Coherent noise suppression in velocity inversion[1]

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ABSTRACT

Data components with well-defined moveout other than primary reflections are sometimes called *coherent noise*. Coherent noise makes velocity analysis ambiguous, since no single velocity function explains incompatible moveouts simultaneously. Contemporary data processing treats the control of coherent noise influence on velocity as an interpretive step. Dual regularization theory suggests an alternative, automatic inversion algorithm for suppression of coherent noise when primary reflection phases dominate the data. Experiments with marine data illustrate the robustness and effectiveness of the algorithm.

INTRODUCTION

Velocity analysis quantifies and parametrizes moveout in terms of velocity functions. In common with most other parts of conventional processing, velocity analysis rests on the linearized model of reflections, which treats short scale components of Earth mechanical structure function as perturbations of the large scale components. In fact, most processing derives from the acoustic version of this model, which predicts only one family of moveout curves, or phases, those of so–called compressional wave primary reflections.

Some seismic data exhibit the characteristics predicted by this *acoustic primaries–only model* to good approximation. Other data exhibit several phases, however; while one of these usually appears to be a compressional wave primary phase, others may represent multiple reflections, mode conversions, 3D reflection phenomena in data treated as 2D, and so forth. These other phases may carry considerable energy. Multiple reflection energy is suppressed to some extent by various multiple removal techniques, but none is universally effective in removing all phases but the primary. Residual non-primary phases pose an obstacle to velocity estimation, in that a single velocity function cannot predict several moveout families simultaneously (within the linearized acoustic approximation). The conventional approach to moveout ambiguity is visual and interpretive: the processor is expected to reject coherent

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noise by interactively updating velocity functions to recognize and flatten selectively the primary events in image gathers, recognize and fit only primary reflection peaks, and so on. It seems odd that the most robust information in seismic data must in the end be teased out by hand, especially as the size of 3D datasets precludes visual inspection of all but a small fraction of the prestack data.

This paper presents an alternative approach to coherent noise rejection, based on a formulation of velocity analysis as an inverse problem, when primary reflection energy dominates the (preprocessed) data. The idea is quite simple. Flatness of image gathers diagnoses the success of a velocity analysis. Image gathers created from data containing multiple phases are impossible to flatten. Therefore creation of flat image gathers requires data perturbation. If the primary phase is dominant, then *the smallest data perturbation permitting flat image gathers will be that which removes the non-primary phases.*

The mathematical embodiment of this idea is the *dual regularization theory* of velocity inversion, introduced in Gockenbach et al. (1995); Gockenbach and Symes (1997). Given a relative noise level σ, we seek the velocity function v and the data perturbation of root mean square relative size at most σ which together yield the flattest image gather. To measure flatness, we use the *differential semblance* criterion, introduced in Symes (1986) and developed in a series of papers [for example Symes (1998b); Chauris et al. (1998)]. In fact, differential semblance is the only semblance measure providing the good theoretical properties needed to ensure the reliability of coherent noise rejection (Kim and Symes, 1998; Symes, 1998a).

Thie next section describes a simple algorithm for solution of this constrained optimization problem. An example using a marine CMP and layered acoustic modeling demonstrates the coherent noise rejection permitted by reasonable estimates of noise level σ. The final section summarizes our conclusions and formulates a few directions for further research.

ALGORITHM

Given a relative noise level σ, we seek the velocity function v and the data perturbation of root mean square relative size at most σ which together yield the flattest image gather. Transformation of this idea into an implementable algorithm requires definition of operators, functions, and optimization methods. This section gives a sketch of these mathematical details.

The *forward map* $F[v]$ is a linear operator depending on a velocity function v. The velocity function depends on all or part of the subsurface coordinates; the examples presented below use depth-dependent velocity. $F[v]$ is a *prestack* forward modeling operator; it takes an image volume or bin-dependent reflectivity as input, and outputs a seismic data volume. In the examples presented below, the data will be a common midpoint gather, each bin will contain a single trace, and the bin parameter is offset. Thus the input reflectivity also has the appearance of a common midpoint gather, and can be identified with the image gather in this setting.

The *inverse map* $G[v]$ is an approximate inverse to $F[v]$. That is, if data d and reflectivity r satisfy $d = F[v]r$, then $r \simeq G[v]d$. For multioffset data and multidimensional models, Beylkin (1985) showed how to build such operators as weighted diffraction sums. For layered modeling, $G[v]$ is essentially moveout correction, after compensation for amplitude and wavelet deconvolution; $F[v]$ inverts these steps.

Differential semblance measures nonflatness by comparing neighboring image bins. That is, if the image is $r = G[v]d$, and the bin index is i, then the differential semblance is the mean square power of $(Dr)_i = \text{const.} \times (r_{i+1} - r_i)$. A convenient notation for root mean square of a field, say r, is $\|r\|$. The dot product of two fields (viewed as vectors of samples), say r_1 and r_2, is $< r_1, r_2 >$. Thus $\|r\|^2 =< r, r >$. The basic ("raw") semblance operator is $W[v] = F[v]DG[v]$. The application of the modeling operator $F[v]$ after formation of the bin difference makes the power of the output independent of amplitude, up to an error which decays with increasing signal frequency. [This trick was discovered by Hua Song (Song, 1994)]. Since the data is differenced in formation of $W[v]d$, we bring its high frequency content back into consistency with that of the data *via* a *smoothing operator* H of order -2 (k^{-2} filter).

The dual regularization objective function J_σ is then

$$J_\sigma[v; d] = \min_r \frac{1}{2} < W[v]r, HW[v]r > \ \text{subj}\ \|r - d\| \leq \sigma$$

Note that the differential semblance objective explored in the above cited references is the special case of this one with $\sigma = 0$. In general, a Lagrange multiplier λ exists for which the solution r satsifies the *normal* and *secular* equations:

$$W[v]^T HW[v]r + \lambda(r - d) = 0, \ \|r - d\| = \sigma$$

These two equations together determine $\lambda = \lambda[v; \sigma]$ and $r = r[v; \sigma]$. Thus J_σ is

$$J_\sigma[v; d] = \frac{1}{2} < W[v]r[v; \sigma], HW[v]r[v; \sigma] >$$

First order perturbation with respect to v, $v \to v + \delta v$, gives

$$\delta J_\sigma[v; d] =< W[v]r[v; \sigma], H\delta W[v]r[v; \sigma] >$$

after simplifications due to the normal and secular equations. From this expression follows a formula for the gradient of J_σ in terms of the first order perturbation of $W[v]$ and its adjoint operator, which may in turn be expressed as products of the operators $F[v]$, D, and $G[v]$, their first order perturbations, and the adjoints of these.

Besides accurate numerical implementations of the operators described above, we require methods for solving the system of normal and secular equations. For the latter, we use the Moré-Hebden algorithm (Björk, 1997) with the linear systems occuring in this method solved approximately by conjugate gradient iteration. The best estimate of v results from gradient-based optimization (a quasi-Newton method) applied to J_σ. In the experiments reported here, we have used the Limited Memory Broyden-Fletcher-Goldfarb-Shanno method as developed in Nocedal (1980).

EXAMPLE: A CMP FROM THE NORTH SEA

Figure 1 shows a common midpoint gather from the Mobil AVO data set (Keys and Foster, 1998). This part of the North Sea covers relatively flat lying sediments to a depth of 2 s, where an unconformity introduces older, more deformed rock which is nonetheless still for the most part flat lying. Therefore layered modeling seems reasonable for this data, at least to perhaps 3 s and as a first approximation. The work reported here views whatever converted wave energy is present in the data as noise, so acoustic modeling is reasonable. Finally, as the range of offsets in this data is modest (2.5 km maximum), the hyperbolic moveout approximation seemed likely to be adequate, at least when combined with an agressive mute as displayed in Figure 1.

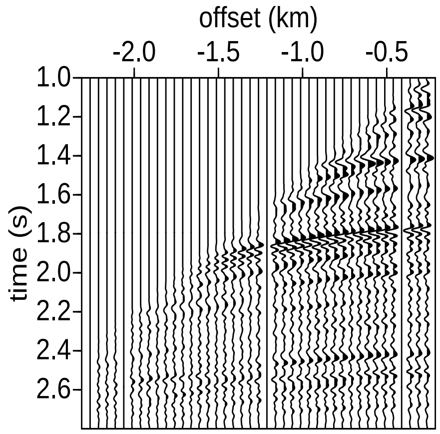

Figure 1: CMP from Mobil AVO data. cmpfig [NR]

Strong surface related multiple energy is characteristic of this region. The main pre-processing steps were hyperbolic Radon filtering and bandpass filtering. The Radon filter suppressed but did not entirely remove coherent noise: it seemed reasonable to hope that primary energy dominates the filtered data, as is required by the dual regularization strategy. The bandpass filter ensured that the data were not spatially aliased, so that the differential semblance could be computed accurately.

Minimization of $J_0[v; d]$ produced the RMS velocity displayed in Figure 2, which exhibits a characteristic feature of the differential semblance function: when faced with contradictory moveout (as for example in the interval 1.8-2.4 s), it averages the apparent velocities to come as close as possible to flattening all events. The moveout corrected data ($G[v]d$ in the notation of the last section) displayed in Figure 3 shows a mixture of overcorrected and undercorrected events. The minimization process (*via* a quasi-Newton algorithm) required approximately 12 s on an SGI Origin2000 processor.

We used the output of the $J_0[v; d]$ minimization as the initial guess for minimization of $J_{0.5}[v; d]$; the latter required approximately 3 min on the Origin. Figure 4 shows that $\sigma = 0.5$ was not a bad guess at the level of coherent noise: the automatic velocity analysis

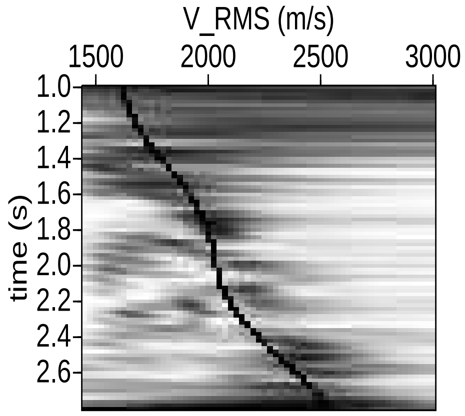

Figure 2: V_{RMS} from velocity inversion, $\sigma = 0.0$, overplotted on velocity spectrum. Note that the estimated RMS velocity navigates between peaks. sigdisplay0 [NR]

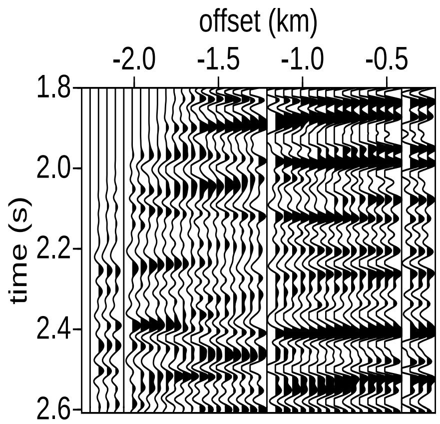

Figure 3: Image gather (= NMO corrected CMP) using V_{RMS} from velocity inversion with $\sigma = 0.0$. Note residual curvature in all events. siginvdata0 [NR]

has now essentially ignored slow (multiple reflection) and smaller fast (steeply dipping or out of plane) phases and placed its estimate of RMS velocity squarely in the main corridor of apparent primary reflection phases, as one also sees in the the conventional image gather (= moveout corrected data $G[v]d$, Figure 5). Dual regularization also produces an *inverted reflectivity* ($r[v;\sigma]$ in the last section, Figure 6) or denoised data, which is superior to the conventional image gather as a basis for further processing.

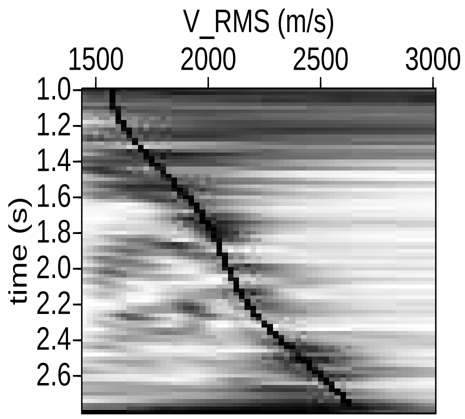

Figure 4: V_{RMS} from velocity inversion, $\sigma = 0.5$, overplotted on velocity spectrum. Note that the estimated RMS velocity picks apparent primary phases. sigdisplay5 [NR]

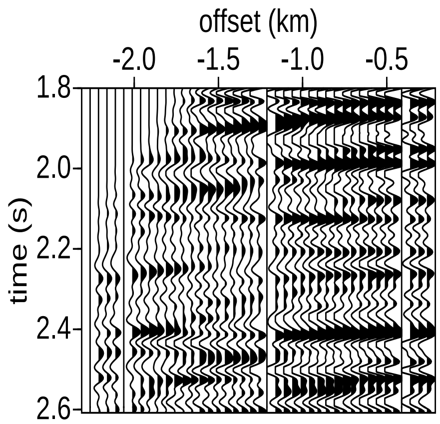

Figure 5: Image gather (= NMO corrected CMP) using V_{RMS} from velocity inversion with $\sigma = 0.5$. Primary reflections are essentially flat. siginvdata5 [NR]

DISCUSSION

The results exhibited in the last section are characteristic of dual regularization: it robustly identifies the dominant moveout trend, so long as the linear solves in the Moré-Hebden

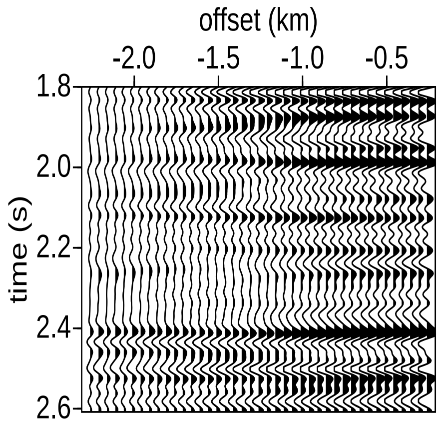

Figure 6: Inverted reflectivity, $\sigma = 0.5$, essentially a cleaned up version of the data, with non-primary phases suppressed. sigestrefl5 [NR]

algorithm are sufficiently precise. It is also a great deal more expensive than "raw" differential semblance (J_0) minimization, often by a factor of 10-15, due to the need to solve linear systems.

Note that dual regularization does not eliminate the need for preprocessing to reduce coherent noise: the desired primary reflection phase must be energetically dominant, whereas very strong multiple reflection phases are common in some areas.

Speed improvements should be possible through better heuristics and algorithmic tuning, and perhaps through more effective constrained optimization. Since differential semblance is also effective in estimating laterally heterogeneous models (Symes and Versteeg, 1993; Chauris et al., 1998), dual regularization can also be applied in that context; of course, algorithmic efficiency will then become even more of an issue.

Dual regularization also implies a strategy for determination of σ: it should assume the smallest value for which the minimum of J_σ is (essentially) zero. Application of this noise level determination algorithm requires a method for estimating a tolerance for this minimum, a matter currently under study.

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