# Automatic wave equation migration velocity analysis by differential semblance optimization

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# Outline

- Method
  - Objective function
  - Gradient calculation
- Low velocity lens example
- Marmousi example
- Conclusions and future works

# Introduction

- **Method of depth extrapolation used in this work:**

$\frac{\partial}{\partial z}R = -\frac{iw}{c}\sqrt{1 + \frac{c^2}{w^2}\nabla_x^2}R$

Implicity finite difference (reference: Ober *et al*, 1997)

(similar treatment by Double-Square-Root equation: see abstract)

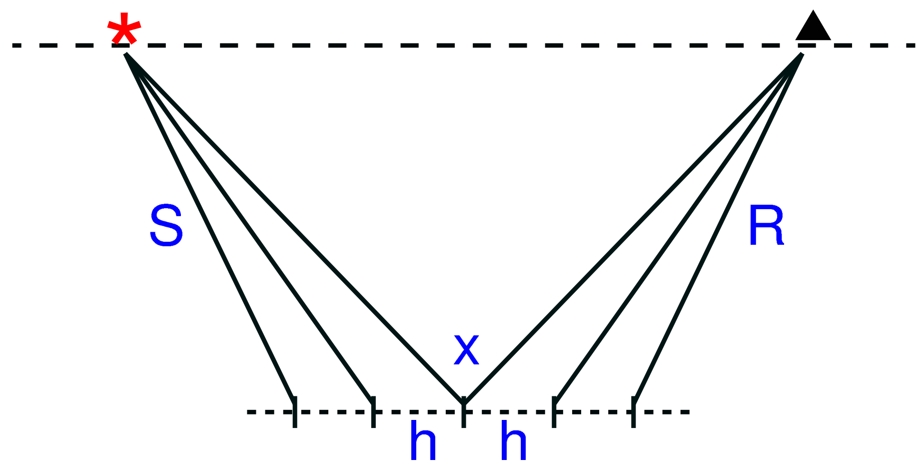
- **Subsurface offset imaging by shot-record migration:**

$I(x,h) = \sum_\omega \sum_s S(x-h;s,\omega)R(x+h;s,\omega,)$

where $S(x;s,\omega)$, $R(x;s,\omega)$ are depth-extrapolated source, receiver fields.

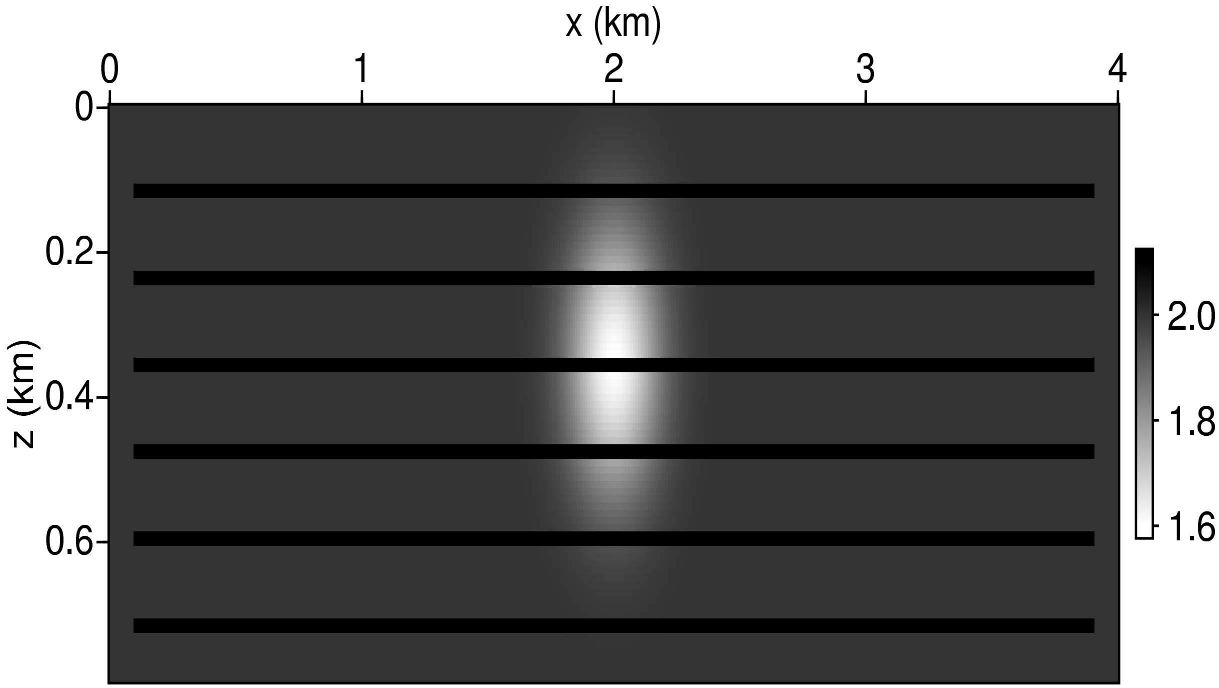
# Semblance principle for subsurface offset depth imaging

**when imaging at correct velocity**



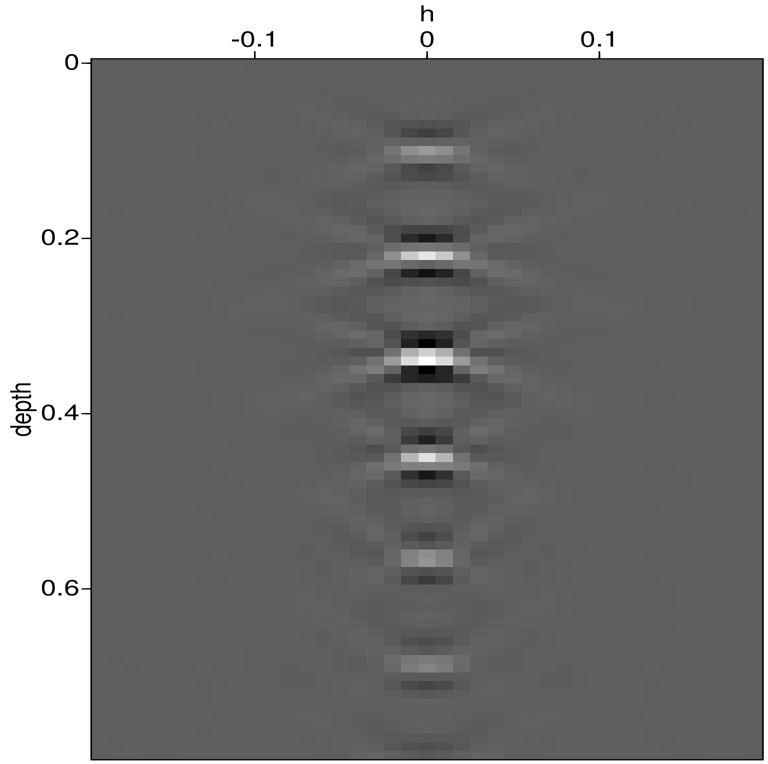

$h$ : half of the correlation distance between source and receiver fields.
$I(\mathbf{x}, h)$ : focused at zero offset.

# Example

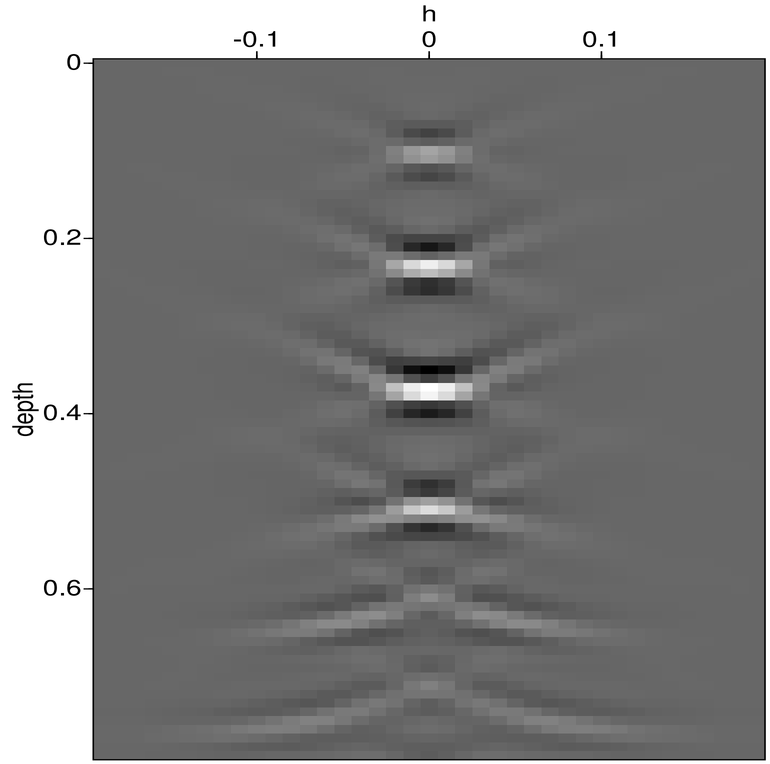



Model: low velocity lens, background velocity at 2 km/sec,
6 horizontal reflectors

# Offset gathers at center




At correct velocity




At constant velocity of 2km/sec

# Differential semblance minimization criterion

**At correct velocity**

$I(\mathbf{x}, h) \approx \delta(h) I(\mathbf{x}) \quad \rightarrow \quad \|hI(\mathbf{x}, h)\|^2$ is minimized

**Differential semblance objective function in offset domain**

$J = \frac{1}{2} \sum_h \sum_x h^2 I(x, h)^2$

**Properties of $J$**

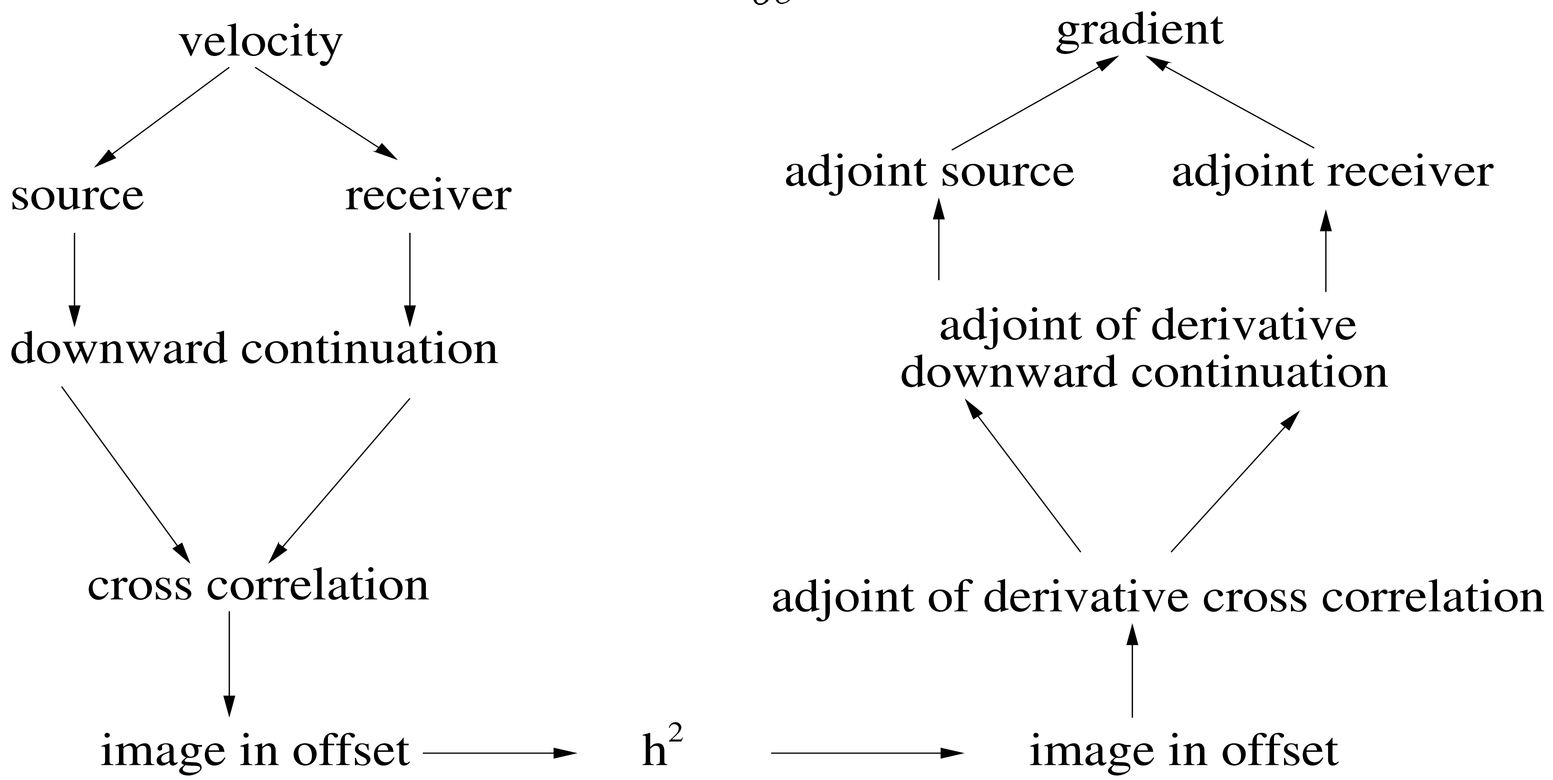
It is (essentially) the unique objective which is

- quadratic in image (implicitly, in data)
- smooth in velocity

Therefore $J$ is (essentially) the only objective suitable for automatic velocity analysis by gradient-based optimization. (Stolk & Symes, 2003)

# Gradient calculation

$$\nabla_c J = \mathrm{real}\{(\frac{\partial I}{\partial c})^* h^2 I\}$$



# Model smoothness

**Problem:**

Focussing property of offset gathers assured when model is smooth (length scale $\gg$ wavelength), but gradient updates have full data bandwidth.

**Solution:**

Confine velocity model to space of B-splines with node spacing $\gg$ wavelength

**Implication:**

Projection onto B-spline space

$$\begin{aligned} B &= \text{interpolation operator} &&: \text{from B-spline grid onto imaging grid} \\ B^* &= \text{projection operator} &&: \text{from imaging grid onto B-spline grid} \end{aligned}$$

# Optimization on B-spline model space

input: $m$, B-spline model parameter vector

c=$Bm$

$J = \frac{1}{2}||PI||^2$

$\nabla_c J = (\frac{\partial I}{\partial c})^* P^* PI$

$\nabla_m J = B^* \nabla_c J,$

construct BFGS update $\delta m$

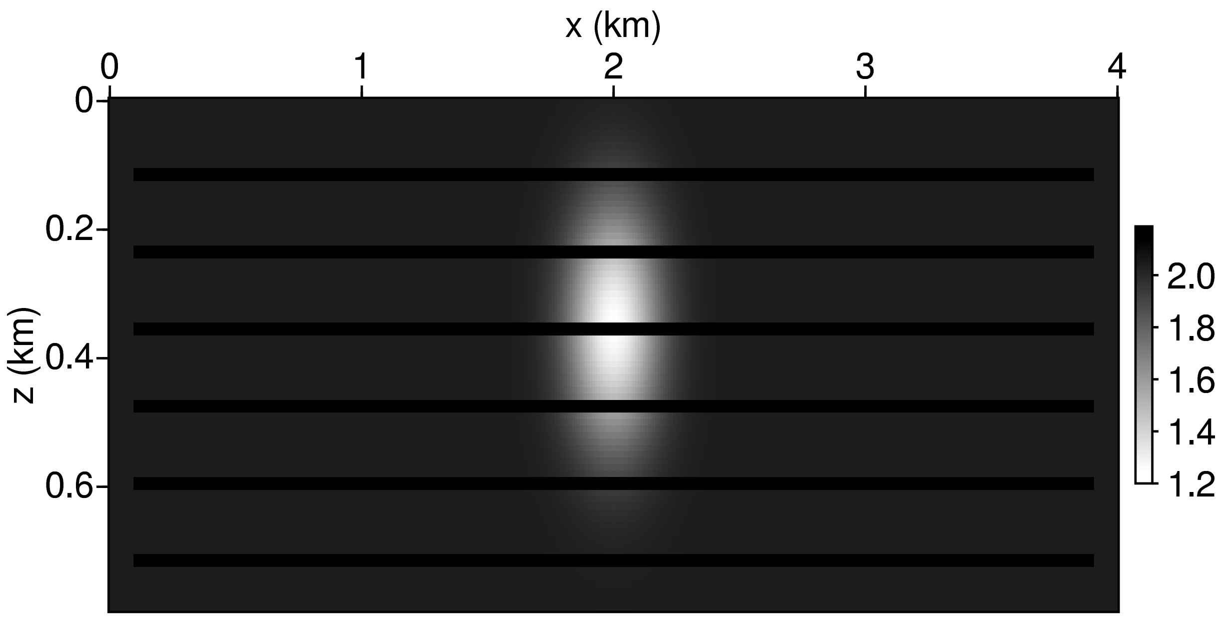
line search $m \leftarrow m + \alpha\delta m$

output: optimized velocity, optimized image

(Reference for BFGS quasi-Newton method: Nocedal & Wright, 2000)

# Lens example

$$v(z,x) = 2 - ae^{-\frac{(x-2)^2+(z-0.3)^2}{0.3}}$$

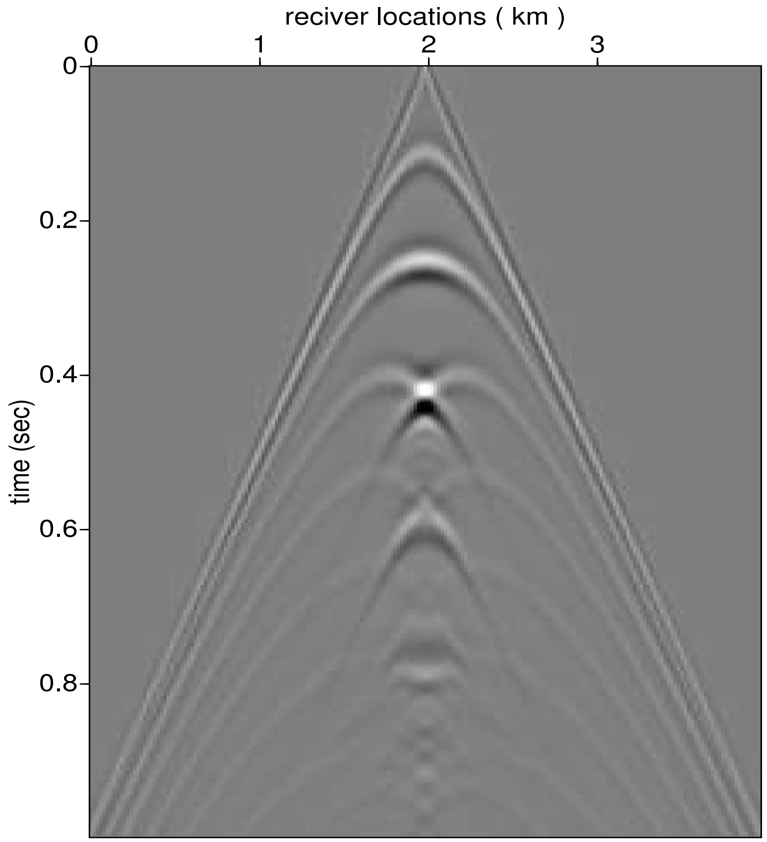



Model: background velocity at 2km/sec, $a = 0.8$.

# Acquisition geometry and setup of optimization

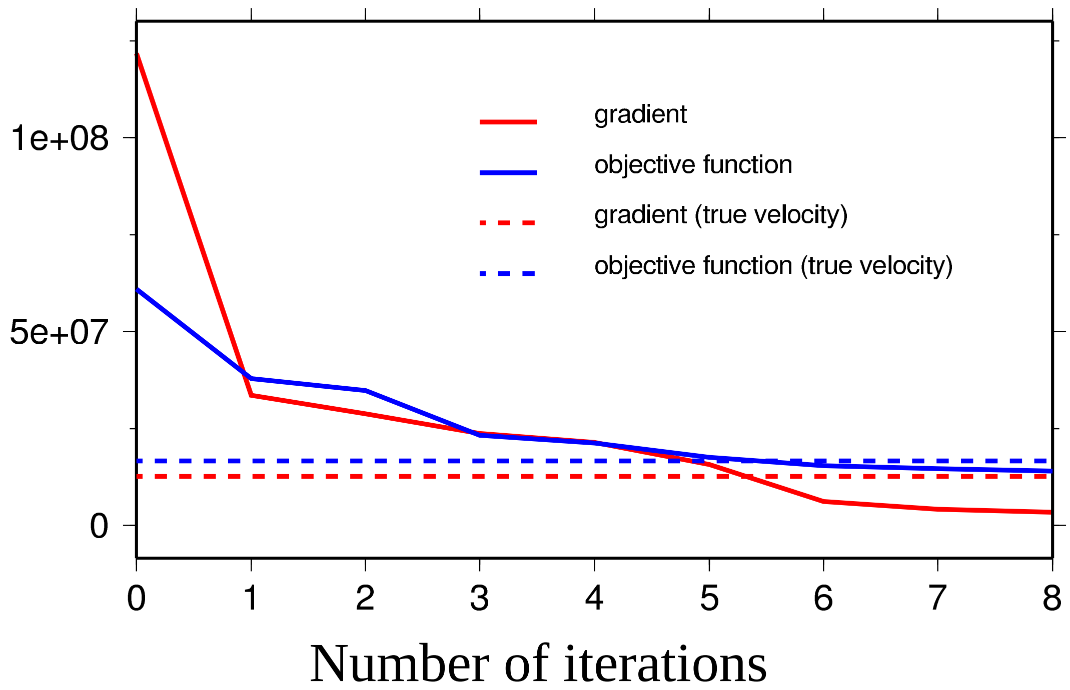
- Acquisition gemetry
  - source spacing = receiver spacing = 0.02 km
  - source wavelet: Ricker wavelet with peak frequency at 18Hz
  - source location ranging from 0.1 km to 3.9 km
  - data received at simulated streamer geometry
- setup of optimization
  - B-spline grid: $\Delta x = 0.18$ km, $\Delta z = 0.1$ km
  - imaging grid: $dx = 0.01$ km, $dz = 0.01$ km
  - initial velocity: 2km/sec, constant background

# Data: strong multipathing



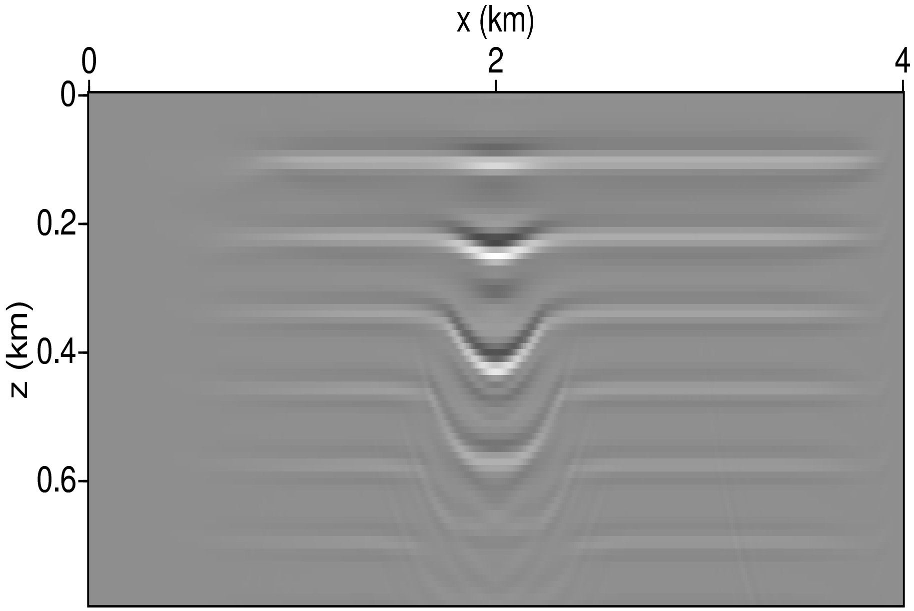

Shot gather at center.

# Progress of BFGS iterations

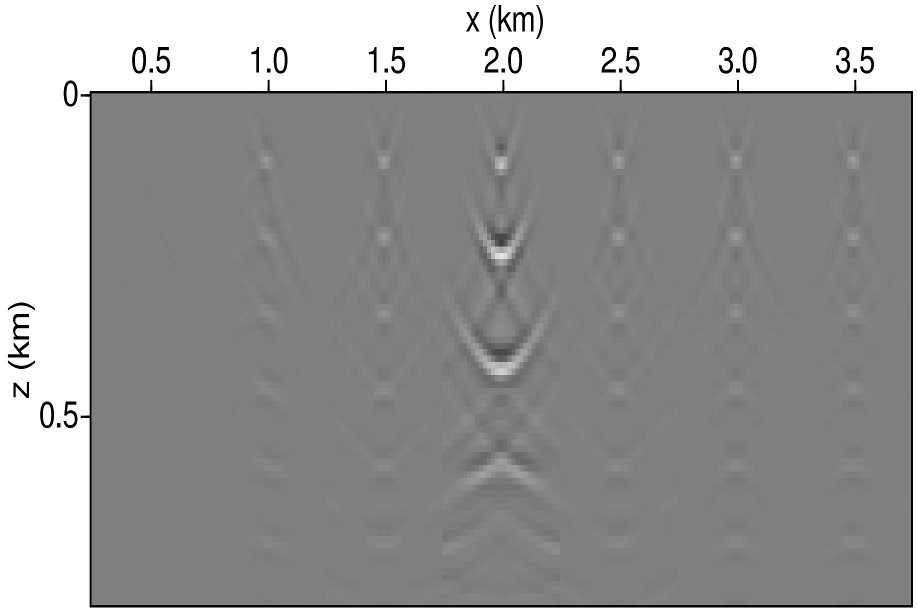



- most significant function value decay in first iteration
- magnitude of gradient $\rightarrow$ “zero”

# Initial images obtained at velocity=2km/sec



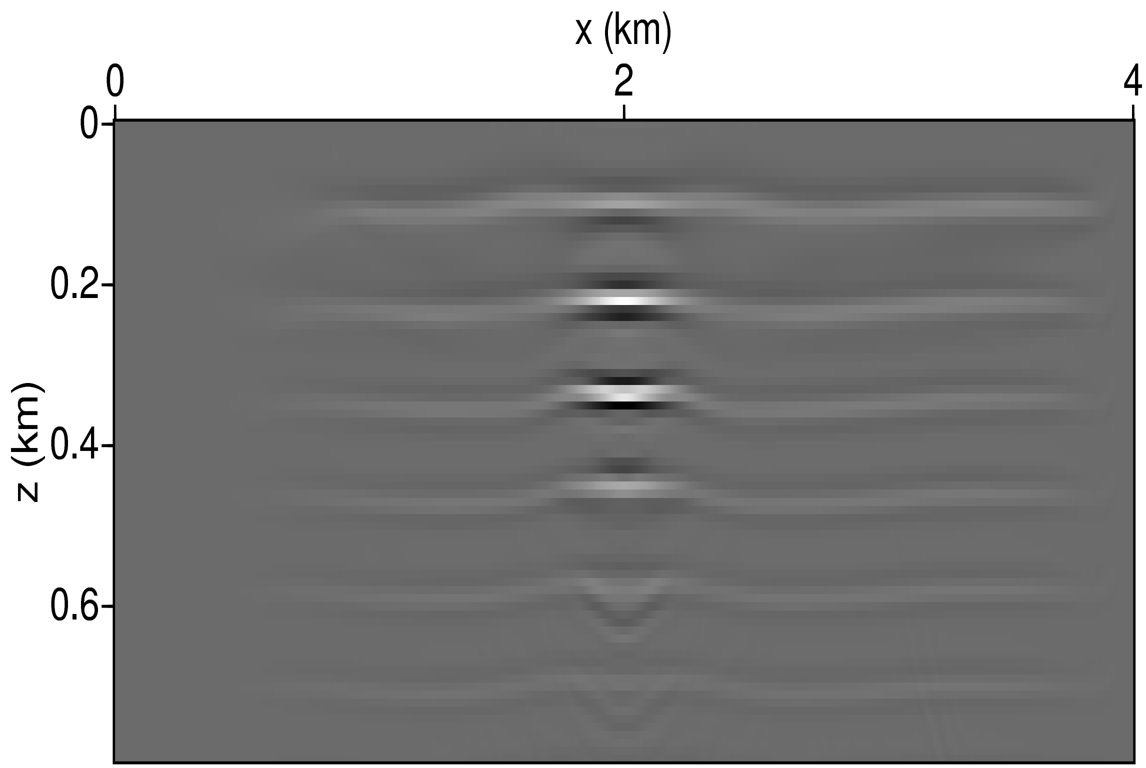

Image




Offset gathers at mid-points 0.5-3.5

- wrong image at constant velocity
- offset gathers are *not focused* at zero offset

# Images at final model




Result of image at 6th BFGS iteration.

# Image gathers at final model

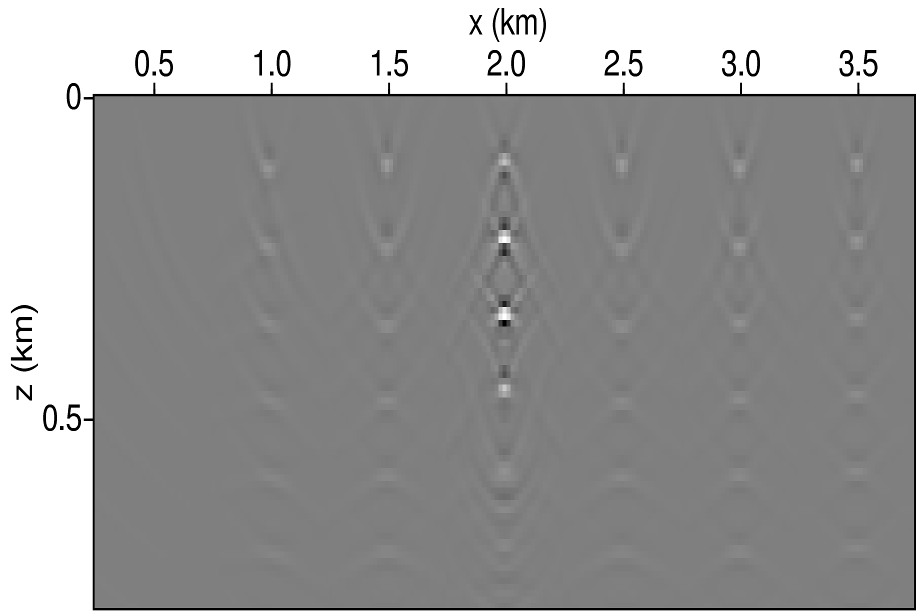



Image gathers at final model

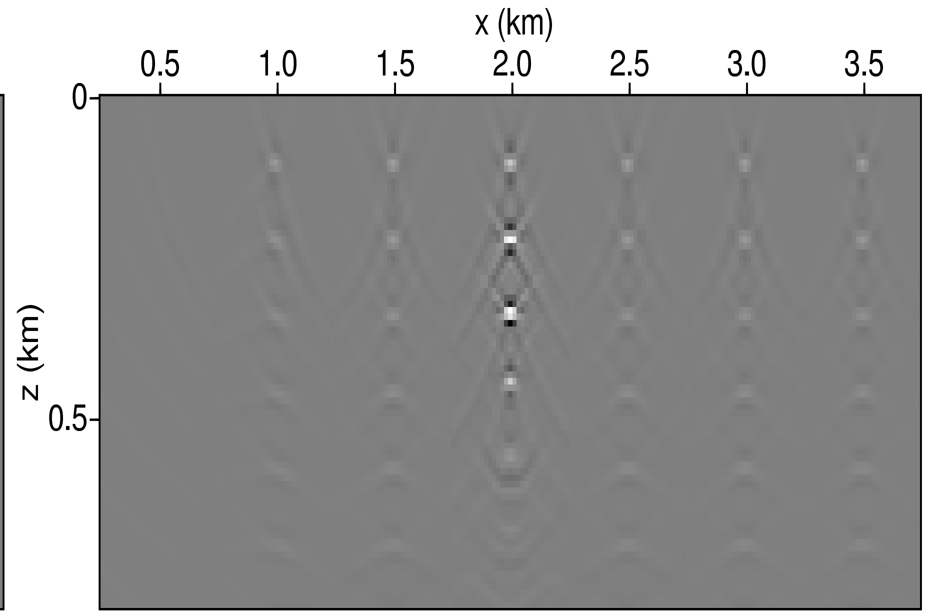
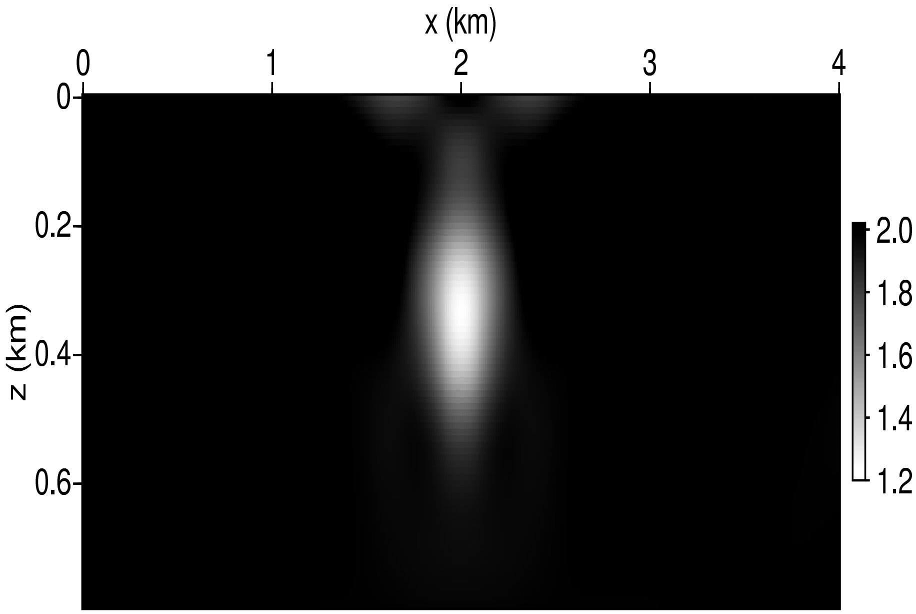


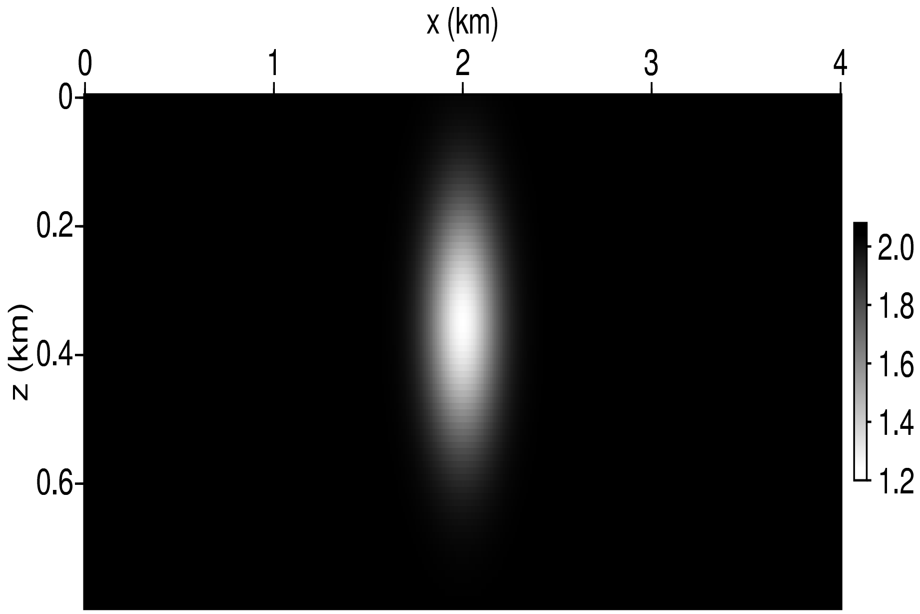

Image gathers at true velocity

- image gathers from optimization become focused at zero offset
- image gathers from optimization resemble those from true velocity

# Final model vs. true model

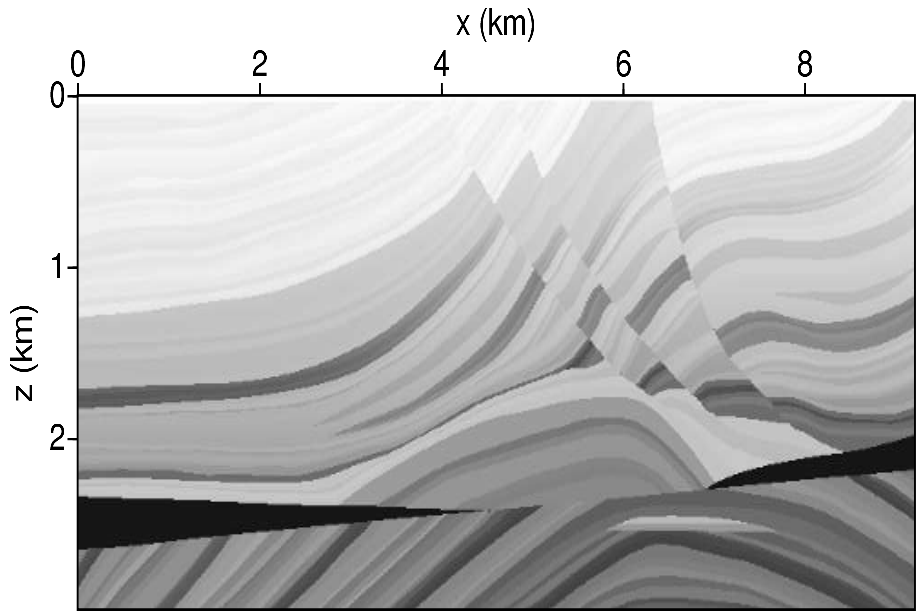



Final model

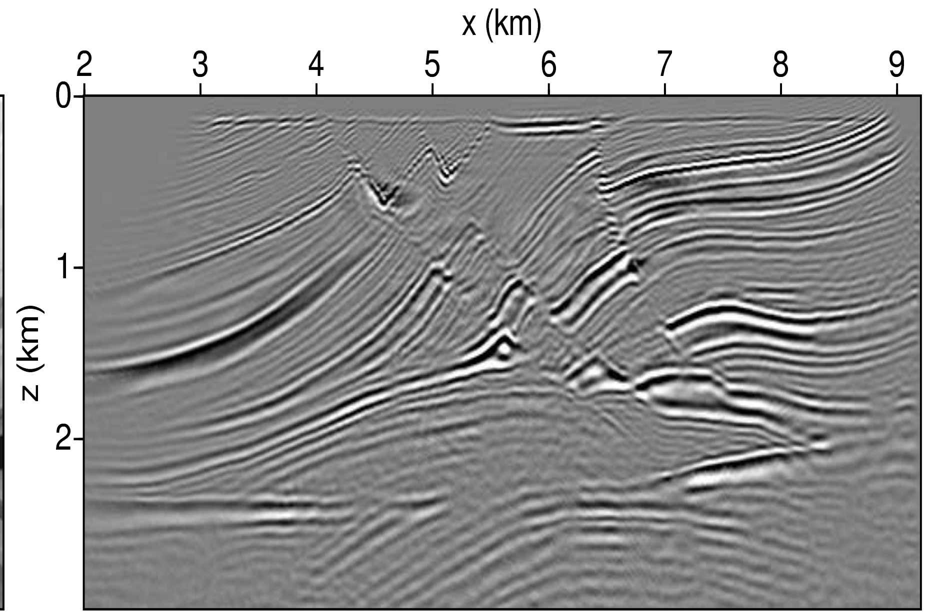



True model

# Marmousi example

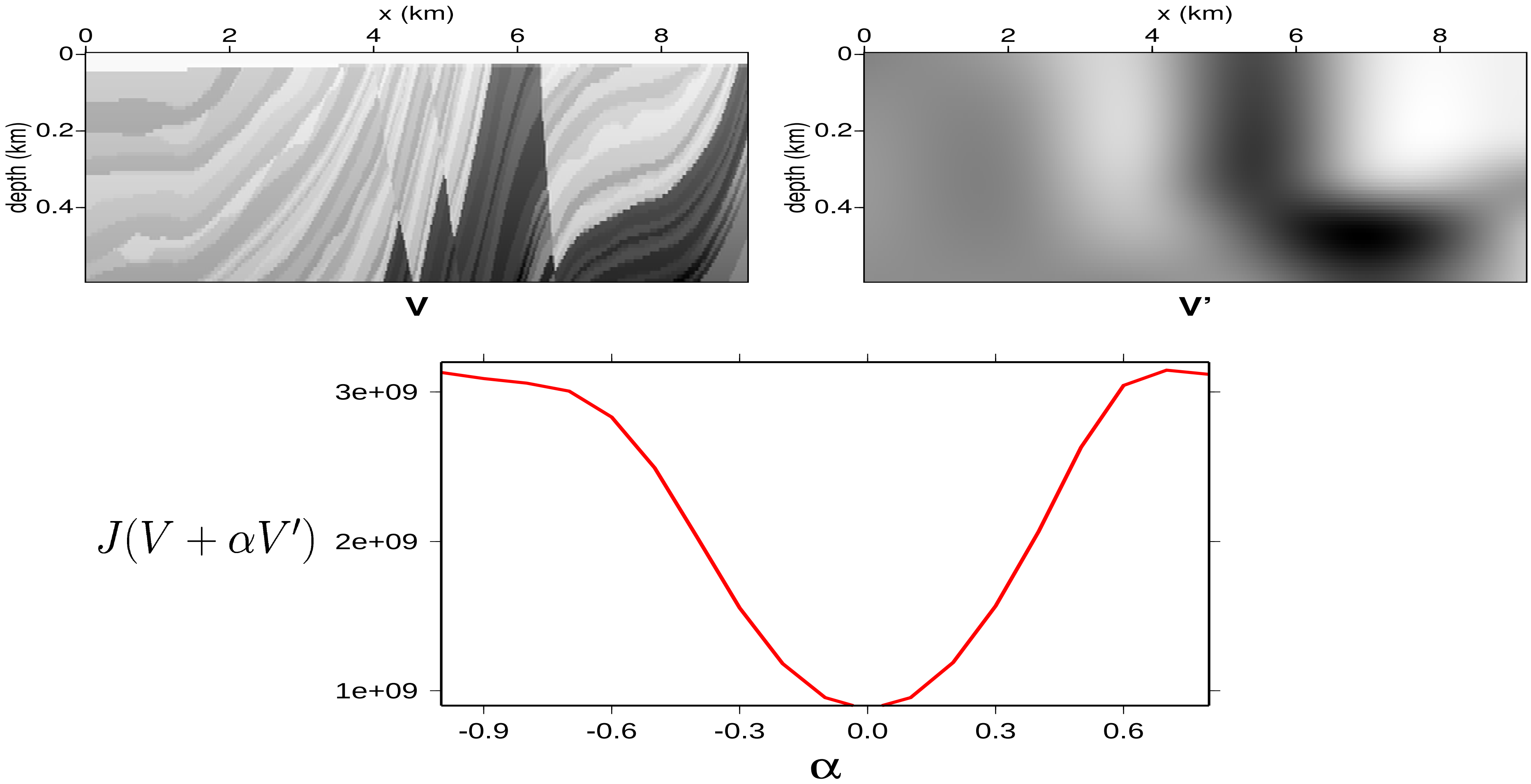



True velocity




Migrated at true velocity

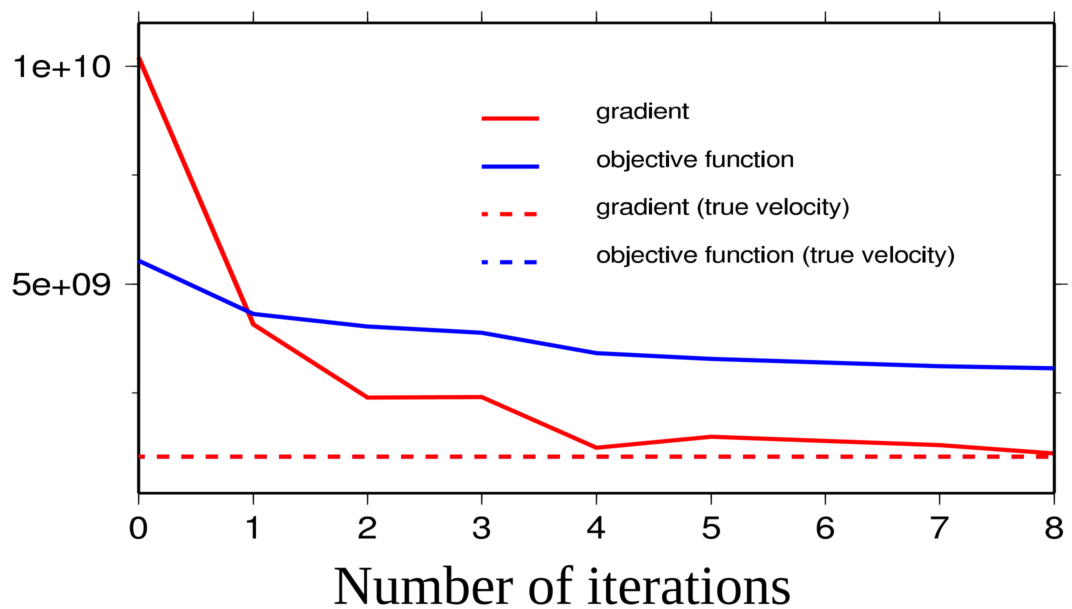
# Objective function



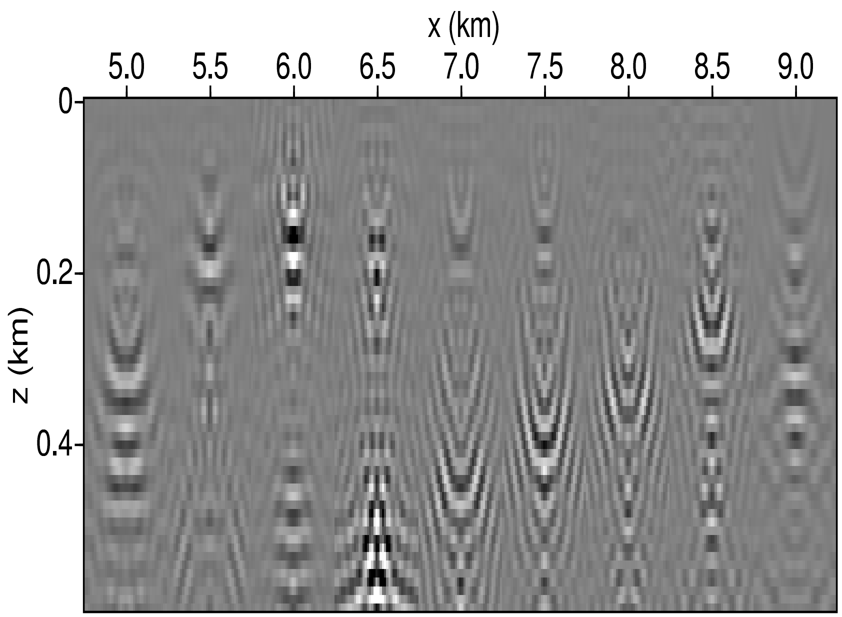

objective function: a wide smooth curve centered at the true velocity

# BFGS iterations

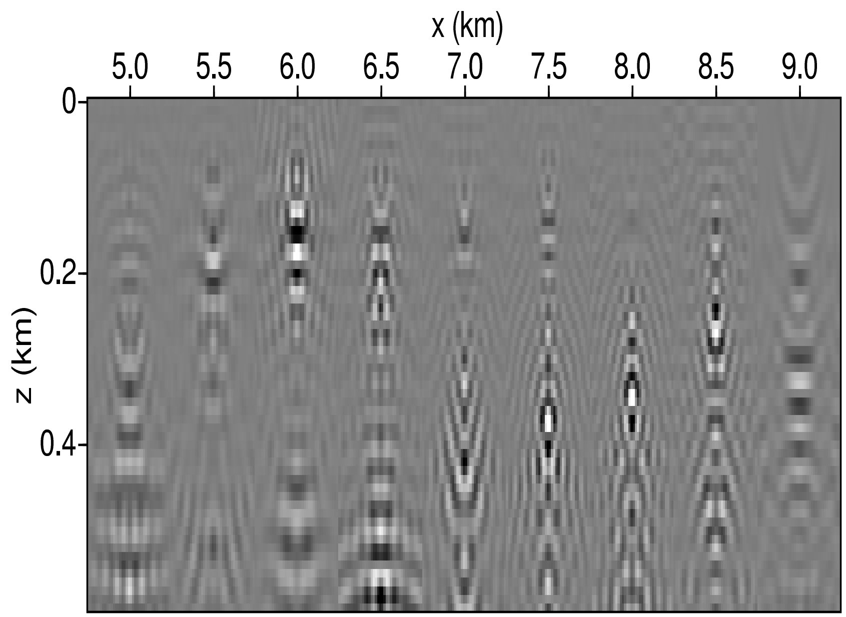
- minimization setups:
  - B-spline grid spacing: $\Delta x = 1.5$km, $\Delta z = 0.1$km
  - imaging grid spacing: $dx = 0.01$km, $dz = 0.01$km
  - starting at constant background velocity: 1.8km/sec
- iterations:



# Offset image gathers



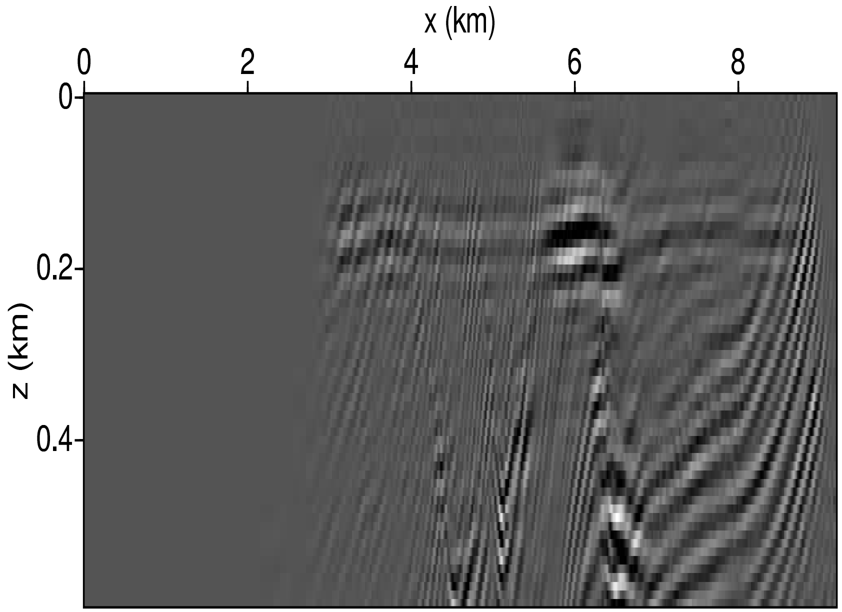

Initial image gathers at 1.8km/sec



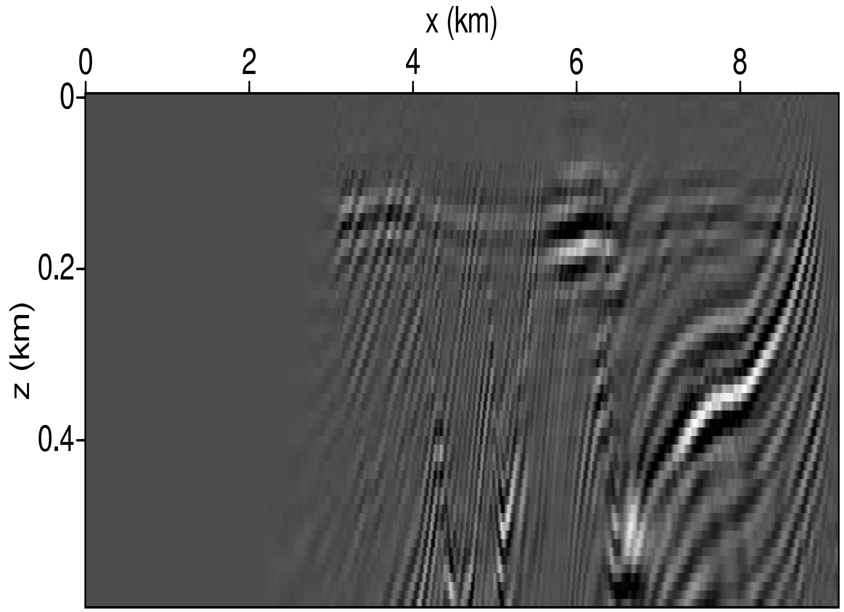

Final image gathers

- initial image gathers not focused
- differential semblance optimization focuses the offset gathers

# Initial image vs. final image

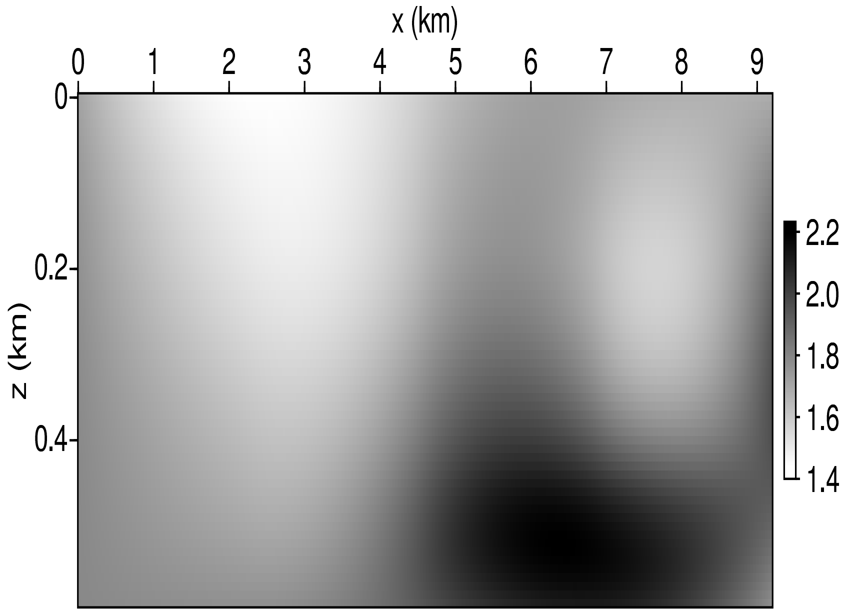



Initial image at 1.8km/sec

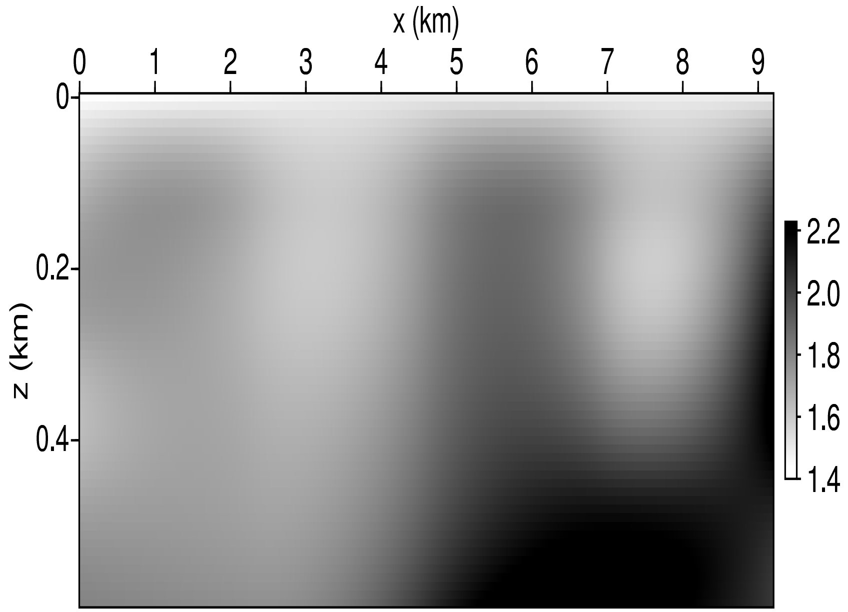



Final image

# Final velocity




by minimizing $||hI||^2$



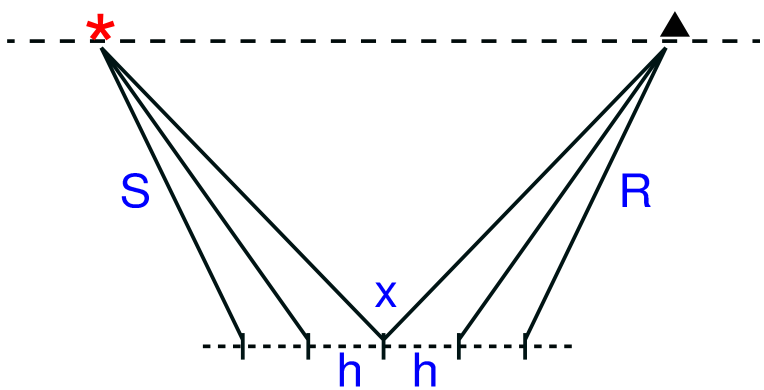

by minimizing $||Bm - V_{true}||^2$

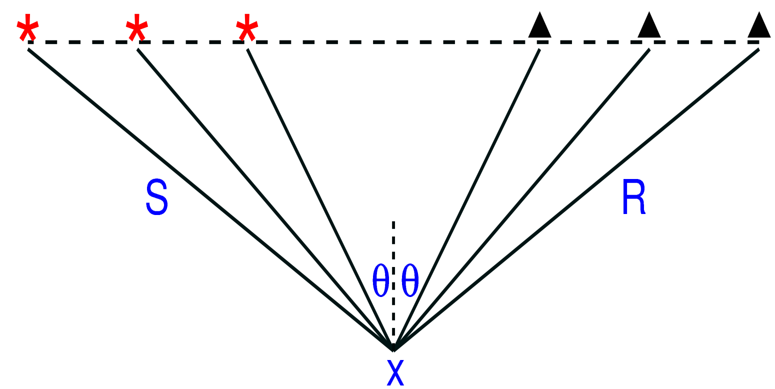
# Conclusions & Future works

- **Concusion:**
  - Optimizing in B-spline space provide velocity update with sufficient smoothness.
  - Offset domain differential semblance objective function is, in general, free of local minima.
  - This technique of offset domain differential semblance velocity analysis works when background velocity is smooth.
- **Furture works:**
  - Understand why optimization may reach beyond the objective of true model.
  - Reduce the degree of smoothness.

# Relationship between angle gather and offset gather

offset gather → Radon transform → angle gather
focused offset gather → Radon transform → flat angle gather




(Sava & Fomel, 2003)

In this work, we only use offset domain version