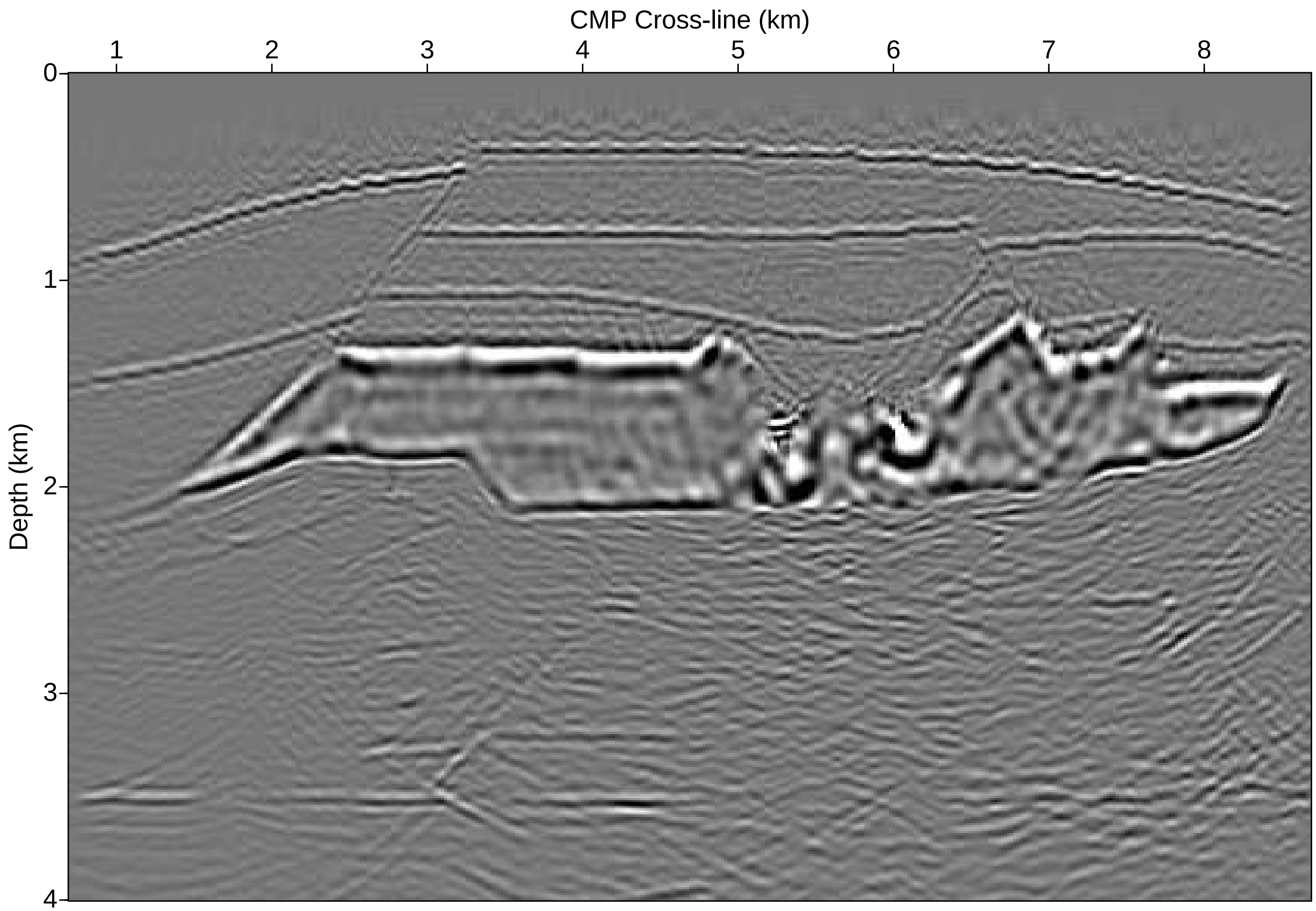
Same section obtained using split COMAZ via the explicit scheme.



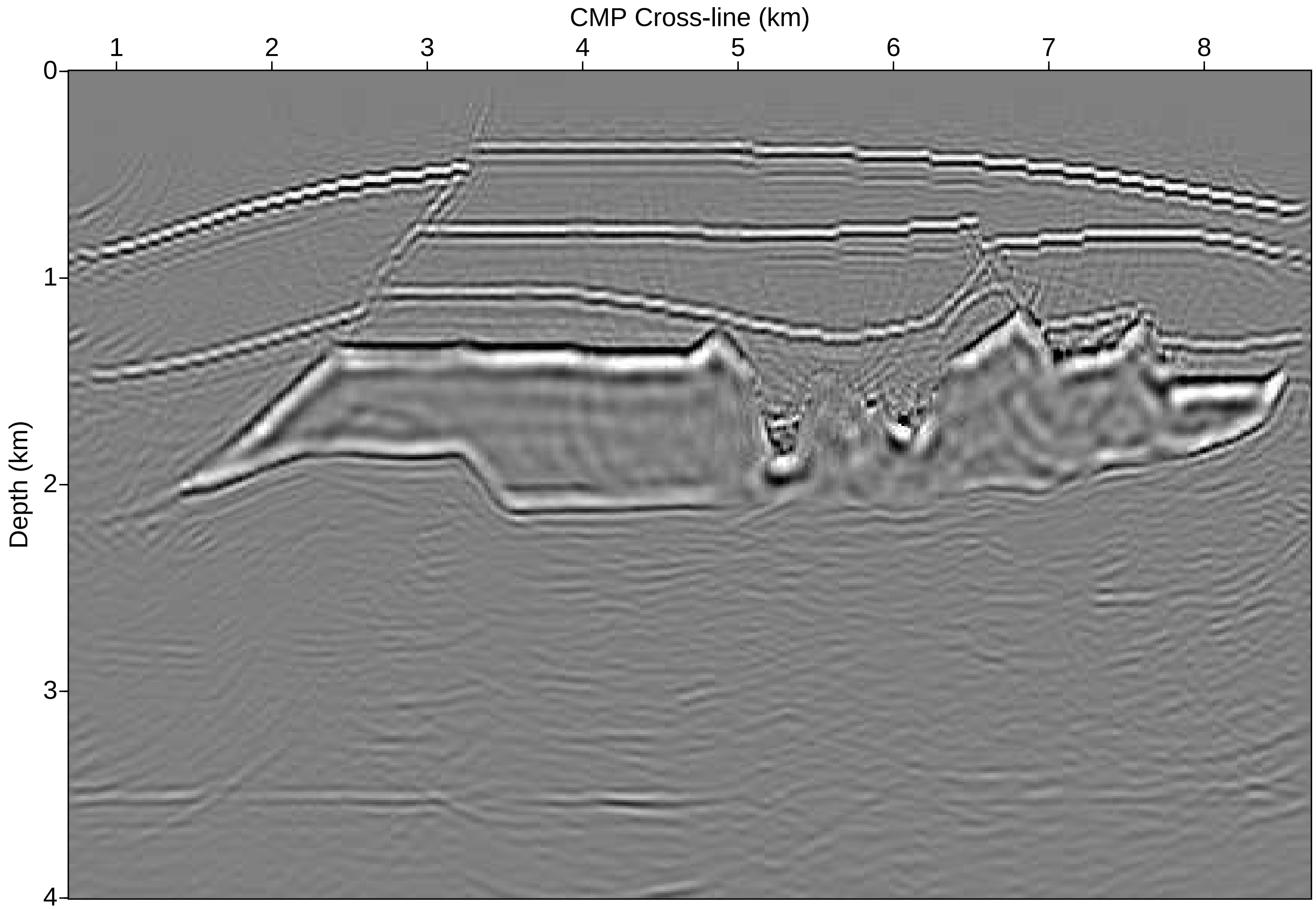
Same section obtained using COMAZ via GSP.



SEG-EAGE salt model: cross-line section at constant in-line coordinate $x = 7,440$m.



Same section obtained using split COMAZ via the explicit scheme.



Same section obtained using COMAZ via GSP.

Summary

- Broad overview of 3-D depth-extrapolation methods.
- Migration of common-offset common-azimuth data with an explicit scheme.
- Future work: extension to the full DSR operator?

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