

An alternative residual-curvature velocity updating method for prestack depth migration

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Summary

Residual-curvature analysis of prestack depth migrated common image gathers (CIGs) is widely used for updating the whole velocity model in areas of complex geology. The velocity is estimated by maximizing the flatness of these events in either the time domain or the depth domain. In order to perform conventional reflection tomography, the reflector positions are needed and are either guessed or estimated. Therefore, the backprojection operator is often incorrectly calculated because of the incorrectly estimated reflector positions. This process may slow down the convergence of inversion. Van Trier (1990) points out that, instead of using reflectors in the depth domain as the reference events, we may use the reference events in the time domain. Based on the principle that ray tracing (modeling) undoes migration, these reference events in the time domain are the true events and don't change with velocity variations. Hence if we can convert the depth deviations into adequate time deviations, there is no need to use reflector positions in the inversion. With this in mind, we use specular ray tracing to obtain all the necessary information. The true reflector positions need not be guessed or estimated: they follow naturally from the migration results. Instead, the backprojection operator incorporates reflector movement and ray-bending effects. A simple synthetic example shows that the algorithm discussed holds promise.

Introduction

In the past several years, many papers on seismic tomography have been published. In general, velocity updating by tomography includes several key elements. These are: a model description (cell, grid, tessellation, spline, etc.); data perturbations and the relationship between the data perturbations and the velocity perturbations (which builds the so-called backprojection operator); and the algebraic solver that is used to obtain the approximate solution. Thus described, the entire process seems pretty simple. Unlike seismic migration, however, tomographic methods are not yet mature. Many factors, such as non-uniqueness and uncertainty, conspire to degrade our results. Model description and inversion routines also play important roles in the final tomographic solution. In this abstract, we limit our discussion to the problems of migration velocity analysis (Stork, 1992) and the generation of backprojection operators.

Migration velocity analysis basically uses the flatness of the CIG as a criterion for velocity quality. If the migration velocity model is not correct, the same reflector will appear

at different depth positions for different constant-offset sections. In principle, one needs to determine the differences between the true and migrated depths of the reflector at each offset and surface location, and convert these depth perturbations into velocity perturbations in the inversion method.

To obtain the depth deviation, most tomographic methods need reflector positions for finding the depth perturbation. Unfortunately, the true position of a reflector is not known, so depth perturbations from it can not be accurately determined. To solve this problem, one may try to invert for an additional unknown apart from velocity, namely the depths of interfaces in the velocity model (Stork 1992). But the velocity-depth ambiguity can lead to instabilities and poor convergence in the optimization. Van Trier (1990) chose not to parameterize both velocity and depth in the optimization. Instead, he obtained the reflector depths from the migrated data, and used them only in computing the backprojection operator.

According to the kinematic condition of zero-time imaging, as long as the same velocity is used in both modeling and migration, ray trace modeling (demigration) will undo the migration result. In the near-offset sections, migrated reflectors can be demigrated and used as a reference for modeled events in the farther-offset sections. The reflector perturbations are thus better determined in the time domain than they are in the depth domains. Since the reference events are the true seismic events and do not move as velocity changes, calculating the backprojection operator will focus only on the offset behavior of the events, which provides the most detailed velocity variations.

This cascaded process (migration and demigration) is similar to Deregowski loop (Deregowski, 1990) but it is more accurate because it is not limited to lateral velocity homogeneity, small offset, and horizontal reflectors.

The velocity analysis is formulated as an iterative optimization process, in which the objective is to minimize discrepancies between migrated events in the different constant-offset sections. The true reflector positions need not be guessed or estimated: they follow naturally from the migration results. Instead, the backprojection operator incorporates reflector movement and ray-bending effects.

Algorithm

The lack of flatness of an event across different offsets on a depth migrated CIG indicates that the velocity is in error.

Residual-curvature velocity update

At each fixed surface location r , the depth perturbation can be represented as

$$\Delta z = z_{true} - z_r, \quad (1)$$

where z_{true} is the depth of the true reflection point below the surface location r , and z_r is the corresponding migrated depth for the reflector with the given surface location, offset (and azimuth in 3D) and migration velocity. If the true depth of the reflector is known, we can use it in the inversion. But in practice, except near well locations, we don't know the true location of the reflector. This makes depth perturbations can not be accurately determined. Some have suggested the use of a floating datum (Woodward et al., 1998; Bednar, 1999) as a partial solution to this problem. The idea behind floating datum is that since we don't know the true reflector position, we can remove the effect of reflector by subtracting the near-offset deviation from the far-offset deviation.

Here we follow Van Trier (1990) to give an alternative solution. In his thesis, Van Trier suggests using a true zero-offset reflection event as a reference, and modeling zero-offset events for the migrated reflectors in each constant-offset section. Since the modeled events in the zero-offset section correspond to true zero-offset events, this approach would seem to provide an ideal first step to a full solution of the inverse problem. Unfortunately, however, we don't have true zero-offset data. To make Van Trier's method feasible, we need to use another, usually near-offset as the reference event.

The deviation between the true two-way traveltime t_0 ($= t_{s0} + t_{r0}$) for the reference near offset and the modeled two-way traveltime t_0^h ($= t_{s0}^h + t_{r0}^h$) for an arbitrary offset can be described as

$$\Delta t^h = t_0 - t_0^h, \quad (2)$$

Because the velocity is in error, the RHS of equation (2) will be not zero. Therefore, an optimization is applied to minimize the traveltime residual curvatures along offset. To find the correct backprojection operator, the gradient calculation must honor the velocity dependence at the image point. By assuming that the reflection point is moving in direction ℓ , with the first-order Taylor expansion, we can obtain the following equation from equation (2):

$$\Delta t = \sum_i \frac{\partial t_0^h}{\partial m_i} \Delta m_i + \sum_j \frac{\partial t_0^h}{\partial \ell} \frac{\partial \ell}{\partial m_j} \Delta m_j, \quad (3)$$

We re-write equation (3) symbolically as:

$$\Delta t = A \Delta m = (B + C \circ D) \Delta m, \quad (4)$$

where $A = B + C \circ D$ is the operator, which will be used to backproject the time deviation into velocity perturbation. Operator B describes, for a fixed reflection point, how the modeled traveltime changes with velocity variations. The operator $C \circ D$ is a composite operator, which denotes a cascaded process that, if the velocity changes, it will influence the reflector movement and the reflector movement will influence the modeled traveltimes. The operator $C = \partial t_0^h / \partial \ell = \ell \bullet \nabla t_0^h$ is the directional

derivative of a two-way traveltime t_0^h along direction ℓ . It describes how the modeled traveltime changes as the reflection point position changes. Using the zero-time imaging condition (Wang et al., 1995), we can show that

$$D = \frac{\partial \ell}{\partial m_j} = - \frac{\partial t^h}{\partial m_j} \bigg/ \frac{\partial t^h}{\partial \ell}, \quad (5)$$

Similarly, $\partial t^h / \partial \ell = \ell \bullet \nabla t^h$ is a directional derivative of a two-way traveltime with respect to direction ℓ . Operator D determines how far a reflector will move along the direction ℓ in response to a velocity change.

Traveltimes in the reference event section correspond to the true near-offset times in the data, and are not affected by changes in the model, which means that for a reference event, operator A should be zero, i.e., the effect of operator B will be cancelled by the effect of composite operator $C \circ D$. This can be easily seen by substituting operators B , C , and D into equation (4).

Given a direction ℓ , each term in the above equations can be obtained from specular ray tracing. Two popular options for the direction ℓ are obtained by assuming either that the reflector moves along surface normal direction n , or that the reflector moves vertically. These alternatives generate the following equations:

$$\frac{\partial t^h}{\partial z} = \frac{\cos \theta_s^h + \cos \theta_r^h}{v}, \quad (6)$$

and

$$\frac{\partial t^h}{\partial n} = \frac{2 \cos \theta^h}{v}, \quad (7)$$

where θ_s^h and θ_r^h represent the angles of incidence rays from the source and from the receiver with the z axis at the image point, respectively. θ^h is the half-opening angle between the incidence rays from the source and the receiver at the image point. v is velocity at the image point.

Residual-curvature velocity update

To summarize, the main procedure involves:

- First, a common-offset depth migration is performed, and the locally coherent events should be picked, either automatically or manually, for reflection points and dips.
- Second, choose a near-offset reflection point as a reference point, and shoot the specular rays using offset and azimuth of this reference section. This will give t_0 .
- Third, for all other individual reflection points related to the above reference point, we again perform specular ray tracing, using the offset and azimuth of the reference point, to mimic near-offset modeling. This will generate the traveltime t_0^h , hence the traveltime deviation in equation (2). We can also obtain the corresponding derivatives, which are the operators B and C .
- Fourth, for the same reflection points, we perform a specular ray tracing again, this time using the current offset and azimuth of this reflection point instead of those of the reference point. This process is used to obtain the operator D .
- Fifth, compose the backprojection operator A according to equation (4).
- Sixth, solve the linear system of equations.

Example

In this section, we compare our results with two kinds of floating-datum methods in the depth domain. For simplicity, we choose a two-layer model with a horizontal reflector. The reflection points are in the middle interval of the grids at a depth of 2050m. The length of the model is 128*100m, and its depth is 30*100m. The correct velocity is 1000m/s, and the migration velocity is chosen as 1200m/s. The depth residuals are computed with the analytical formula. Figure 1 shows the results of these three kinds of inversion. In Figure 1, the left column shows how to find the backprojection operators for given reference point B and individual reflection point A in a far-offset section. The right column shows the relative errors of velocity corresponding to different algorithms shown at the left. The top plot is our result. Note that the reference near-offset ray and the arbitrary ray start from the same reflection point. The middle plot is the result from one kind of the floating-datum inversion. Here the rays come from different reflection points. In these two methods, the

specular rays are found by shooting from their own individual reflection points and are both mathematically correct. The bottom plot shows the result of a kind of technique, in which the specular rays are found by shooting not from the individual far-offset reflection point but from the reference point. This will introduce errors as shown in the right figure of the bottom plot. The synthetic test shows that, for this example, our time domain method has more useful velocity coverage and is more stable than floating-datum methods. The total number of iterations is 10,000 using a sparse matrix LSQR solver without any regularization.

Conclusions

Residual tomographic inversion is an effective tool for estimation of velocity structure. Proper formulation of the method produces an accurate and rapidly converging algorithm.

Reference

- Bednar, J.B., L. Vinson, and Jeff Thorson, 1999, Residual tomographic updating: 69th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1251—1254.
- Deregowski, S. M., 1990, Common offset migrations and velocity analysis: First break, 8, 224-234.
- Stork, C., 1992, Reflection tomography in the postmigrated domain: Geophysics, 57, 680-692.
- Van Trer, J., 1990, Tomographic determination of structural velocities from depth-migrated seismic data: Ph.D thesis, Stanford University.
- Wang, B. K. Pann, and R.A. Meek, 1995, Macro velocity model estimation through model-based globally-optimized residual-curvature analysis: 65th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1084—1087.
- Woodward, M., Farmer, P., Nichols, D., and Charles, S., 1999, Automated 3D tomographic velocity analysis of residual moveout in prestack depth migrated common image point gathers: 69th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts.

Acknowledgment

We would like to thank Dr. Sam Gray for his kind review of the paper.

Residual-curvature velocity update

