# Kinematics of Reverse Time S-G Migration

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Agenda: explore *prestack focussing properties* of RT S-G migration, proper definition of *image volume*, using ray theory.

- "Standard" PSDM (CO, CS, CSA) exhibits *kinematic artifacts* in complex structure (TRIP,...): image gathers *not flat* when velocity is correct.

- Stolk-deHoop '01: no artifacts in prestack S-G migration (perfect focussing of offset image panels at zero offset, even in complex velocity structures). Limitations: reflector dip subhorizontal, rays do not turn ("DSR assumption")

- RT formulation permits arbitrary reflector orientation, propagation. Image volume combining horizontal, vertical offsets focusses near zero offset.

Outline, Part I:

- Born approximation, extended models, common offset, angle PSDM

- kinematic image artifacts: why image gathers may not be flat at correct velocity

- double reflector model, double reflector PSDM,

- relation to S-G migration via DSR equation.

- reverse time adjoint computation

Outline, Part II:

- kinematics of double reflector model, horizontal offsets and focussing property under DSR assumption
- why horizontal offset is insufficient; combining horizontal and vertical offset: filtered coordinate image volumes
- derivation of focussing property, limitation to small offset corridor
- some implementation details: how to make RT S-G as fast as standard RT

S-G modeling - begin with a *extension* of Born modeling, different from extension underlying standard CO, CS, CSA migration:

$$\bar{F}[v]R(\mathbf{x}_r, t; \mathbf{x}_s) =$$

$$\frac{\partial^2}{\partial t^2} \int dy \int dh \int d\tau \, \frac{2R(\mathbf{y}, \mathbf{h})}{v^2(\mathbf{y})} G(\mathbf{y} + \mathbf{h}, t - \tau; \mathbf{x}_r) G(\mathbf{y} - \mathbf{h}, \tau; \mathbf{x}_s)$$

Looks similar to common offset extension, BUT:

- "offset" parameter $\mathbf{h}$ is *not* same as surface offset $(\mathbf{x}_r - \mathbf{x}_s)/2$ - *two* reflection points $\mathbf{y} \pm \mathbf{h}$ - *double reflector model*

- each output point $(\mathbf{x}_r, t; \mathbf{x}_s)$ depends on all model points $(\mathbf{x}, \mathbf{h})$

- Same as Born modeling - $\bar{F}[v]R = F[v]r$ - when $R(\mathbf{x},\mathbf{h}) = r(x)\delta(\mathbf{h})$, i.e. (double) reflectivity *focussed* at offset zero (rather than flat as in common offset extension).

Alternate representation: *source-receiver parametrization* $\bar{R}(\mathbf{y}_r, \mathbf{y}_s) = R(\mathbf{y}, \mathbf{h})$ where $\mathbf{y} = (\mathbf{y}_r + \mathbf{y}_s)/2$, $\mathbf{h} = (\mathbf{y}_r - \mathbf{y}_s)/2$ ("sunken" mid-point, offset). Rewrite

$$\bar{F}[v]\bar{R}(\mathbf{x}_r, t; \mathbf{x}_s) =$$

$$\frac{\partial^2}{\partial t^2} \int dy_r \int dy_s \int d\tau \, \frac{2\bar{R}(\mathbf{y}_r, \mathbf{y}_s)}{v^2(\mathbf{y})} G(\mathbf{y}_r, t - \tau; \mathbf{x}_r) G(\mathbf{y}_s, \tau; \mathbf{x}_s)$$

Adjoint operator (S-G migration):

$$\bar{F}^*[v]d(\mathbf{y}_s, \mathbf{y}_r) =$$

$$\frac{2}{v((\mathbf{y}_s + \mathbf{y}_r)/2)} \int dx_s \int dx_r \int dt \int d\tau \, G(\mathbf{y}_r, t - \tau; \mathbf{x}_r) G(\mathbf{y}_s, \tau; \mathbf{x}_s) d(\mathbf{x}_r, t; \mathbf{x}_s)$$

RT representation mentioned last time, later today.

*Kinematics:* relation between (double) reflectors, reflection events in data.

Phase space description: reflector has *location* $(\mathbf{y}_r, \mathbf{y}_s)$ and *dip* $(\mathbf{k}_r, \mathbf{k}_s)$. Means: near $(\mathbf{y}_r, \mathbf{y}_s)$ reflectivity $\bar{R}(\mathbf{y}'_r, \mathbf{y}'_s)$ has significant energy in plane wave component $\exp i(\mathbf{y}'_r \cdot \mathbf{k}_r + \mathbf{y}'_s \cdot \mathbf{k}_s)$.

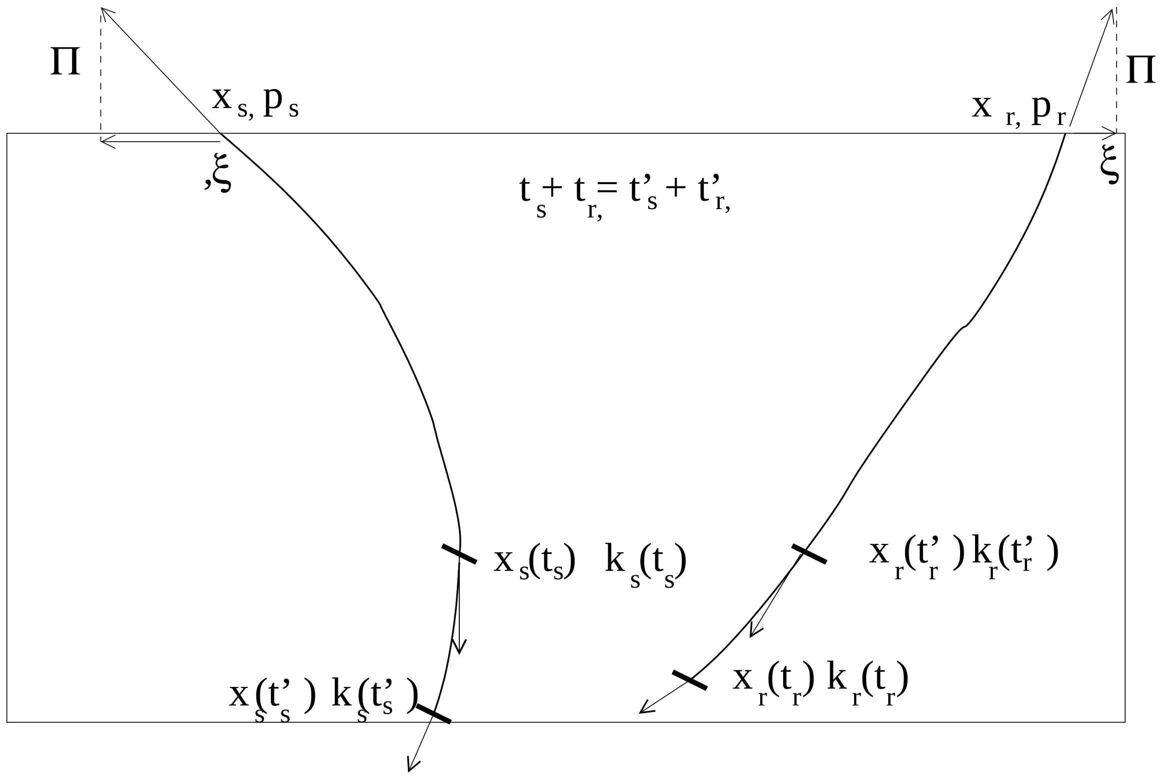
Similarly, reflection event in data at location $(\mathbf{x}_r, t; \mathbf{x}_s)$ and dip $\omega(\mathbf{p}_r, 1; \mathbf{p}_s)$. Event slownesses $\mathbf{p}_r, \mathbf{p}_s$ determined by data for "true 3D", otherwise many data-compatible slownesses (eg. for idealized streamer geometry).

**Kinematic Relation** of S-G modeling/migration: reflection event $(\mathbf{x}_r, t; \mathbf{x}_s), \omega(\mathbf{p}_r, 1; \mathbf{p}_s)$ occurs $\Leftrightarrow$ reflector exists at $\mathbf{y}_r, \mathbf{y}_s, \mathbf{k}_r, \mathbf{k}_s$ **and**

- a ray begins at $\mathbf{x}_s$ with takeoff slowness $\mathbf{p}_s$ and reaches $\mathbf{y}_s$ with arrival slowness $\mathbf{k}_s/\omega$, in time $t_s$;

- a ray begins at $\mathbf{x}_r$ with takeoff slowness $\mathbf{p}_r$ and reaches $\mathbf{y}_r$ with arrival slowness $\mathbf{k}_r/\omega$, in time $t_r$;

- $t_s + t_r = t$



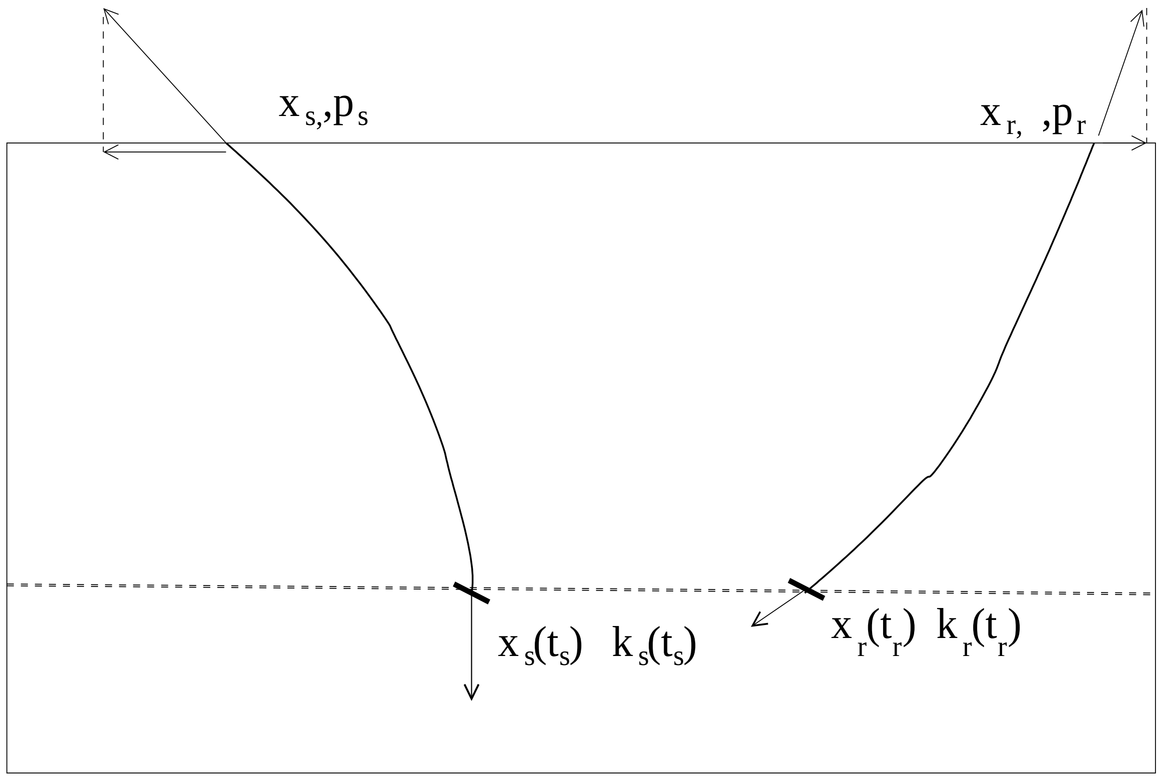

Kinematic relation of S-G modeling/migration

Note: for any given reflection event in data, *many corresponding (double) reflectors*: all points on rays from source, receiver with correct total time.

$\Rightarrow$ gross imaging ambiguity

[Recall: double reflection model has too many parameters!]

The "traditional" fix: (1) DSR assumption, i.e. no turning rays; (2) "sunken offset" vector *horizontal*




Kinematic relation of S-G modeling/migration + DSR + horizontal offset: NO IMAGING AMBIGUITY (Stolk-deHoop 2001)

Recall: S-G modeling same as Born modeling when

$$\bar{R}(\mathbf{y}_r, \mathbf{y}_s) = r\left(\frac{\mathbf{y}_r + \mathbf{y}_s}{2}\right)\delta\left(\frac{\mathbf{y}_r - \mathbf{y}_s}{2}\right)$$

i.e. reflector energy *focussed at zero offset*.

When (i) velocity is correct and data is noise free, and (ii) DSR and horizontal offset assumptions are enforced, Stolk-deHoop $\Rightarrow$ energy must focus at zero offset because (a) that is one of the locations in reflectivity phase space corresponding to the event, and (b) there can be only one such location!

Translation: no imaging artifacts in S-G migration under these assumptions.

Relation between double and ordinary reflectors: if

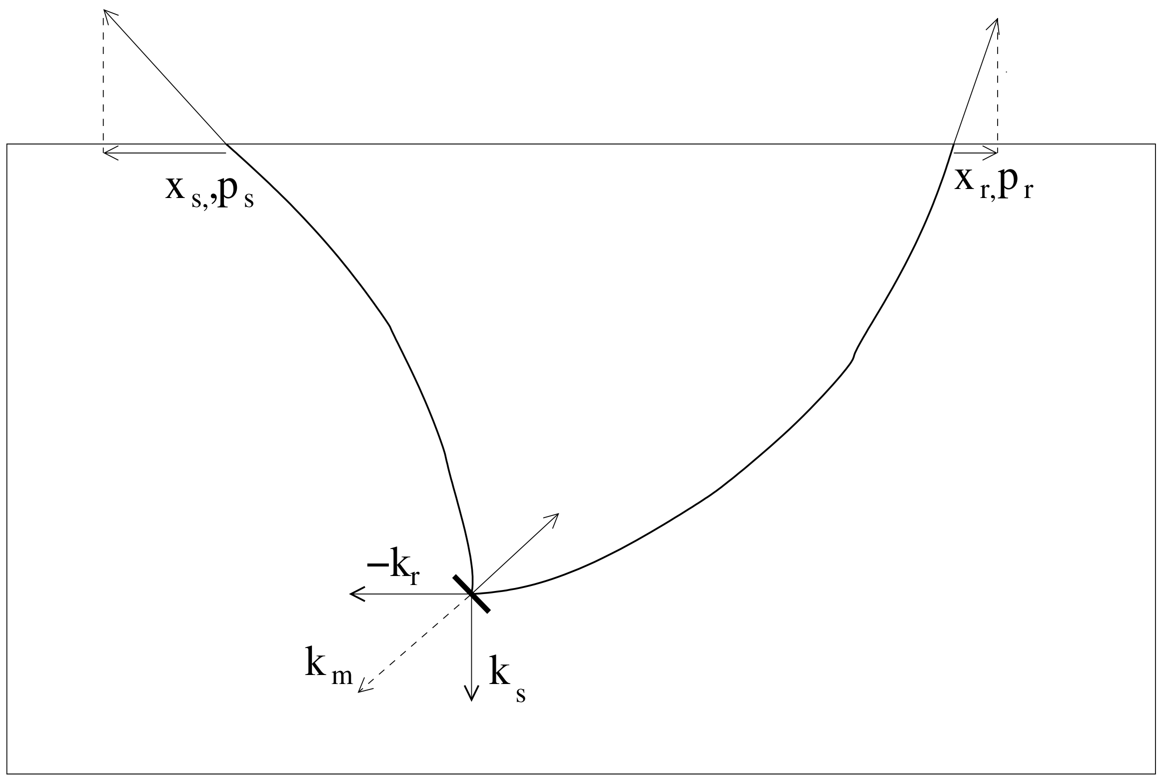
$$\bar{R}(\mathbf{y}_r, \mathbf{y}_s) = r\left(\frac{\mathbf{y}_r + \mathbf{y}_s}{2}\right)\delta\left(\frac{\mathbf{y}_r - \mathbf{y}_s}{2}\right)$$

then plane wave components on LHS must come from RHS: these have the form

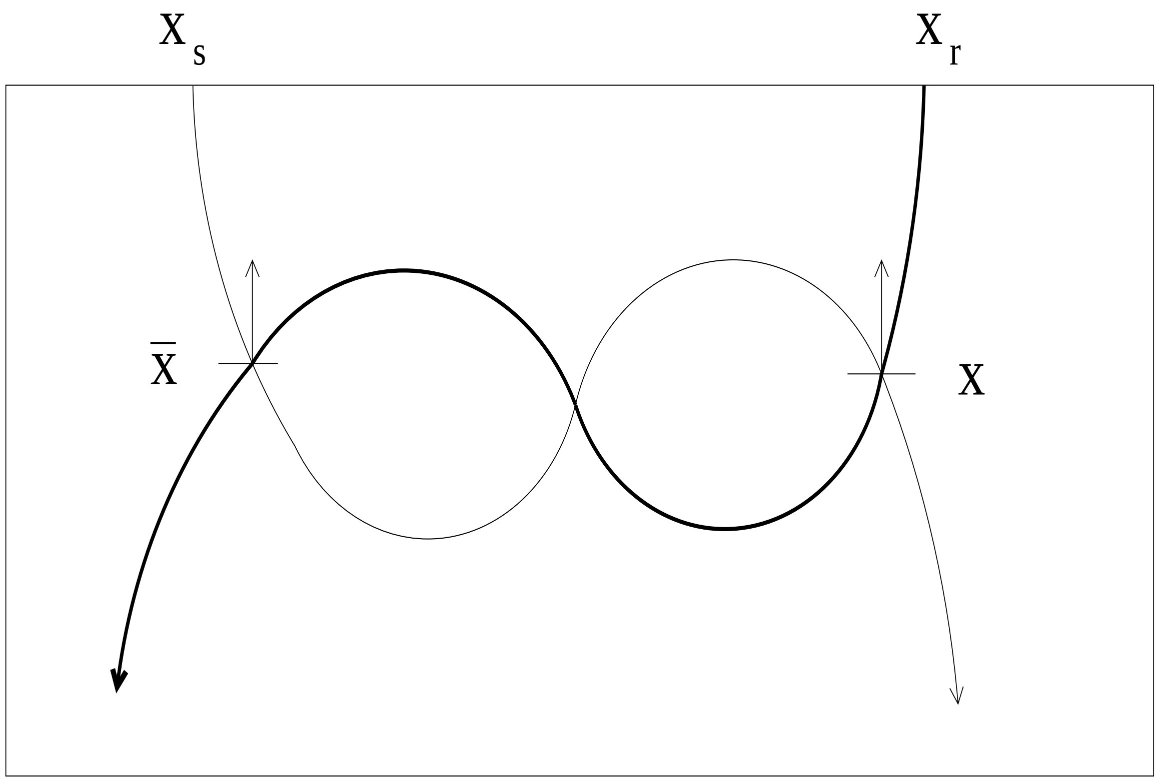
$$e^{i\mathbf{k}_m\cdot(\mathbf{y}_r+\mathbf{y}_s)} \cdot e^{i\mathbf{k}_h\cdot(\mathbf{y}_r-\mathbf{y}_s)}$$

$\Rightarrow$ $\mathbf{k}_m = \mathbf{k}_s + \mathbf{k}_r$; $\mathbf{k}_s, \mathbf{k}_r$ must have same length (both are ray parameters at $\mathbf{y}_r = \mathbf{y}_s$!)

$\Rightarrow$ Snell's law (but must also assume *TIC: a physical reflector is uniquely determined by the incident and reflected rays and the total travel time.*)




Snell's law - focussed case of S-G migration = usual reflection kinematics




TIC can be a nontrivial assumption in complex velocity structure.

How do you prove stuff like this?

Idea 1: use Green's function representation, asymptotic representation of Green's function. *Bad idea:* rep. breaks down at caustics, exactly where this gets intersting.

Idea 2: use PDE expression of S-G modeling: $\bar{F}[v]\bar{R}(\mathbf{x}_r, t; \mathbf{x}_s) = u(\mathbf{x}_r, t; \mathbf{x}_s)$,

$$\left(\frac{1}{v(\mathbf{x})^2}\frac{\partial^2}{\partial t^2} - \nabla^2_{\mathbf{x}}\right) u(\mathbf{x}, t; \mathbf{x}_s) = w_s(\mathbf{x}, t; \mathbf{x}_s)$$

$$\left(\frac{1}{v(\mathbf{y})^2}\frac{\partial^2}{\partial t^2} - \nabla^2_{\mathbf{y}}\right) w_s(\mathbf{x}, t; \mathbf{y}) = \bar{R}(\mathbf{x}, \mathbf{y})\delta(t)$$

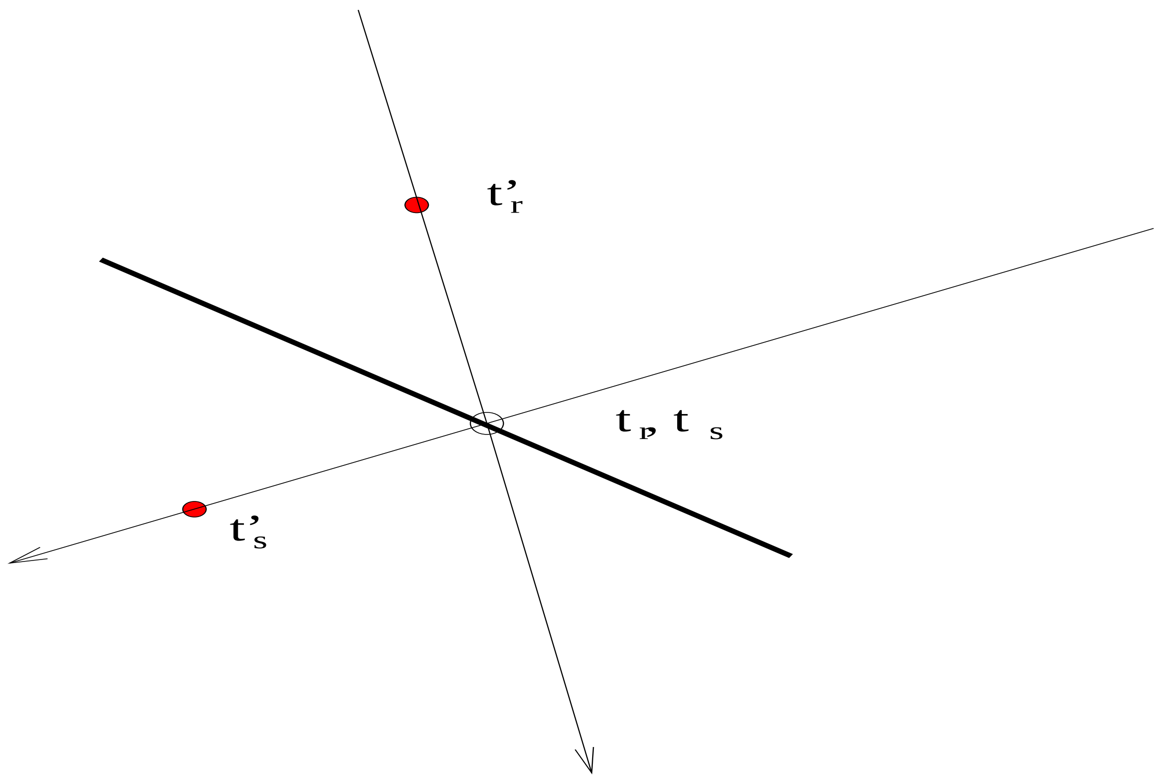
plus harmonic analysis of singularities - follows Rakesh's 1988 analysis of ordinary reflection. See WWS, Stolk, Biondi, 2002 TRIP AR (`www.trip.caam.rice.edu`)

Q. Why drop DSR? A. Because in complex structure, rays turn.

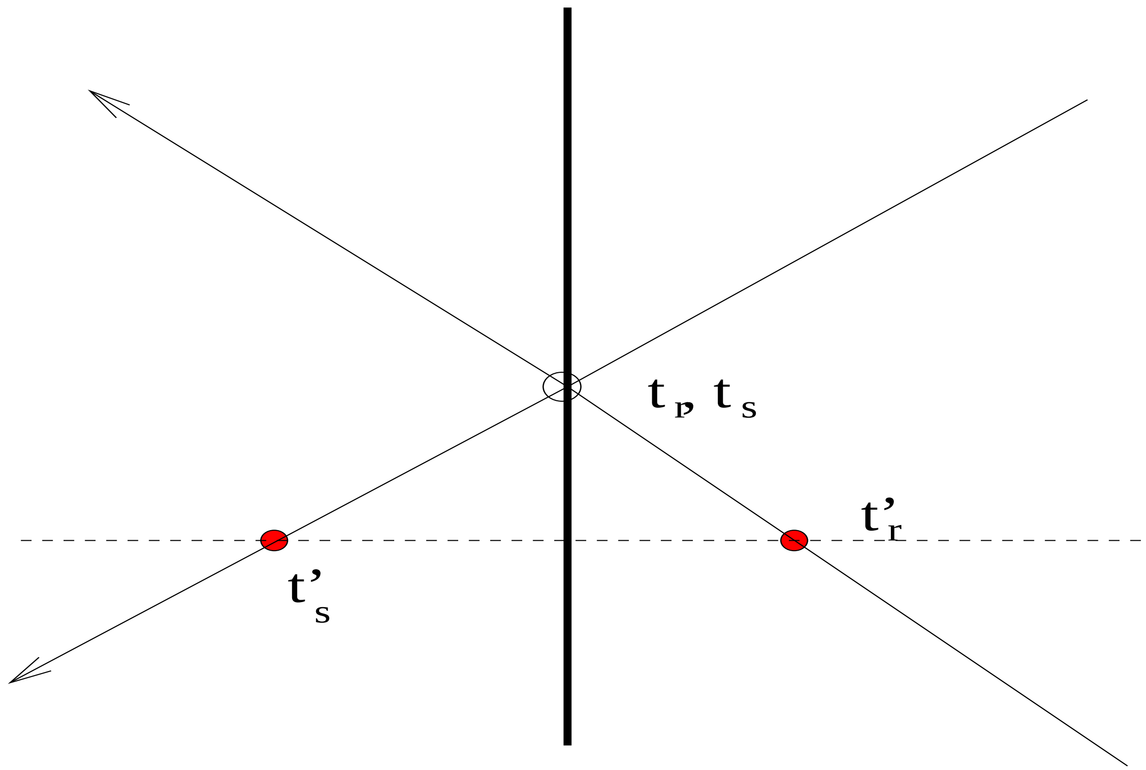
Q. Why drop horizontal offsets? A. Because reflectors structures may be vertical or near-vertical, and then horizontal offset images will be *smeared* (i.e. ambiguous reflector locations!)

Nonvertical reflector $\Rightarrow$ total traveltime determines reflection point uniquely when velocity is correct and *horizontal* offset assumed.

Vertical reflector $\Rightarrow$ many different (double) reflectors correspond to single physical reflector, all having same traveltimes and horizontal offset.




Nonvertical reflector: $t_r + t_s = t'_r + t'_s$, but depths can *only* be the same at one point (which must be the physical reflection point, if velocity is correct, by S-deH).




(Near) vertical reflector: $t_r + t_s = t'_r + t'_s$, and depths can be the same at a continuum of points, besides the physical reflection point $\Rightarrow$ reflector is smeared, location ambiguous.

How to admit non-horizontal offsets, 2D version (for 3D see paper):

Define *projector filters* $\Pi_z, \Pi_x$, smooth functions of $(k_z, k_x)$ satisfying:

- $0 \leq \Pi_z, \Pi_z \leq 1$, $\Pi_z + \Pi_x \equiv 1$
- $\Pi_z(0, k_x) = 0$, $\Pi_x(k_z, 0) = 0$

Define *injection operators* in (midpoint, offset) coordinates:

$$Q_z r_z(\mathbf{y}, \mathbf{h}) = \Pi_z[r_z(\mathbf{y}, h_x)\delta(h_z)], Q_x r_x(\mathbf{y}, \mathbf{h}) = \Pi_x[r_x(\mathbf{y}, h_z)\delta(h_x)],$$

for horizontal offset and vertical offset reflectivity volumes $r_z(\mathbf{y}, h_x), r_x(\mathbf{y}, h_z)$ respectively.

Define the *bidirectional double reflection modeling operator* $\bar{\mathbf{F}}[v]$ by

$$\bar{\mathbf{F}}[v](r_z, r_x) = \bar{F}[v]Q_z r_z + \bar{F}[v]Q_x r_x$$

- $\bar{\mathbf{F}}[v]$ has a 1-1 kinematic relation - no imaging ambiguities - when $h_x$ (for $r_z$) and $h_z$ (for $r_x$) are *not too big*.

- Double reflectivity volume $(r_z, r_x)$ output by *bidirection al DR modeling operator*

$$\bar{\mathbf{F}}[v]^* d = (Q_z^* \bar{F}[v]^* d, Q_x^* \bar{F}[v]^* d)$$

- Efficiency: $Q_z = E_z \hat{\Pi}_z$ where $E_z r_z(\mathbf{y}, \mathbf{h}) = r_z(\mathbf{y}, h_x)\delta(h_z)$ and $\hat{\Pi}_z$ = filter on $(\mathbf{y}, h_z)$, filters out components where the horizontal offset wavenumber is too large relative to the midpoint wavenumber, similar for $r_x$.