Tomographic Residual Curvature Analysis: The Process and its Components

Hongbo Zhou, Samuel H. Gray, Jerry Young, Don Pham, and Yu Zhang, Veritas DGC Inc.

Summary

The image quality of 3D pre-stack depth migration in areas of complex geology depends strongly on the accuracy of the velocity model. Velocity updating by seismic tomography, in the form of either traveltime tomography or residual curvature analysis (RCA), has become an important component of the depth imaging process. In this paper, we describe the components of tomography, discussing RCA in detail. These components include: building of the tomographic updating equations; regularization in both data and model spaces; and application of the least-squares solver. Numerical examples show that the RCA algorithm works well, producing velocity models that improve the quality of depth-migrated images over models produced by vertical updating schemes.

Introduction

3-D prestack depth migration has become a standard seismic imaging tool, but it requires an accurate velocity model to image subsurface structures accurately. Simple velocity updating methods, such as the Deregowski (1990) loop, are based on the concept of vertical updating, which analyzes each location with a local assumption of lateral invariance. More advanced seismic tomographic procedures seek either to match observed traveltimes on unmigrated data or to flatten events on depth-migrated gathers (CIG's). Although these methods are more advanced than vertical updating methods, they are based on a simple rule: when the velocity is correct, there will be no time or depth discrepancy. A satisfactory model will predict traveltimes that agree with observed traveltimes (traveltime tomography), or it will yield a depth migration sections with no residual moveout on CIG's (RCA). Seismic tomography holds the promise of providing accurate velocity model in areas of complex geology, where vertical updating methods may fail.

Given an initial velocity model and data (picks), the seismic tomography process consists of three components. First is the choice of model parameterization. Second is building a linear system of tomographic updating equations that incorporate raypaths traced through the initial model with the data residual errors, which come either from discrepancies between observed traveltimes and raytraced traveltimes through the initial model (traveltime tomography) or from residual moveout on migrated CIG's (RCA). Third is solving the equations by back-projecting these errors along the raypaths, either to minimize the traveltime discrepancies or to flatten the events in either depth or time (Bishop et al., 1985; Van Trier, 1991, Woodward et al, 1999; Zhou et al, 2001).

In this abstract, we describe the tomography process as it applies to RCA. First, we categorize the methods for building tomographic updating equations. Next, we show how to regularize both the model and the data to reduce the effects of noisy data and inadequate model parameterizations. This regularization is essential to the reconstruction of velocity models that satisfy the algebraic equations of tomography while, at the same time, retaining geologic plausibility. Finally, we present a solver that is a variant of the least-squares conjugate gradient (LSCG) method.

Synthetic and production tests have shown that velocity models produced by a carefully crafted RCA process yield improved depth-migrated images over models produced by vertical updating techniques.

The components of RCA

- **Building the tomographic equations**

Tomography relies heavily on raypaths traced through the initial velocity model. Depending on the approach taken, this may require knowledge of reference reflectors or events. In RCA, specular raypaths from these reflector locations and the velocities they encounter provide all the information needed to build equations that convert the depth or time errors to velocity perturbations. The equations are some expression of the relationship among distance, velocity, and time, and the inverted velocity perturbations are distributed, or back-projected, along the raypaths.

A standard tomography approach assumes that the reflector positions are fixed (Bishop, et al., 1985). These locations are estimated somehow (e.g., in RCA by picking events on the stacked image) before the tomography begins. Although the reflector locations are updated during the inversion, they remain fixed during the raytracing. Ray pairs for all offsets are traced from this fixed location (Figure 1 left). We refer to this approach as the fixed-reflector method. This approach ignores the fact that velocity errors cause a migrated horizon to appear at different locations for different offsets, e.g., locations A and B on Figure 1. This error in estimated reflector positions will cause errors in the back-projection operator, and may slow down the convergence of inversion. Two alternative approaches avoid this error; we call these the floating-reflector method (Woodward et al., 1999) and the fixed-event method (Van Trier, 1990; Zhou et al., 2001). Both are based on the principle that ray tracing (modeling) undoes migration. The floating-reflector method assumes that true reflector locations are used as reference reflectors, and it reduces the error in estimated reflector locations by tracing a ray pair for a given offset from the migrated depth for that offset. As in Figure 1 (middle), this method will trace the nonzero-offset ray pair from location A and the zero-offset ray from location B. However, this method, like the fixed-reflector method, still assumes the existence of known true reflector locations, leading to some loss of accuracy (Woodward et al., 1999).

The fixed-event method goes further than the floating-reflector method in correcting the error in reflector location. This method uses events in the time (unmigrated) domain as its reference events. It traces normal rays from a given reflection event for all offsets (Figure 1 right). Even if the migration was performed with an incorrect velocity, the true traveltime t_{n0} along the normal ray from migrated event B on the zero-offset section will be fully recovered and will be one-half the correct zero-offset time for that event. If the velocity is correct, then the traveltime t_{nh} along the normal ray from the corresponding event A on the *nonzero*-offset section will also be one-half the correct *zero*-offset time, since the zero-offset and nonzero-offset reflection events coincide in this case. However, if the velocity is incorrect, then the normal-ray traveltime from A will be in error, and this error ($t_{nh} - t_{n0}$) can be used along with the depth residuals to invert for velocity perturbations. Doing this allows the back-projection operator to be more accurate than for either the fixed-reflector or the floating-reflector method since both true reference events and the original specular ray pairs are used.

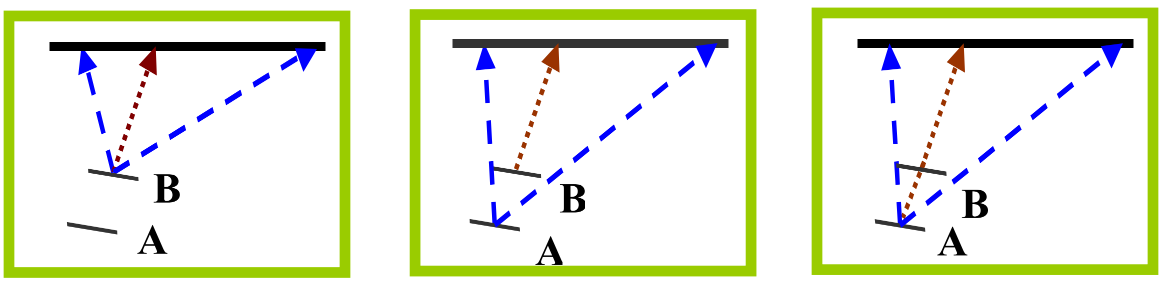

Figure 1. Three tomographic updating strategies. In these figures, **A** represents a reflector segment from a nonzero-offset migration, and **B** is the corresponding reflector segment from a zero-offset migration. Since **A** and **B** are different, the CIG's contain residual moveout. The dashed line is the specular ray pair for the nonzero offset image and the dotted line represents the zero-offset ray. On the left, the specular rays from both zero-offset and nonzero-offset image points are traced from **B** (fixed-reflector method). On the middle (floating-reflector method) and right (fixed-event method), the nonzero-offset ray pair is traced from **A** and the zero-offset ray is traced from **B**. However, in the fixed-event method, an additional zero-offset ray is traced from nonzero-offset image point **A**.

- **Regularization with differential operators in both data and model space**

High-quality solutions to geophysical inverse problems require appropriate data and model regularization (Bube and Langan, 1994; Zhou et al., 2002). In general, regularization in data space helps to reduce the effect of pick outliers. Regularization in model space helps stabilize the solution, and provides a means of applying *a priori* information into the inversion so that a model is constructed with certain user-defined characteristics, such as smoothness. Derivatives, as a measure of model or data "roughness", are often used for regularization. The "distance" in model or data space, defined by norms (e.g., L^p), can be selected according to our prior knowledge of the model and data. As a rule of thumb, L^1-like norms tend to preserve edges (intra-region smoothing), and L^2-like norms tend to smooth across the edges (inter-region smoothing). Certainly real-world tomography problems may require several combinations of simple data and model regularizations, of which Table 1 provides representative examples.

- **Least-squares conjugate gradient solver with left and right preconditioning**

The quality of the data and model parameterization has a great influence on the final result of tomographic velocity analysis (Zhou et al. 2002). A reasonable solution can be obtained only by properly constraining the equations to attenuate the influence of bad data or poor model parameterization. Applying these constraints leads to the following preconditioned equations:

$$\begin{cases} LFRx' = Ld, \\ x = Rx'. \end{cases}$$

Here F is a linear operator that maps model perturbations into data residuals (a matrix containing derivatives of depth or traveltime residuals with respect to velocities), x is the updated velocity, and d is the data (picks). In this equation, L is a left preconditioning operator that is used to reduce the harmful effects of inconsistent or extreme picks in the data space, whereas R is a right preconditioning operator, typically a smoothing operator to regularize the solution in model space. Application of both L and R will typically accelerate the convergence and stabilize the solution.

To solve these equations, traditional algorithms typically require operator *LFR* in the above equation to be used as one single composite operator. This works well mathematically, but it provides no physical meaning. In this abstract, we propose a variation of a least–squares conjugate gradient algorithm with left and right preconditioning operators treated separately. This can be written as:

$$
\begin{aligned}
& r = d - Fx_0; \\
& \bar{r} = \mathbf{L}^T\mathbf{L}\, r; \quad \beta = 0; \\
& iterate\{ \\
& \quad \Delta x = F'\bar{r}; \quad z_1 = \mathbf{R}\mathbf{R}^{\mathrm{T}}\Delta x; \\
& \quad \textit{if not the first iteration}, \ \beta = \frac{(\Delta x, z_1)}{\gamma}; \quad \gamma = (\Delta x, z_1); \\
& \quad s = z_1 + \beta\ s; \quad \Delta r = Fz_1 + \beta\ \Delta r; \\
& \quad z_2 = \mathbf{L}^T\mathbf{L}\,\Delta r; \quad \alpha = \frac{\gamma}{(\Delta r, z_2\ )}; \\
& \quad x = x + \alpha\ s; \quad \bar{r} = \bar{r} - \alpha\ \ z_2. \\
& \}
\end{aligned}
$$

Here, z_1 is the preconditioned variable for the model perturbations; the preconditioning helps shape and guide the solution Δx. Variable z_2 is the preconditioned variable for Δr in data space; in the data space, the preconditioning helps remove extreme picks. Note the scale parameters α, β, γ are the inner products of the vectors and have been properly modified corresponding to the preconditioned variables. Compared to traditional least-squares conjugate gradient solver, the discussed algorithm has more physical meaning and, if appropriate, it is much easier to replace the corresponding preconditioning operators with new operators.

Example

Figures 2 show stacked migrated images using an initial smoothing velocity (left) and tomographic velocities (right). The artifacts in the left figure (arrows) are caused by errors in the initial model. While the image on the right is not perfect, the artifacts have mostly been eliminated. The bottom two figures show the corresponding CIG's (left, vertical updating; right, RCA). RCA has greatly reduced the residual moveout that dominates the initial CIG's.

Conclusions

We have described the component parts that make up a complete process of RCA tomography. Isolating the component parts has allowed us to describe optimizations such as the use of fixed events in time and the use of left and right preconditioners. It has also allowed us to show how the components, and the optimizations, work together to form an integrated process. Our simple numerical example has illustrated the improvement of RCA over traditional vertical updating techniques.

References

Bishop, T., Bube, K., Cutler, R., Langan, R., Love, P., Resnick, J., Shuey, R., Spindler, D., and Wyld, H., 1985, Tomographic determination of velocity and depth in laterally varying media: Geophysics, 50, 903-923.

Bube, K. P. and Langan, R. T., 1994, A continuation approach to regularization for traveltime tomography: 64th SEG Expanded Abstracts: 980-983.

Deregowski, S., 1990, Common offset migrations and velocity analysis: First Break, 8, 224-234.

Van Trier, J.,1990, Tomographic determination of structural velocities from depth-migrated seismic data: Ph.D thesis, Stanford University.

Woodward, M., Farmer, P., Nichols, D., Charles, S., 1999, Automated 3D tomographic velocity analysis of residual moveout in prestack depth migrated common image point gathers: 69th SEG Expanded Abstracts, 1218-1221.

Zhou, H., Guo, J., and Young, J., 2001, An alternative residual-curvature updating method for prestack depth migration, 71th SEG Expanded Abstracts, 877-880.

Zhou, H., Zhang, Y., Gray, S. H., and Zhang, G., 2002, Regularization algorithms for seismic inverse problems: 72nd SEG, Expanded Abstracts.

Residual curvature tomographic analysis

Space	Model Space			Data Space
	Regularization		Preconditioning	
Norm	Order of derivative	Effect		
L^2 Norm	$R(u)=\int \lvert\nabla u\rvert^2 dx$	Inter-region smoothing (linear)	$-\nabla \bullet \nabla u = 0$	$F(r)=\lvert r\rvert^2$
	$R(u)=\int \lvert\Delta u\rvert^2 dx$	Inter-region smoothing (cubic spline)	$\Delta^2 u = 0$	
L^1 Norm	$R(u)=\int \lvert\nabla u\rvert dx$	Intra-region smoothing	$-\nabla \bullet \frac{\nabla u}{\lvert\nabla u\rvert} = 0$	$F(r)=\lvert r\rvert$

Table 1. Examples of model and data regularizations

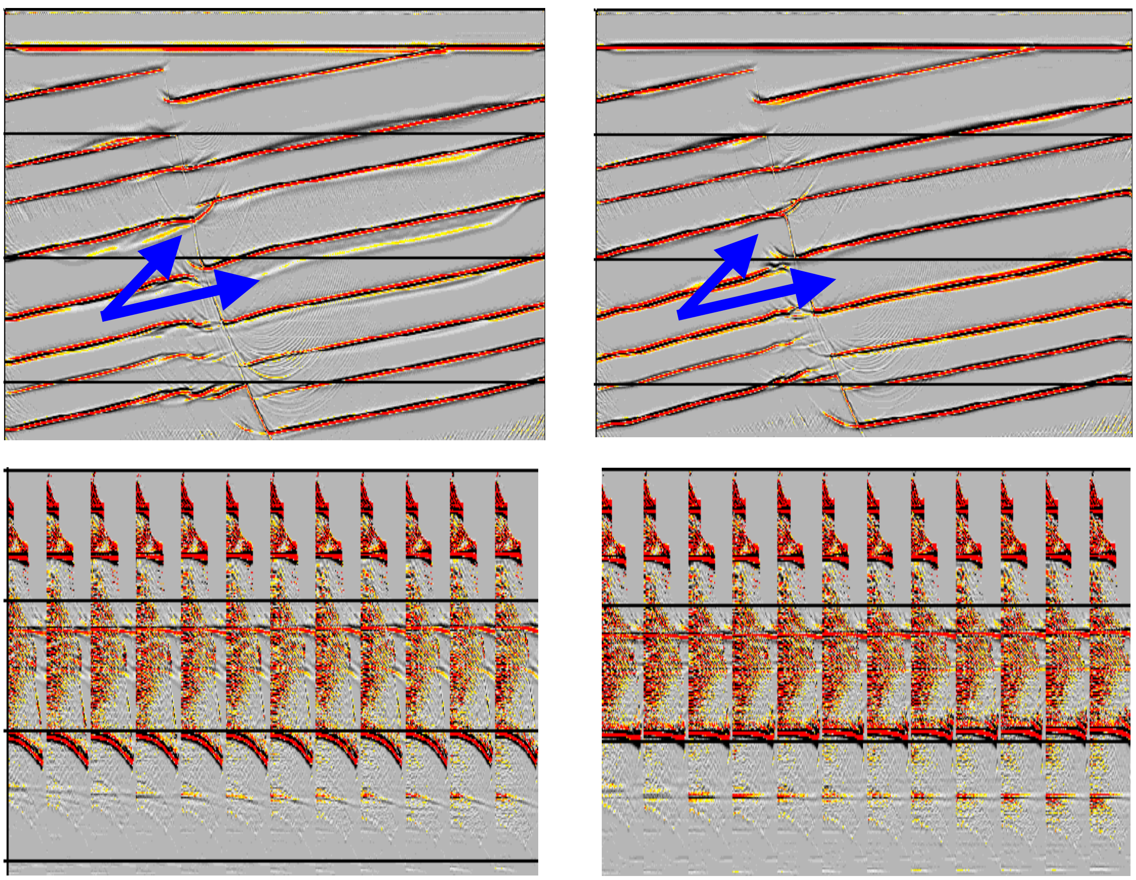

Figure 2 Top: stacked migrated results, initial velocity (left) and tomographic velocity (right). Bottom: CIG's, initial velocity (left) and tomographic velocity (right). RCA has improved the image, and has greatly reduced the residual moveout in the gathers.