Determining Depth Function for Marine Streamer via Interior-Point Optimization

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ABSTRACT

Accuracy of data processing for offshore seismic exploration experiments can be enhanced with improved characterizations of marine streamer (cable) depth functions. Using the data set from a shot record, we formulate the cable depth function problem as a convex optimization problem. This optimization problem seeks to minimizes the curvature of the desired cable depth function subject to bounds imposed by depth measurements and physical boundaries. We employ an interior-point optimization algorithm to solve the optimization problem, resulting in physically accurate cable depth function characterizations. The constrained optimization approach proposed here overcomes some of the problems of interpolation methods, such as cable depth functions with unrealistic "kinks", "wiggles", and constraint violations. Numerical approximations will be illustrated.

1. INTRODUCTION

Marine streamer (cable) characterization is a very important aspect of off-shore seismic data processing. The streamers contain hydrophones and depth measurement devices which are towed by a ship. Since these cables contain source-receiver positioning for seismic experiments, it is important to have an accurate mathematical description of the cables.

2. PROBLEM FORMULATION

We consider a marine streamer which contains 240 seismic channels with depth measurement devices at every 16th channel (i.e., channel 16, 32, 48, ...). The depth readings are supplied to the nearest meter (integer data). The depth reading for channel 1, however, is unavailable due to equipment malfunction, so the far offset depth is not directly constrained, injecting an element of extrapolation into the problem.

Given streamer depth measurements, d_i, at control points, X_i, $i = 1, ..., L$ (offset or channel number...), we seek to find a cable depth function $u(x)$, discretely on a fine grid,

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x_j, $j = 1, ..., N$ so that curvature (or something similar) of the streamer is minimized, and control point data are fit to within prescribed measurement precision. The optimization problem is given by

$$\min \quad \widehat{J}(u) = \int_{x_{\min}}^{x_{\max}} dx \, |\frac{d^2u}{dx^2}|^2 \tag{1}$$

subject to

$$\begin{array}{ll} & |u(X_i) - d_i| \leq \varepsilon, \\ \text{and} & \\ & \min_i\{d_i\} - \varepsilon \leq u(x) \leq \max_i\{d_i\} + \varepsilon, \end{array} \tag{2}$$

where $i = 1, ..., L$, depth function u is assumed continuous with at least two derivatives, N = total number of discrete grid points, and L = total number of control points. The constraint, $|u(X_i) - d_i| \leq \epsilon$, represents the measurement precision at the ith control point. The control nodes, X_i, are a subset of the grid, x_j. Furthermore, regular spacing of the control points is not a requirement of the formulation or the solution.

2.1. Discretization of Optimization problem

To obtain an approximation of the optimization problem defined by (1) and (2), we discretize the problem. For the objective (1) the domain is partitioned so that

$$x_{\min} = x_1 < x_2 < \cdots < x_{N-1} < x_N = x_{\max},$$

where $x_j = x_{\min} + (j-1)\,h$ and $h = \frac{x_{\max} - x_{\min}}{N-1}$. The integrand is approximated using finite difference

$$\begin{aligned} D_{xx}u|_{x_1} &\simeq \frac{1}{h^2}\left(u_1 - 2u_2 + u_3\right), \\ D_{xx}u|_{x_j} &\simeq \frac{1}{h^2}\left(u_{j-1} - 2u_j + u_{j+1}\right), \qquad j = 2 : N-1 \\ D_{xx}u|_{x_N} &\simeq \frac{1}{h^2}\left(u_{N-2} - 2u_{N-1} + u_N\right). \end{aligned}$$

where $u_j \equiv u(x_j)$. There is no requirement in the formulation for "endpoint curvature" estimates. Using the finite difference approximation of the integrand, the objective value is then approximated using trapezoidal quadrature:

$$\begin{aligned} \widehat{J}(u) &= \int_{x_{\min}}^{x_{\max}} dx \, |\frac{d^2u}{dx^2}|^2 \\ &\simeq \frac{h}{2}|D_{xx}u\Big|_{x_1}^{2} + h\sum_{i=2}^{N-1}|D_{xx}u\Big|_{x_j}^{2} + \frac{h}{2}|D_{xx}u\Big|_{x_N}^{2} \\ &= J(u) \end{aligned}$$

To express the given optimization problem (1) and (2) in the form of a standard convex programming problem (Fiacco and McCormick, 1990), we consider the first constraint expression in (2),

$$|u(X_i) - d_i| \leq \varepsilon, \quad i = 1, ..., L.$$

For X_i a control node, this constraint enforces the depth measurement precision producing $2L$ bound constraints,

$$d_i - \varepsilon \leq u(X_i) \leq d_i + \varepsilon \quad i = 1, ..., L. \tag{3}$$

The second constraint expression in (2) is a constraint on the discrete approximation $u(x_j)$ of $u(x)$. Thus the constraint can be written as

$$\min_i\{d_i\} - \varepsilon \leq u(x_j) \leq \max_i\{d_i\} + \varepsilon, \quad j = 1, ..., N \tag{4}$$

where i ranges over the control nodes. If x_j is a control node, (3) and (4) imply

$$\max\{\min_i\{d_i\} - \varepsilon, d_j - \varepsilon\} \leq u(x_j) \leq \min\{\max_i\{d_i\} + \varepsilon, d_j + \varepsilon\}.$$

Else, if x_j not a control node, (3) and (4) imply

$$\min_i\{d_i\} - \varepsilon \leq u(x_j) \leq \max_i\{d_i\} + \varepsilon.$$

Thus, all the constraints can be written as

$$m_j \leq u(x_j) \leq M_j \quad j = 1, ..., N$$

or

$$\begin{aligned} g_1(u_j) &= u(x_j) - m_j \geq 0 \\ g_2(u_j) &= M_j - u(x_j) \geq 0, \end{aligned}$$

where

$$m_j = \max\{\min_i\{d_i\} - \varepsilon, d_j - \varepsilon\} \text{ or } \min_i\{d_i\} - \varepsilon$$

and

$$M_j = \min\{\max_i\{d_i\} + \varepsilon, d_j + \varepsilon\} \text{ or } \max_i\{d_i\} + \varepsilon.$$

The discrete form of the optimization problem can thus be written as

$$\begin{aligned} &\min\ J(u_i) \\ s.t.& \\ &g_1(u_j) = u(x_j) - m_j \geq 0 \quad j = 1, ..., N, \\ &g_2(u_j) = M_j - u(x_j) \geq 0 \quad j = 1, ..., N. \end{aligned}$$

Letting $g(u) = [g_1(u_i); g_2(u_i)]$, i.e., $g : \mathbf{R}^N \to \mathbf{R}^{2N}$, we reduce the problem to the following standard form convex programming problem:

$$\begin{aligned} &\min\ J(u_i) \\ s.t.& \\ &g(u_i) \geq 0 \quad i = 1, ..., N. \end{aligned} \tag{5}$$

2.2. Algorithm to Solve Optimization problem

In this section we formulate the primal-dual Newton interior-point method for the standard form convex programming problem (5). Our approach will be a damped Newton method applied to the perturbed-Karush-Kuhn-Tucker (PKKT) conditions (El-Bakry et al., 1996).

The Lagrangian associated with problem (5) is

$$l\left(u,z\right)=J\left(u\right)-z^{T}g$$

and the KKT (or first order necessary) conditions are

$$\begin{aligned} \nabla_{u}l\left(u,z\right) &= 0 \\ g\left(u\right) &\geq 0 \\ Zg\left(u\right) &= 0 \\ z &\geq 0 \end{aligned}$$

where $\nabla_{u}l\left(u,z\right)=\nabla F\left(u\right)-\nabla gz$ and $Z\equiv diag\left(z\right)$. The slack variable form of the KKT conditions is

$$F\left(u,s,z\right)=\begin{pmatrix} \nabla_{u}l\left(u,z\right) \\ g\left(u\right)-s \\ Zs \end{pmatrix}=0,\quad (s,z)\geq 0. \tag{6}$$

There exist numerical instabilities in applying a Newton method to the KKT conditions, which derive from the complementarity equation, $Zs=0$, in (6). In fact, in any Newton's method formulation, we deal with linearized complementarity,

$$Z\Delta s+S\Delta z=-SZe$$

where $e\equiv(1,1,\ldots,1)^{T}$, and $S\equiv diag\left(s\right)$. The algorithmic properties of linearized complementarity was observed by Tapia (1980) for the general nonlinear programming problem. The linearized complementarity equation that results from applying Newton's method to the KKT conditions has the flaw that global convergence may be precluded (El-Bakry et al., 1996). That is, if a component $\left[s_{k}\right]_{i}$ of a current iterate becomes zero and $\left[z_{k}\right]_{i}>0$, then from the linearized complementarity equation, we see that $\left[s_{k}\right]_{i}=0$ for all $p>k$; i.e., this component will remain zero in all future iterations. A fix for this problem is to provide a means for zero variable to become nonzero in subsequent iterations. This is accomplished by replacing the complimentarity equation in (6) with a perturbed complimentarity equation, $Zs=\mu e$, $\mu>0$. This, modification of the KKT conditions results in the following perturbed-KKT conditions

$$F_{\mu}\left(u,s,z\right)=\begin{pmatrix} \nabla_{u}l\left(u,z\right) \\ g\left(u\right)-s \\ Zs-\mu e \end{pmatrix}=0,\quad (s,z)\geq 0 \tag{7}$$

for a perturbation parameter μ.

We apply a damped Newton method to the perturbed-KKT

$$F'_\mu(v)\,w = -F_\mu(v)$$

to obtain the update $v_+ = v + \alpha w$, where $v = (u, s, z)^T$ and $w = (\Delta u, \Delta s, \Delta z)^T$. The parameter μ is decreased at each iteration, and in the limit the PKKT conditions approximate the KKT conditions of the original problem. The Jacobian of $F_\mu(u, s, z)$ is given by

$$F'_\mu(u, s, z) = \begin{pmatrix} \nabla_u^2 l(u, z) & 0 & \nabla g \\ \nabla g^T & I & 0 \\ 0 & Z & S \end{pmatrix}.$$

The sequence of iterates generated by this method converge to the solution of (5).

3. RESULTS

In an offshore seismic exploration experiment, we want a marine streamer depth function for each individual shot. The following result shows problem data for one shot along with the characterization obtained from the algorithm described in Section 2.2. The problem data are given in Table 1. The control node separation, is given in terms of channels, which happen to be spaced slightly nonuniformly at about 12.5 meters separation. The solution of the optimization problem problem is given in Figure 1.

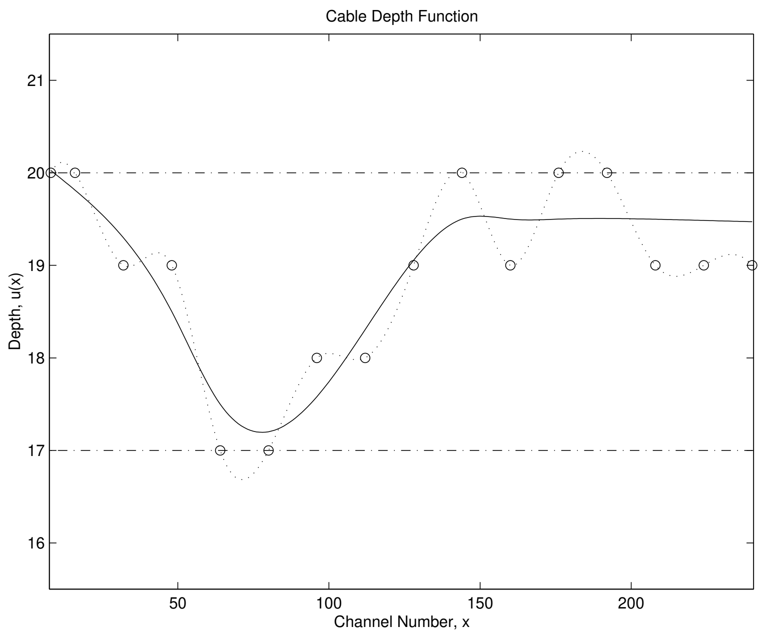

FIG. 1. Marine streamer depth function function characterization for a streamer with 240 channels and depth measurements at every 16th channel. Solid line is the constrained optimization solution and the dotted line is a cubic spline approximation.

Depth Function Problem Data	
Channel	Depth
8	114*
16	20
32	19
48	19
64	17
80	17
96	18
112	18
128	19
144	20
160	19
176	20
192	20
208	19
224	19
240	19

Table 1. Depth Measurements. The control node seperation, is given in terms of channels, which happen to be spaced slightly nonuniformly at about 12.5 meters separation. The optimization problem measurement precision $\epsilon = 0.5$ ft.

4. DISCUSSION

The numerical results of the paper suggest that the marine streamer depth function problem is not a simple interpolation problem, since interpolation schemes fail to provide satisfactory characterizations of the marine streamer. For example, linear interpolation (connecting the dots) is not physical, as it implies unrealistic kinks in the streamer. This is illustrated in Figure 2.

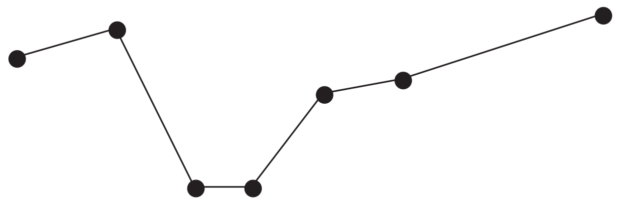

FIG. 2. Illustration of linear interpolation scheme for marine streamer depth measurements.

A spline interpolation of the measurements gives a smooth, more physically reasonable interpolation/extrapolation. However the resulting marine streamer characterization tends to be "too-wiggly." That is, there exist lots of overshoots and undershoots between control points, in some cases placing the streamer above the surface of the sea or below the seafloor! This is, of course, a well-known problem with splines, inherent, more or less, in the polynomials used for the fitting process. Figure 3 provides a schematic of the overshoot and undershoot involved in spline interpolation.

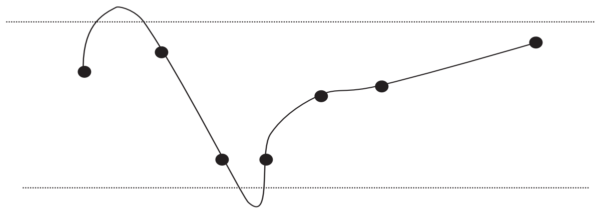

FIG. 3. Illustration of spline interpolation scheme for marine streamer depth measurements.

Splines under tension and related splines based on covariance allow for measurement noise and errors, though not directly for measurement precision as imposed with our convex optimization problem. These modified spline interpolation methods allow one to place L_2 or L_1 error bars at each measurement and, after some iteration, fit a smoother spline that has a compatible distribution of errors.

In the case of the marine streamer depth function problem, our error bars are measurement precision, i.e. L_∞ error bars, which is directly addressed in our formulation of the problem. A direct comparison of the cubic spline interpolating function to the solution of the constrained convex optimization model is illustrated in Figure 1.

5. CONCLUSION

We have shown that casting the marine streamer depth function problem as a constrained optimization problem yields a good characterization of the marine streamer. The resulting discrete approximation of the marine streamer depth function is physically realistic, and satisfies error-bars for depth measurements.

Future directions for the marine streamer depth function characterization problem include: incorporate statistical constraints on the distribution of the data residuals, incorporate constitutive model for streamer (i.e., elastic body in fluid), and proof of principle. The proof of principle involves showing that ignoring accurate marine streamer depth function characterization yield sub-optimal seismic data processing (e.g., incorrect traveltimes, incorrect reflector positions, etc...).

In terms of future extensions, an intriguing possibility for further improvement and stabilization of the estimated cable depths is to formulate a “fuzzy” surface-consistent constraint expressing the notion that the body of the towed cables tends to follow the trail “blazed” by the head of the cable.

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