3-D tomographic velocity analysis in transversely isotropic media

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Summary

Most current anisotropic velocity analysis deals with non-hyperbolic normal-moveout (NMO), with application to prestack time processing. In contrast, we present a general algorithm for 3-D velocity updating with tomographic velocity analysis, with application to 3-D prestack depth imaging. First, we outline the key components of tomographic velocity analysis in transversely isotropic media with arbitrary axis of symmetry (TTI media). Second, we derive an explicit representation of traveltime derivatives in weak transversely isotropic media and present the corresponding analytic formulae. Finally, we demonstrate the effectiveness of this procedure with a simple synthetic data example.

Introduction

Because velocity information is needed for depth migration, accurate knowledge of seismic interval velocities is essential for imaging subsurface structures. With correct velocities, 3-D prestack depth migration positions reflected seismic events at their correct subsurface locations. The presence of seismic anisotropy (angle-dependent velocity) makes it necessary to generalize traditional isotropic velocity analysis to account for anisotropy. A number of approaches to anisotropic traveltime inversion and velocity analysis have been developed (Tsvankin and Thomsen, 1995; Alkhalifah and Tsvankin, 1995; Baan and Kendall, 2002). Most of these approaches analyze higher-order terms in the NMO equation in order to improve images obtained in time processing.

Such traditional non-hyperbolic NMO velocity analysis is often used for building initial velocity/anisotropy models for prestack depth migration. Then the interval velocities required by depth imaging are obtained from these initial models by a layer-stripping process using the Dix formula. This layer stripping can accumulate errors with increasing depth during the inversion process. As an alternative, we propose a tomographic velocity analysis method, in which interval velocities and anisotropy parameters are obtained by globally solving a linear system of equations. This partially solves the problem of error accumulation intrinsic to the layer stripping process with the Dix formula. In addition to avoiding the error accumulation problem, the tomographic method applies ray tracing to recover the specular ray paths, accommodating the ray-bending effect due to velocity heterogeneity that Dix-based methods ignore.

Analysis of depth-migrated gathers (common image gathers, or CIG's), is the basis of most current interval velocity estimation techniques. These gathers should consist of flat events when the velocity/anisotropy model used for the migration is correct. Inaccurate estimates for either velocity or anisotropy parameters will result in nonzero residual moveout in CIGs. The goal of interval velocity analysis is to estimate velocities and anisotropy parameters that flatten events in the gathers, and seismic tomography uses depth migration as a tool to that end.

In this abstract, we use the fixed-time event tomography method (van Trier, 1990; Zhou et al., 2001) to build updating equations for parameters in transversely isotropic media. This method works by using true zero-offset or near-offset events in the time (unmigrated) domain as its reference events, which are recovered based on the principle that ray tracing (modeling) undoes migration whether the migration velocity is accurate or not. The method also uses a specular ray pair for each offset; these raypaths are traced from the migrated depth for the given offset. We extend the fixed-time method to the case of 3-D anisotropic velocity analysis by incorporating anisotropy into the raytracing, and by providing a method for estimating the anisotropy parameters.

We examine the effectiveness of the algorithm with a simple synthetic example.

Algorithm

Prestack depth migration produces common image gathers at specified surface locations. Each trace in the CIG represents the result of depth-migration for a small range of offsets. Given the picked image locations and their set of offsets, we build the linear system of tomography equations for transversely isotropic media with the following steps:

- Shoot the rays up from the given image location along a reflector to find the corresponding incident and reflected ray pairs associated with this location and the given offset. The specular ray pairs should satisfy (anisotropic) Snell's law.
- Calculate the derivatives of traveltime with respect to the unknown parameters, which include velocities and anisotropy parameters (Thomsen, 1986) in each layer.
- Build the linear system of equations with each row related to its corresponding specular ray pairs using the fixed-time event algorithm discussed below.
- Solve the linear equation using least-squares method to obtain the solution of the model such as velocity perturbations.

- Update to get new image locations.
- Repeat the above procedure if necessary.

In the following subsections, we will discuss some of the key components for carrying out the above steps.

- **Finding the reflection angle with Snell's law**

Tomographic velocity analysis depends on finding the specular ray pairs at the given image locations. To obtain the shooting phase angle of the reflected ray (θ_2), we need to apply Snell's law, which is written as (see Figure 1):

$$p = \frac{\sin(\theta_1)}{v(\overline{\theta_1})} = \frac{\sin(\theta_2)}{v(\overline{\theta_2})} \tag{1}$$

where *p* is known as the ray parameter or "wavefront parameter", which we can obtain from the given incident phase angle θ_1 (measured from the normal to the reflector) and the phase velocity $v(\overline{\theta_1})$. Here, $\overline{\theta_1}$ is the angle between the symmetry axis and the incident phase slowness vector. Similarly, $v(\overline{\theta_2})$ is the phase velocity of the reflected ray at the reflection point with $\overline{\theta_2}$ defined as the angle between the symmetry axis and the reflected phase slowness vector. Since $v(\overline{\theta_2})$ is a nonlinear function of the unknown phase angle θ_2 (measured from normal to the reflector) of the reflected ray, we need to devise a numerical scheme to solve for θ_2. In general, there might be more than one value of θ_2 satisfying equation (1). In this case, we select the one whose group direction is on the same side of the reflector as that of the incident ray.

- **Traveltime changes with respect to reflector movement**

We can describe the traveltime changes with respect to reflector movement in normal direction as:

$$\frac{\partial t^h}{\partial n} = \frac{\cos(\theta_1)}{v(\overline{\theta_1})} + \frac{\cos(\theta_2)}{v(\overline{\theta_2})} \tag{2}$$

Here, the variables are defined the same as above. Since the data are often picked along the vertical (z) direction, we are most interested in traveltime changes with respect to reflector movement in the z direction. To obtain this, we can project $\partial t^h / \partial n$ to the z-axis, resulting in the following expression:

$$\frac{\partial t^h}{\partial z} = \cos\theta_0 \frac{\partial t^h}{\partial n} \tag{3}$$

Here, θ_0 is the dip angle of the reflector or the angle between the normal direction of the reflector and the z-axis (see Figure 1).

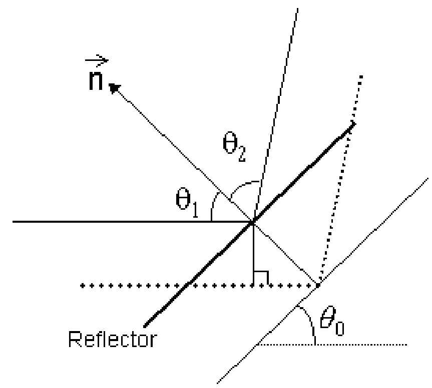

Figure 1. Reflection of a ray: $\vec{n}$ is the normal to the reflector. θ_1 and θ_2 are the phase angles (measured from the normal direction of the reflector) of the incident and reflected rays respectively, and θ_0 is the dip angle of the reflector.

- **Reflector changes with respect to changes of anisotropy parameters**

Based on the zero-time imaging principle (Wang et al., 1995; Zhou, et al., 2001), it is easy to derive the relations between the reflector movement and the changes of anisotropic parameters, which are:

$$\begin{cases} \dfrac{\partial z_h}{\partial v_i} = -\dfrac{\partial t^h}{\partial v_i} \bigg/ \dfrac{\partial t^h}{\partial z_h} \\[2ex] \dfrac{\partial z_h}{\partial \delta_i} = -\dfrac{\partial t^h}{\partial \delta_i} \bigg/ \dfrac{\partial t^h}{\partial z_h} \\[2ex] \dfrac{\partial z_h}{\partial \varepsilon_i} = -\dfrac{\partial t^h}{\partial \varepsilon_i} \bigg/ \dfrac{\partial t^h}{\partial z_h} \end{cases} \tag{4}$$

where v_i, δ_i, and ε_i are the anisotropy parameters (Thomsen, 1986) that we invert for. The index ($i = 1, \ldots, n$) represents the indices of the unknown parameters in the *n* layers or cells encountered/sampled by the specular ray pairs. These equations measure how far the reflector will move when anisotropy parameters are perturbed.

- **Velocity updating equations (Fixed-time event)**

The fixed-time event method works first by finding specular ray pairs for all of the given offsets and picked reflection image locations. Next, this method traces normal-incidence rays (or near-offset rays if zero-offset data are not available) for all these reflection image locations (Zhou et al., 2001). Although we perform the migration with an incorrect velocity, the two-way traveltime along the normal incidence ray from a migrated event on the zero-offset section is still the correct zero-offset time for that event. However, corresponding image locations on the same CIG at other (nonzero) offsets are shallower or deeper than the zero-offset event; if we shoot normal-incidence rays from those locations, the zero-offset times will not be the correct zero-offset times when the velocity is incorrect. This difference can be used to invert for perturbations in velocity v_0, ε, and δ.

Specifically, let t_{h_0} represent the two-way traveltime for half-offset h_0, called the reference offset, at location z_{h_0}. Then, according to the demigration principle, t_{h_0} should be recovered exactly even when the migration velocity is incorrect. Now let $t_{h_0}^h$ represent the modeled two-way traveltime for the same reference offset at location z_h. In principle, if the migration velocity is not correct, $t_{h_0} \neq t_{h_0}^h$. We can then use the difference between t_{h_0} and $t_{h_0}^h$ to get the velocity information. We apply a Taylor expansion to this difference in traveltime to obtain the following expression:

$$t_{h_0} - t_{h_0}^h = \sum_i \frac{\partial t_{h_0}^h}{\partial p_i} \Delta p_i + \frac{\partial t_{h_0}^h}{\partial z_h} \left[\sum_i \frac{\partial z_h}{\partial p_i} \Delta p_i \right] \quad (5)$$

Here, p, $i= 1,...,n$, represents the parameters to be inverted for such as Thomsen parameters v_i, δ_i, and ε_i.

Substituting equations (3) and (4) into equation (5) yields the final updating equation:

$$t_{h_0} - t_{h_0}^h = \sum_i \left(\frac{\partial t_{h_0}^h}{\partial v_i} \Delta v_i + \frac{\partial t_{h_0}^h}{\partial \delta_i} \Delta \delta_i + \frac{\partial t_{h_0}^h}{\partial \varepsilon_i} \Delta \varepsilon_i \right)$$
$$- \frac{\partial t_{h_0}^h}{\partial z_h} \Big/ \frac{\partial t^h}{\partial z_h} \left[\sum_i \left(\frac{\partial t^h}{\partial v_i} \Delta v_i + \frac{\partial t^h}{\partial \delta_i} \Delta \delta_i + \frac{\partial t^h}{\partial \varepsilon_i} \Delta \varepsilon_i \right) \right] \quad (6)$$

with the left-side denoting the traveltime residuals. Although equation (6) is a complete equation for 3-D tomographic velocity updating, it may be insufficient to recover all the vertical velocity and anisotropy parameters (ε and δ) simultaneously. The reason is the trade-off between the vertical velocity and anisotropy coefficients; this trade-off cannot be overcome by using P-wave seismic information alone even if long spreads (e.g., twice the reflector depth) are used (Tsvankin and Thomsen (1995)).

So far, we have discussed all the terms in equation (6) except the traveltime derivatives, which can be computed numerically during ray tracing. Alternatively, as we show next, we can also derive an explicit analytic expression for these derivatives in weak transversely isotropic (TI) media.

- **Traveltime derivatives in weak TI media**

In general, the traveltime can be represented as the following integral along the raypath:

$$t = \int \frac{d\ell}{v_g} \quad (7)$$

where v_g is the group velocity of the ray. In weak TI media, similar to phase velocity, the group velocity v_g can be represented as (Grechka, 1998)

$$v_g = v_0 \left(1 + \delta \sin^2 \varphi \cos^2 \varphi + \varepsilon \sin^4 \varphi \right) \quad (8)$$

where φ is the angle between the raypath and the symmetry axis.

According to Fermat's principle which states that the raypath between any fixed points will be that with minimum traveltime. In other words, the raypath is stationary and will not change with small changes in unknown parameters. Therefore, perturbations of traveltime with respect to small changes of the unknown parameters (v_0, δ and ε) can be derived from equations (7) and (8), after some algebraic manipulations, as

$$\begin{aligned}\delta t &= \int \delta\left(\frac{1}{v_g}\right) d\ell \\ &= -\int \frac{\delta(v_0)}{v_g v_0} d\ell - \int \frac{v_0 \sin^2\varphi \cos^2\varphi}{v_g^2}\delta(\delta)\, d\ell \\ &\quad - \int \frac{v_0 \sin^4\varphi}{v_g^2}\delta(\varepsilon)\, d\ell\end{aligned} \qquad (9)$$

With all of these preparations, we have completed the description of the algorithm. Next, we test our algorithm with a synthetic example.

Synthetic Data Example

We have created a simple vertically transversely isotropic (VTI) synthetic data set to test the above formulation. As shown in Figure 2, the true model is a horizontally layered model, with vertical velocity defined as a continuous piecewise linear function, and with constant δ and ε, defined for each layer. Recognizing the trade-off between the vertical velocity and anisotropy parameters (Tsvankin and Thomsen, 1995), we fix the vertical velocities, and invert only for the anisotropy parameters (δ, ε). To simulate an actual application of tomography, we pick the residual curvatures on the CIG's rather than computing them analytically. The true anisotropy model, initial anisotropy model, and inversion results are shown in Figure 2. The inversion results indicate that although there are still tradeoffs between δ and ε, they can be inverted reasonably well (within a few percent) as long as there is enough far offset data available.

Conclusions

We described a tomographic velocity and anisotropic parameter estimation method for TTI media. We then derived an explicit expression for weak TI media. Finally, we demonstrated its effectiveness with a synthetic VTI data example.

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References

Alkhalifah, T. and Tsvankin, I., 1995, Velocity analysis for transversely isotropic media: Geophysics, 60, 1550-1566.

Mirko van der Baan and Kendall, J., 2002, Estimating anisotropy parameters and traveltimes in the τ-p domain: Geophysics, 67, 1076-1086.

Deregowski, S.M., 1990, Common offset migrations and velocity analysis: First break, 8, 224-234.

Grechka, V., 1998, Transverse isotropy versus lateral heterogeneity in the inversion of P-wave reflection traveltimes: Geophysics, 63, 204-212.

Thomsen, L., 1986, Weak elastic anisotropy: Geophysics, 51, 1954-1966.

Tsvankin, L. and Thomsen, L., 1995, Inversion of reflection traveltimes for transverse isotropy: Geophysics, 60, 1095-1107.

Van Trier, J., 1990, Tomographic determination of structural velocities from depth-migrated seismic data: PhD thesis, SEP.

Wang, B., Pann, K., and Meek, R.A., 1995, Macro velocity model estimation through model-based globally-optimized residual-curvature analysis: 65th Ann. SEG, Expanded Abstracts, 1084-1087.

Zhou, H., Guo, J., and Young, J., 2001, An alternative residual-curvature velocity updating method for prestack depth migration: 71st SEG, Expanded Abstracts.

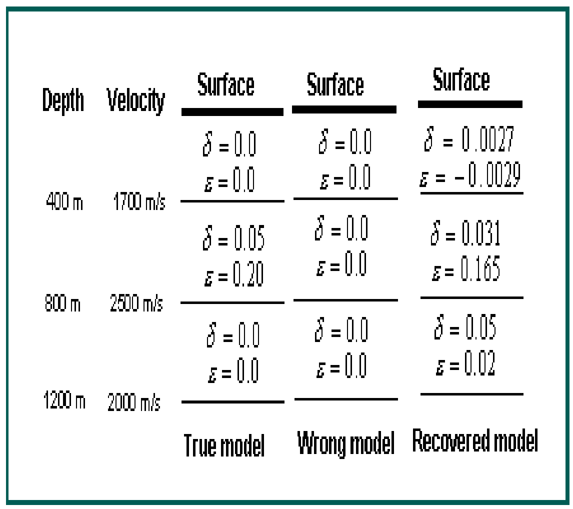

Figure 2. Model for the synthetic test. The top layer is isotropic, the second layer is anisotropic with anisotropy parameters $\delta = 0.20$ and $\varepsilon = 0.05$. The third layer is isotropic.