

Amplitude Preserving $v(z)$ Pre-stack Kirchhoff Migration, Demigration and Modeling

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Summary We analyze the true amplitude weights in time migration and demigration based on Bleistein's Kirchhoff inversion and modeling formulas. In addition, a geometrical spreading factor is derived for any dipping reflector in a $v(z)$ medium. Finally, we propose proper anti-aliasing formulas honoring sampling theory to preserve fidelity for 3-D migration and demigration.

Introduction Kirchhoff prestack time migration is widely used in 3-D seismic data processing while Kirchhoff demigration is a useful tool to obtain a zero-offset data volume after Kirchhoff prestack time migration and stack. Arbitrary weights could be applied to input trace samples during their contribution to the image but, depending on the goal of the migration and demigration, some choices of weights are better than others. For example, a migration algorithm using unit weights is unsuitable for stratigraphic imaging because the lack of proper weights causes migration artifacts to appear on the image, hindering stratigraphic interpretation. Also, improper weights in demigration fail to recover the pre-migration amplitudes. By applying stationary phase to true amplitude demigration, we obtain a geometrical spreading expression for any dipping reflector in a $v(z)$ medium. We apply this formula in modeling and use it to test our true amplitude time migration algorithm.

Another factor affecting amplitudes in migration and demigration is anti-aliasing. Based on discussions in Zhang et al.'s (2001), we find that some anti-aliasing formulas in the literature are too aggressive and unnecessarily reduce the frequency content of the migrated image. In this abstract, we discuss the proper anti-aliasing formulas to be used for both migration and demigration in a $v(z)$ medium.

Migration and demigration in a $v(z)$ medium Bleistein et al. (2001) present general formulas for 3-D Kirchhoff migration and demigration:

$$R(x, y, z) \sim \frac{1}{8\pi^3} \iiint i\omega e^{-i\omega(\tau_s + \tau_r)} W_m \hat{U}(\xi, \eta; \omega) d\xi d\eta d\omega \quad (\text{migration}) \quad (1)$$

$$\hat{U}(x, y; \omega) \sim \iiint i\omega e^{i\omega(\tau_s + \tau_r)} W_d R(x, y, z) dx dy dz \quad (\text{demigration}) \quad (2)$$

where (x, y, z) is the imaging point, W_m and W_d are weights for migration and demigration, respectively, i.e.

$$W_m = \frac{h}{A_s A_r |\nabla(\tau_s + \tau_r)|^2}, \quad (3)$$

$$W_d = A_s A_r |\nabla(\tau_s + \tau_r)|. \quad (4)$$

In (3) and (4), A_s (A_r) is the amplitude of the Green's function from the source (receiver) to the image point, τ_s (τ_r) is the traveltime between source (receiver) and image point, h is the

Beylkin determinant. For 3-D common-offset and common-azimuth case, we define the source and receiver pair as $(x_s, y_s, 0) = (\xi + h_x, \eta + h_y, 0)$ and $(x_r, y_r, 0) = (\xi - h_x, \eta - h_y, 0)$.

When the velocity varies only with depth ($v(z)$), we obtain the following expression for all the factors that make up weights in (3) and (4):

$$|\nabla(\tau_s + \tau_r)| = 2 \cos \theta / v, \quad (5)$$

$$A_s = \sqrt{\frac{v v_0}{\cos \alpha_{s0} \psi_s \sigma_s}} \quad \text{and} \quad A_r = \sqrt{\frac{v v_0}{\cos \alpha_{r0} \psi_r \sigma_r}}, \quad (6)$$

where α_{s0} and α_{r0} are the ray angles for source and receiver relative to the vertical at the surface, and θ is the reflection angle (Figure 1). In (6), ψ and σ are in-plane and out-of-plane spreading terms. In Zhang et al., 2000, an explicit formula for 3-D common-offset, common-azimuth was given as

$$W_m = \frac{\sqrt{\cos \alpha_{s0} \cos \alpha_{r0}}}{v_0} \left[\left(\sqrt{\frac{\psi_s}{\psi_r}} + \sqrt{\frac{\psi_r}{\psi_s}} \right) \left(\sqrt{\frac{\sigma_s}{\sigma_r}} + \sqrt{\frac{\sigma_r}{\sigma_s}} \right) + \frac{\sin^2 \gamma}{2 \cos^2 \theta} L \right] \quad (7)$$

and

$$L = (\cos \alpha_s + \cos \alpha_r) \left(\sqrt{\frac{\psi_s \psi_r}{\sigma_s \sigma_r}} + \sqrt{\frac{\sigma_s \sigma_r}{\psi_s \psi_r}} \right) - (1 + \cos \alpha_s \cos \alpha_r) \left(\sqrt{\frac{\psi_s \sigma_r}{\psi_r \sigma_s}} + \sqrt{\frac{\psi_r \sigma_s}{\psi_s \sigma_r}} \right), \quad (8)$$

where γ is the angle between the projections of source and receiver rays to the surface (Fig. 1).

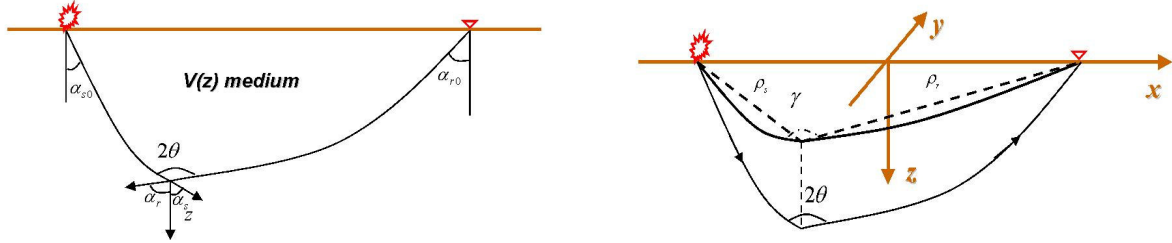


Figure 1: Ray paths in a $v(z)$ medium and parameters definitions in 3-D amplitude weight.

Geometrical spreading for a dipping reflector The geometrical spreading from a flat reflector in a $v(z)$ medium has been discussed in literature, for example, Ursin (1990). Here we point out that demigration formula (2) can be used to derive geometrical spreading when the subsurface reflector is dipping.

Substituting the dipping reflector $R(x, y, z) = \delta\left(\frac{z - ax - by - c}{\sqrt{1 + a^2 + b^2}}\right)$ into (2) and applying stationary phase, we obtain the amplitude of surface reflection

$$G = \frac{A_s A_r |\nabla(\tau_s + \tau_r)|}{\sqrt{|T|}} \sqrt{1 + a^2 + b^2}, \quad (9)$$

where T is the Hessian matrix of traveltime at the stationary point.

Anti-aliasing in migration and demigration Anti-aliasing is an important factor for amplitude preservation in both migration and demigration. The 3-D anti-aliasing formula of migration derived in Zhang et al. (2001) is based on sampling theory. In a $v(z)$ common-offset migration, the maximal un-aliased frequency is given by

$$f_m = \frac{1}{2 \max(|(x - x_s)/\sigma_s + (x - x_r)/\sigma_r| \Delta\xi, |(y - y_s)/\sigma_s + (y - y_r)/\sigma_r| \Delta\eta)}. \quad (10)$$

Based on a similar analysis, the following anti-aliasing formula is proper for a time demigration

$$f_d = \frac{\cos \alpha_s + \cos \alpha_r}{2 \max(|(x - x_s)/\sigma_s + (x - x_r)/\sigma_r| \Delta x, |(y - y_s)/\sigma_s + (y - y_r)/\sigma_r| \Delta y)}. \quad (11)$$

Numerical results We first test amplitude-preserving time migration on the flat model in which we used nine flat reflectors with equal reflectivities of one. The velocity function is linear: $v = 2000 + 0.3z$. We migrate input data up to 7km with different migration weights and show normalized AVO curves in Figure 2. As we see, all the AVO curves line up around 100% with less than $\pm 3\%$ error when we use true amplitude weight (7) in migration (left). On the other hand, using unit migration weight on the same data results in great deviations (right).

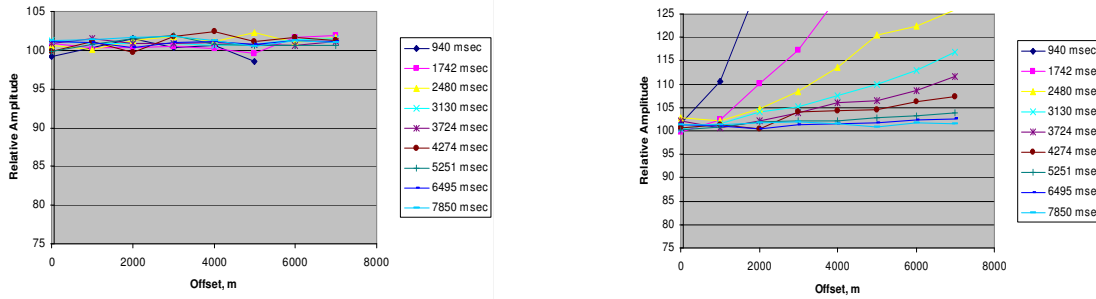


Figure 2: AVO curves of events after prestack time migration. Each curve represents a seismic interface on different depths. Left: The true-amplitude time migration lined all the events up well from very shallow to deep. Right: The results of prestack time migration using unit weight.

In the second test, we construct a fan model (figure 3a), which includes four dipping events (0° , 10° , 20° and 30°). The velocity function is $v = 1500 + 0.15z$. We first applied modeling formula (9) to create seismic data (figure 3b). Then migrated it with and without true amplitude weights. Figure 3c is the result of true amplitude prestack time migration, which shows very good consistency in recovering amplitudes. Whereas, Figure 3d - the result of migration using unit weight - indicates incorrect treatment of amplitudes.

To illustrate the accuracy of our anti-aliasing, we compare 3-D phase-shift time migration result with theoretical prediction. Figure 4 shows the peak amplitudes along the respective impulse response. When trace spacings Δx and Δy are reduced to 8m, anti-aliasing does not affect the amplitude (Figure 4a). Anti-aliasing attenuates amplitude progressively as we increase trace spacings to 12.5m, 25m and 40m, see Figure 4b-d. The figures also show that the amplitude from numerical experiments agree with theoretical prediction using antialiasing formula (10).

Conclusions We have presented true-amplitude weights and anti-aliasing formulas for prestack Kirchhoff time migration and demigration. Numerical results show that applying the proposed algorithms preserves fidelity for prestack migration and demigration in a $v(z)$ medium.

References

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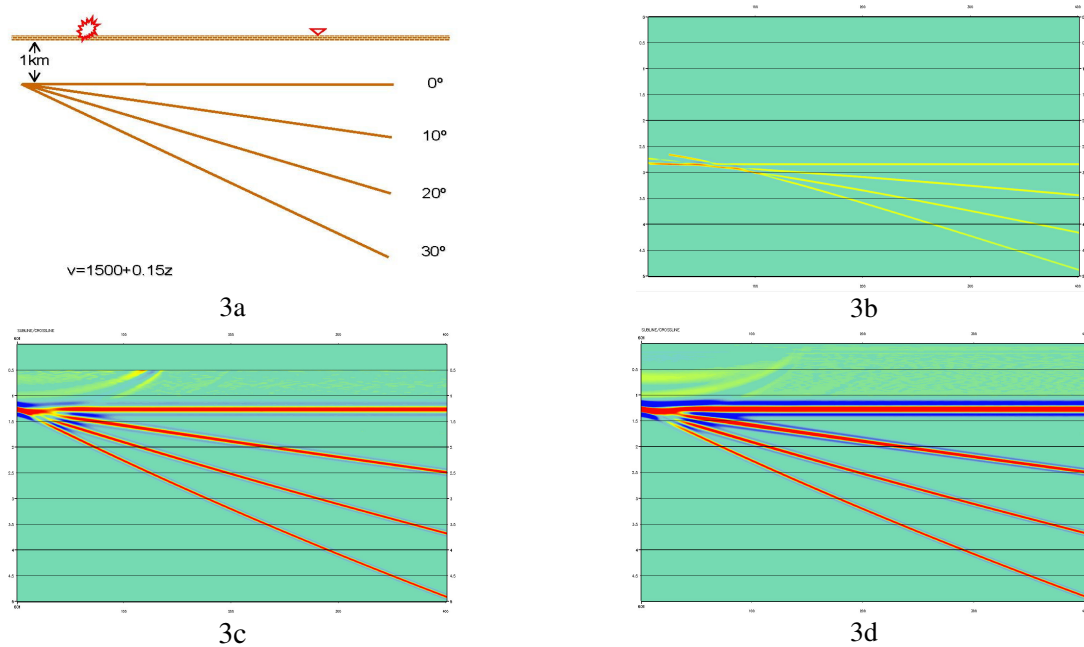


Figure 3: 3a: Five dipping events, 0°, 10°, 20° and 30°. 3b: Common offset (4km) seismic data from modeling. 3c: Prestack time migration output using true amplitude weight. 3d: Prestack time migration output using unit weight.

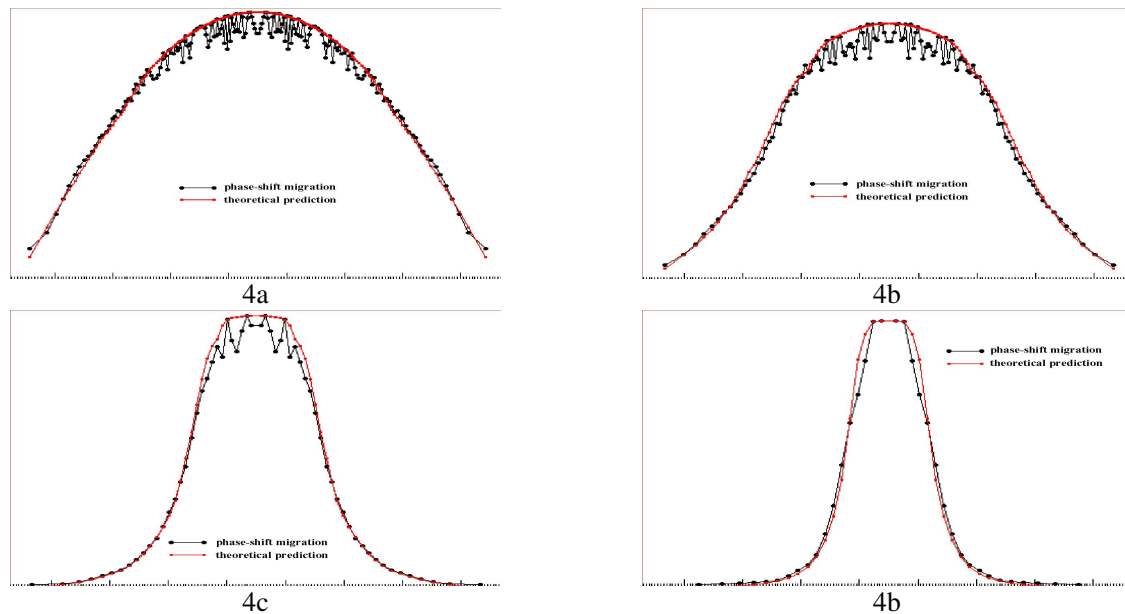


Figure 4: Comparison of phase-shift anti-aliasing with theoretical prediction. In phase-shift migration, velocity is $v = 1500 + 0.3z$. From 4a to 4d, migration trace spacings are 8m, 12.5m, 25m and 50m, respectively.