The Rice Inversion Project

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Annual Project Review, 2002-3
Sponsors, 2003

Amerada Hess
ConocoPhillips
Landmark Graphics
Sensorwise (new!)
Shell Research
Western Geco
Agenda

0900-1200:

(1) WWS: Reverse Time S-G Migration and Differential Similarity
(2) PS: DS Velocity Analysis via Depth Extrapolation
(3) CS: Analysis of Phase Screen Depth Extrapolation

1200-1330:

Lunch, Cohen House

1330-1430:

Wrapup session
Reverse Time S-G Migration and Differential Semblance

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TRIP Review
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www.trip.caam.rice.edu
**Partially linearized seismic inverse problem** ("velocity analysis"): given observed seismic data $S^{\text{obs}}$, find smooth velocity $v(x)$ oscillatory reflectivity $r(x)$, functions of $x \in X$ so that

$$F[v]r \simeq S^{\text{obs}}$$

Scattering operator $F$ defined by acoustic "partially linearized" model: acoustic potential field $u$ and its perturbation $\delta u$ solve

$$\left(\frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) u = \delta(t)\delta(x - x_s), \quad \left(\frac{1}{v^2} \frac{\partial^2}{\partial t^2} - \nabla^2\right) \delta u = 2r\nabla^2 u$$

plus suitable bdry and initial conditions.

$$F[v]r(x_s, x_r, t) = \frac{\partial \delta u}{\partial t}(x_s, x_r, t)$$

where source positions $= \{x_s\}$, receiver positions $= \{x_r\}$. 
Agenda:

- How common offset and Claerbout’s survey sinking or shot-geophone migration are similar, and how they are different

- How to perform shot-geophone migration as a sequence of two-way reverse time shot-profile (“RTSG”) migrations

- How RTSG migration avoids kinematic artifacts

- How RTSG images arbitrary dips

- A new variant of differential semblance
Common Offset vs. Shot-Geophone.

Common features: both involve a prestack image or reflectivity volume $\tilde{X} = \text{many copies of subsurface } X$ parametrized by a bin parameter $h$ (half-offset)

*Physical* reflectivity volume produced from physical reflectivity by an *extension operator* $\chi$.

Prestack migration operator $\tilde{G}[v] = \text{adjoint of prestack modeling operator } \tilde{F}[v]$ (or closely related operator), parametrized by velocity function $v(x)$.

Reformulation of inverse problem = **velocity analysis**: given prestack data $d^{\text{obs}}$, find $v$ so that $\tilde{G}[v]d^{\text{obs}}$ is physical, i.e. lies in the range of $\chi$ (comes from a physical reflectivity).
Common offset prestack image volume: $X = \text{subsurface volume}$, $\Sigma_h = \text{set of half-offsets in data}$, $\tilde{X} = X \times \Sigma_h$, $\chi[r](x, h) = r(x)$.

Extended forward modeling op, applied to prestack reflectivity $\tilde{r}(x, h)$:

$$\tilde{F}[\nu]\tilde{r}(x_s, t, x_r) = \int dx \tilde{r}(x, h) \int ds \, g(x_m + h, t - s; x) g(x_m - h, s; x)$$

where $g(x_s, t; x)$ is acoustic Green’s function for source at $x_s$, or close relative, and $x_r$ is receiver coord, $x_m = \frac{1}{2}(x_r + x_s)$, $h = \frac{1}{2}(x_r - x_s)$.

If $\tilde{r}$ is physical, i.e. independent of $h$, then this reduces to usual integral representation (“Lippman-Schwinger equation”) of Born forward modeling.

**NB:** note that $\tilde{F}[\nu]$ is “block diagonal” - family of operators parametrized by $h$. 
\[ \tilde{G}[v] = \text{adjoint of } \tilde{F}[v]: \]

\[ \tilde{G}[v]d(x, h) = \]

\[ \int \, dx_s \int \, dt \, d(x_s, t, x_s + 2h) \int \, ds \, g(x_s + 2h, t - s; x)g(x_s, s; x) \]

Replace \( g \) by its usual h. f. asymptotic expansion

\[ g(x_s, t; x) \simeq A(x_s, x)\delta(t - T(x_s, x)) \]

and you have prestack Kirchhoff common offset migration. Add some more amplitude terms and you have Kirchhoff inversion (Beylkin 1985, Bleistein 1987).
Shot-geophone prestack image volume: $\Sigma_d = \text{somewhat arbitrary set of vectors near 0 ("depth half-offsets")}, \bar{X} = X \times \Sigma_d$

Physical reflectivity volumes $\chi[r](x, h) = r(x)\delta(h)$

Prestack forward modeling op, applied to prestack reflectivity $\bar{r}(x, h)$:

$$\bar{F}[v] \bar{r}(x_s, t, x_r) =$$

$$\int dx \int dh \bar{r}(x, h) \int ds g(x_s, t - s; x - h)g(x_r, s; x + h)$$

If $\bar{r}$ is physical, reduces to usual Born forward model.
Computing $\mathcal{G}[v]$: could produce Kirchhoff formula as in common offset case - nonstandard.


(1) assume *double square root* (“DSR”) hypothesis: all rays carrying significant energy are downgoing between source and reflection point or upcoming from reflection point to receiver.

(2) restrict offsets to be horizontal, i.e. $\mathbf{h} = (h_x, h_y, 0)$, and correspondingly restrict $\mathcal{F}$ to reflectivity volumes of the form

$$\tilde{r}_z(\mathbf{x}, \mathbf{h}) = \tilde{r}_z(\mathbf{x}, h_x, h_y) \delta(h_z)$$

Restricted operator $= \mathcal{F}_z[v] \tilde{r}_z$
Stolk and deHoop, TRIP 2001: up to a factor affecting amplitudes (neglected in standard implementations), (1) and (2) \[ \bar{F}_z[v]^*d(x, h) = w(x - h, x + h, 0) \] where \( w(y_s, y_r, t) \) solves 1-way wave equations in \( z \) and \( y_s, t, z \) and \( y_r, t \) resp.

This is the **survey-sinking** method of Claerbout: downward continue sources, downward continue receivers to same depth, read off image at \( t = 0 \).

Standard implementations in frequency, various one-way wave equation approximations (parabolic, phase screen, ...).

(Slightly different derivation: CIME notes, www.trip.caam.rice.edu)
Summary: comparison of common offset, shot-geophone migration operators

- both are adjoins of prestack modeling operators

- bin parameter is offset - restricted to surface data offsets for common offset, *unrestricted* for S-G (conventionally horizontal)

- physical prestack reflectivity volumes are different: independence from $h$ vs. focusing in $h$.

- Kirchhoff is available for shot-geophone (but never used!), *mandatory* for common offset
Reverse Time Shot-Geophone Migration

Based on wave equation solved by integral representation of modeling operator:

$$\bar{F}[\nu] \vec{r}(x_r, t; x_s) = \frac{\partial}{\partial t} \delta \bar{u}(x, t; x_s)|_{x=x_r}$$

where

$$\left( \frac{1}{v(x)^2} \frac{\partial^2}{\partial t^2} - \nabla_x^2 \right) \delta \bar{u}(x, t; x_s) = \int_{x+2\Sigma_d} dy \, \bar{r}(x, y) g(y, t; x_s)$$

(that’s the same $g$ as before, i.e. the causal Green’s function).
Specify adjoint field $w(x, t; x_s)$ as in standard reverse time prestack migration:

$$
\left( \frac{1}{v(x)^2} \frac{\partial^2}{\partial t^2} - \nabla^2_x \right) w(x, t; x_s) = \int dx_r \ d(x_r, t; x_s) \delta(x - x_r)
$$

with $w(x, t; x_s) = 0, t >> 0$. Then

$$
\tilde{G}[v]d(x, h) = \int dx_s \ \int dt \ g(x + 2h, t; x_s) w(x, t; x_s)
$$

i.e. exactly the same computation as for reverse time prestack, except that crosscorrelation occurs at offset $2h$ rather than 0. (Equivalent: Biondi and Shan, SEG 2002).
Implementation issues:

(1) Restricted offsets: simply set \( h_z = 0 \) in output (this is adjoint of \( \tilde{r} \mapsto \tilde{r} \)) to get \( \tilde{G}_z[v] \).

(2) Implementation using finite difference method: no additional expense over standard reverse time prestack, except for additional loop over offsets - one correlation of \( g, w \) per offset. Expense equivalent to one additional timestep per offset sample.

(3) For restricted offsets, eg. \( h_z = 0 \), simply don’t compute correlations for \( h_z \neq 0 \).
What should be the character of the image when the velocity is correct?

Hint: for simulation of seismograms, the input reflectivity had the form \( r(x)\delta(h) \).

Therefore guess that when velocity is correct, image is concentrated near \( h = 0 \).

Examples: 2D finite difference implementation of reverse time method. Correct velocity \( \equiv 1 \). Input reflectivity used to generate synthetic data: random! For output reflectivity (image of \( \tilde{F}_z[v]^* \)), constrain offset to be horizontal: \( \tilde{r}(x, h) = \tilde{r}_z(x, h_x)\delta(h_z) \). Display CIGs (i.e. \( x = \text{const.} \) slices of \( \tilde{r}_z \)).
Two way reverse time S-G image gathers of data from random reflectivity, constant velocity. From left to right: correct velocity, 10% high, 10% low.
Kinematics of reverse time S-G Migration

Advantage of “standard” (common shot) two way reverse time migration: images energy which violates DSR assumption (turning rays, overturned reflectors) - standard “survey-sinking” migration using depth extrapolation does not (see eg. recent TLE article by Lines et al.).

Same advantage accrues to reverse time shot-geophone migration (Biondi and Shan, SEG 2002).

Need to understand how events in data are imaged as reflectors in reflectivity volume $\bar{r}(x,h)$.

Mathematics = propagation of singularities, following Rakesh 1988; see WWS, Stolk, Biondi TRIP 2002.
Convenient domain for expression of kinematics: source receiver parametrization

\[ \bar{R}(y_s, y_r) = \bar{r}\left(\frac{y_s + y_r}{2}, \frac{y_r - y_s}{2}\right) \]

Events, reflectors as points in phase space:

Event ("element") in data: \((x_s, x_r, t, \omega p_s, \omega p_r, \omega)\)

Reflector in subsurface: \((y_s, y_r, k_s, k_r)\)
Imaging relation:

- source ray \((X_s, P_s), X_s(0) = x_s, P_s(0) = p_s\)

- receiver ray \((X_r, P_r), X_r(t) = x_r, P_r(t) = p_r\)

- at imaging time \(=\) time \(t_s\) along source ray, rays match reflecting element:
  - \(X_s(t_s) = y_s, \omega P_s(t_s) = -k_s\)
  - \(X_r(t_s) = y_r, \omega P_r(t_s) = k_r\)
\[ t_s + t_r = t \]
\[ t'_s + t'_r = t \]

\[ X_s(t'_s), P_s(t'_s) = y_s, -k_s/\omega \]

\[ X_r(t'_s), -P_r(t'_s) = y_r, -k_r/\omega \]
Obvious imaging ambiguity: given data event, correspondingrays, can choose any $t_s$ between 0 and $t$!

Convenient method to remove ambiguity (WWS, Stolk, Biondi,
TRIP 2002, see also Biondi and WWS, SEP 112 for another,
similar approach): restrict offset direction, as in original Claerbout
S-G.

Horizontal offsets: $h_z = 0$, i.e.

$$
\bar{r}_z(x, h) = \bar{r}_z(x, h_x, h_y) \delta(h_z)
$$

or in source-receiver coords

$$
\bar{R}_z(y_s, y_r) = \bar{R}_z \left( y_s, x, y_s, y_r, x, y_r, y, \frac{y_r, z + y_s, z}{2} \right) \delta(y_r, z - y_s, z)
$$

Implies phase space constraint: reflector lies in reduced phase
space of $\bar{R}$, wave vector $= (k_{s,x}, k_{s,y}, k_{r,x}, k_{r,y}, k_z)$ and $z$-imaging
condition is $X_{s,z}(t_s) = X_{r,z}(t_s), \omega(P_{s,z}(t_s) - P_{r,z}(t_s)) = k_z$. 

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Stolk and deHoop, TRIP 2001: **SUPPOSE**: DSR assumption: all significant energy to be imaged travels on downgoing source rays ($P_{s,z} > 0$) and upcoming receiver rays ($P_{r,z} < 0$). **NB**: must assume to use depth extrapolation in S-G migration.

**THEN**: Each event is imaged in *exactly one* reflector in the horizontal offset reflectivity volume $\tilde{R}_z$, whether the velocity is correct or not.

**PROOF**: obvious (picture).

**COROLLARY**: If the velocity is correct, and DSR holds, then S-G image gathers will be focussed (i.e. S-G version of semblance criterion will hold) - regardless of the complexity of the velocity field.
\( x_s, p_s \)

\( x_r, p_r \)

\( X_r(t_s), P_r(t_s) \)

\( X_s(t_s), P_s(t_s) \)

\( t_s + t_r = t \)
Why this is remarkable: analogous statement for common offset is \textit{false} i.e. common offset image gathers \textit{may not be flat} even when velocity is correct (Stolk, Stolk and WWS - TRIP 2001, after Nolan TRIP 1995 for common source).

Example: Gaussian lens over flat reflector at depth \( z \) (\( r(x) = \delta(x_1 - z) \), \( x_1 = \text{depth} \)).
Common offset migration of lens data. **Left:** image at offset $h = 0.3$ km **Right:** CIG at $x = 1.0$ km - not smooth in $h$!
S-G migration of lens data. **Left:** image \((h = 0\) section) **Center:** CIG at \(x = 1.0\,km\) **Right:** Angle CIG (Radon of CIG in \(h, z\))

[Thanks: Biondo Biondi]
Imaging arbitrary dips

DSR assumption, horizontal offset reflectivity incompatible with imaging reflecting elements with $k_z = 0$ (i.e. vertical reflectors): imaging condition is

$$\omega (P_{s,z}(t_s) - P_{r,z}(t_s)) = k_z$$

but DSR requires

$$P_{s,z} > 0, \quad P_{r,z} < 0$$

and these are incompatible with $k_z = 0$ unless $\omega = 0$. In practice: $k_z$ small $\Rightarrow$ low-frequency artifacts (“smearing”), see Biondi and WWS SEP 112, Biondi and Shan SEG 02.
Imaging (near-) vertical reflectors ⇒ give up DSR, permit *vertical offsets* \( \mathbf{h} = (0, h_z) \) (2D for simplicity - 3D similar), and correspondingly restrict \( \mathcal{F} \) to reflectivity volumes of the form

\[
\tilde{r}_x(x, \mathbf{h}) = \tilde{r}_x(x, h_z) \delta(h_x)
\]

Restricted operator = \( \mathcal{F}_x[v] \tilde{r}_x \)

As before, to get adjoint \( \tilde{\mathcal{G}}_x[v] \) simply set \( h_x = 0 \) in output of \( \tilde{\mathcal{G}}[v] \).

Two image volumes: \( \tilde{\mathcal{G}}_z[v]d \), smeared near vertical reflectors, and \( \tilde{\mathcal{G}}_x[v]d \), smeared near horizontal reflectors.
A solution (Stolk, WWS, Biondi, 2003 - see Biondi and WWS SEP 112 for another approach): introduce dip filters $\Pi_x, \Pi_z$ with

$$\Pi_x(0, k_s, z, k_r, z) = 0, \quad \Pi_z(k_s, x, k_r, x, 0) = 0$$

and define a total forward map on pairs of reflectivity volumes

$$\tilde{F}_t[v] (\tilde{r}_x, \tilde{r}_z) = \tilde{F}_x[v](\Pi_x\tilde{r}_x) + \tilde{F}_z[v](\Pi_z\tilde{r}_z)$$

Adjoint $\tilde{G}_t[v]$ outputs filtered restricted offset reflectivities with smearing removed. But that is not all...
For correct velocity, images focus (source, receiver rays intersect) at \( h = 0 \) at imaging time \( t_s \). S-G imaging condition reduces to usual Snell’s law at these points.

Because of imaging condition, rays focusing at \( k_z \neq 0 \) must have \( P_{r,z} - P_{s,z} \neq 0 \) \( \Rightarrow \) depth components of source, receiver rays must separate immediately, i.e. \( h_z = 0 \) is violated for times near \( t_s \). Leads to generalization of Stolk-deHoop theorem:

**Local Focussing Theorem:** If the velocity is correct, the filtered image volumes are focussed at \( h_z = 0 \) resp. \( h_x = 0 \) within a corridor of width \( h_c \), i.e. \( |h_x|, |h_z| < h_c \).

[Does energy focus outside the corridor? Probably. Stay tuned.]
Differential semblance

Quantifying the semblance principle: devise operator $W$ for which $W\bar{r} \simeq 0$ is equivalent to $\bar{r}$ being physical, at least approximately.

Then minimize w.r.t. $v$ a suitable norm
\[ J[v] = \frac{1}{2} \| W\bar{G}[v] d^{\text{obs}} \|^2 \]

Given size of these problems, want to use if possible descent-based methods, which require smoothness of objective.

Stolk and WWS TRIP 2002 (published in IP, 2003): The only operators $W$ which work are pseudodifferential = compositions of differential operators and $|k|^p$ filters.
For common offset, physical $= \text{ does not depend on offset, so only choice of } W \text{ is}

\[ W = P \nabla_h \]

with $P$ a $\Psi$DO of order $-1$. Hence name of this technique: 
* differential semblance *

For S-G, physical $= \text{ focussed at } h = 0$, hence necessarily

\[ W = Ph \]

with $P$ a $\Psi$DO of order 0 (Stolk 2000, Stolk & deHoop 2001).
Ongoing Work

(1) implementation of DSR-based DS using one-way propagators (Shen, Stolk), demonstration of Stolk-deHoop focussing property and VA in presence of multipathing

(2) implementation of RTSG-based DS using FD WE solvers (WWS)

(3) design of noise suppression, antialiasing for these operators (Shen, WWS)

(4) further study of one-way propagators (Stolk)

(5) theoretical study of S-G based DS (WWS)