Lecture 5: Theory of differential semblance, migration, and multidimensional velocity estimation

- Differential semblance - examples, second installment

- Sketch of the theory for layered media

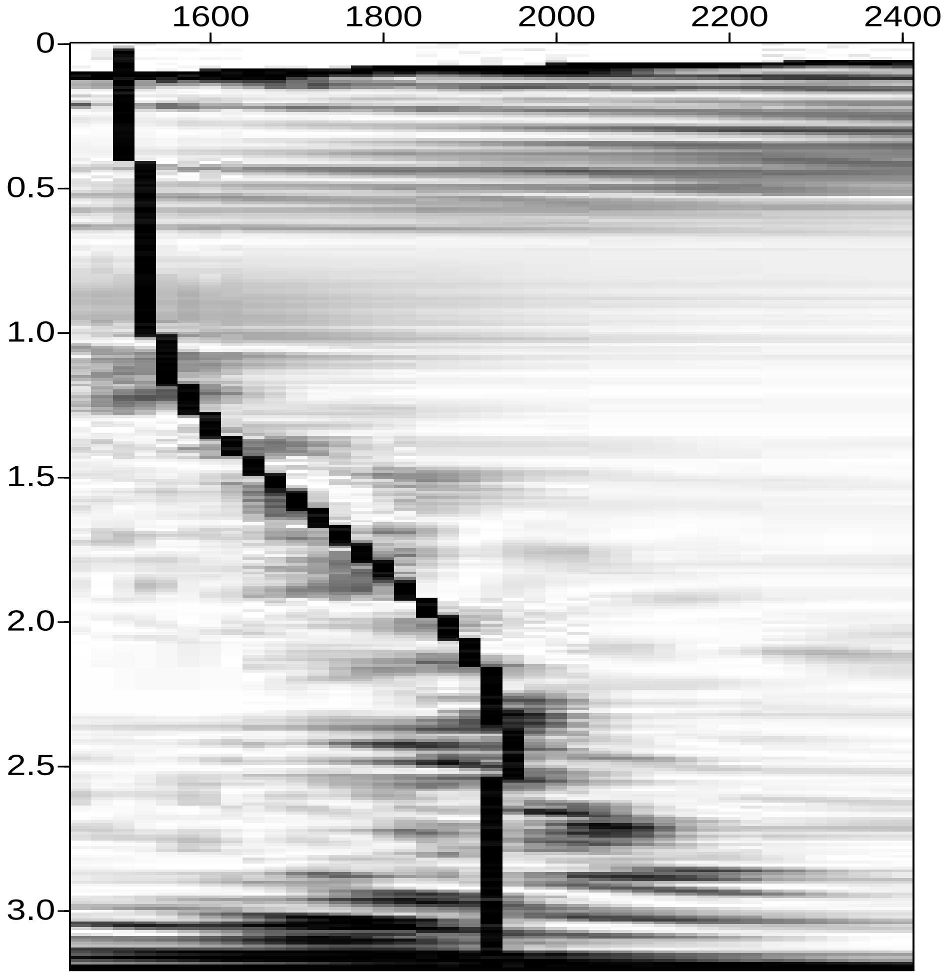
- Multidimensional reflection modeling, migration, velocity estimation

Conclusions from several dozen applications of DS velocity estimation based on convolutional model to field data:

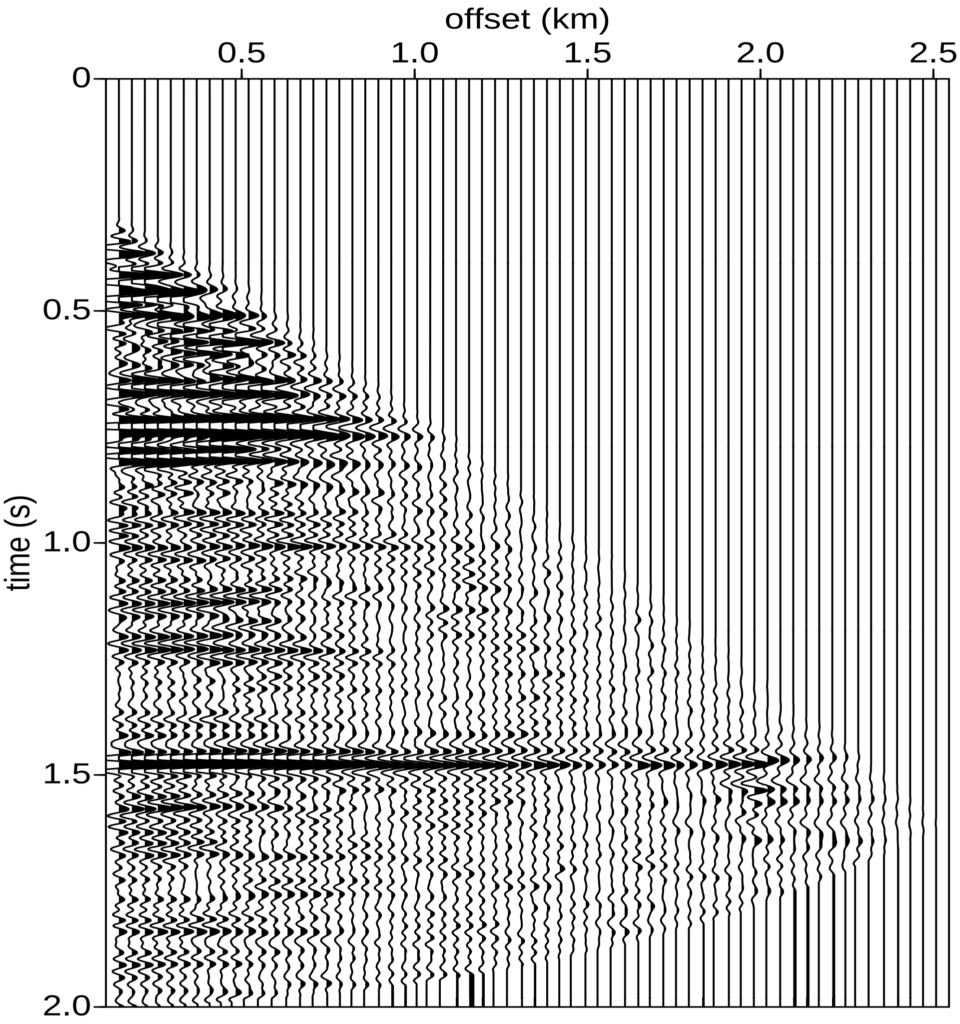
- typically 5 - 20 quasiNewton steps required to achieve optimal result, each one involving NMO and inverse NMO.

- when semblance peaks are well-defined (so when conventional VA yields unambiguous results), DS succeeds: NMO corrected CMP gathers are flat, V_{RMS} passes through semblance peaks,...

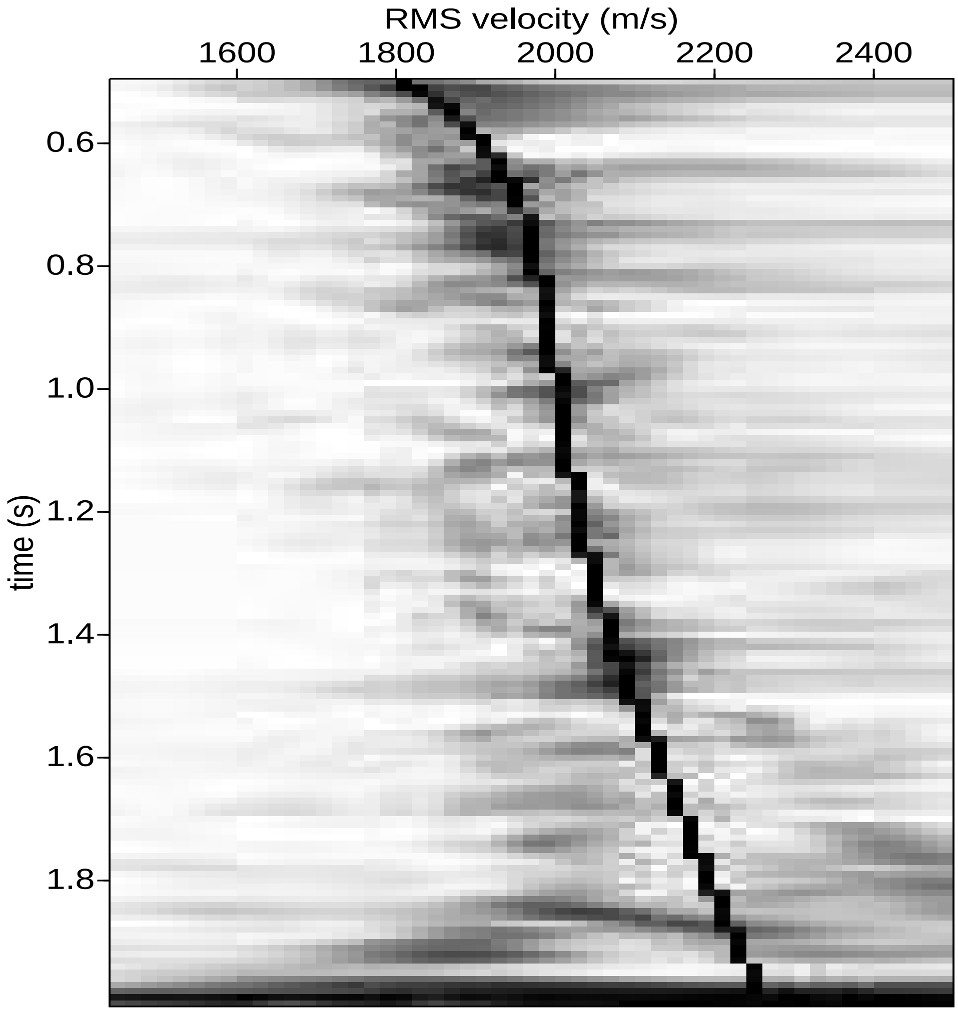
- when conventional VA would have trouble (eg. significant multiple reflection energy in data), DS output is degraded.



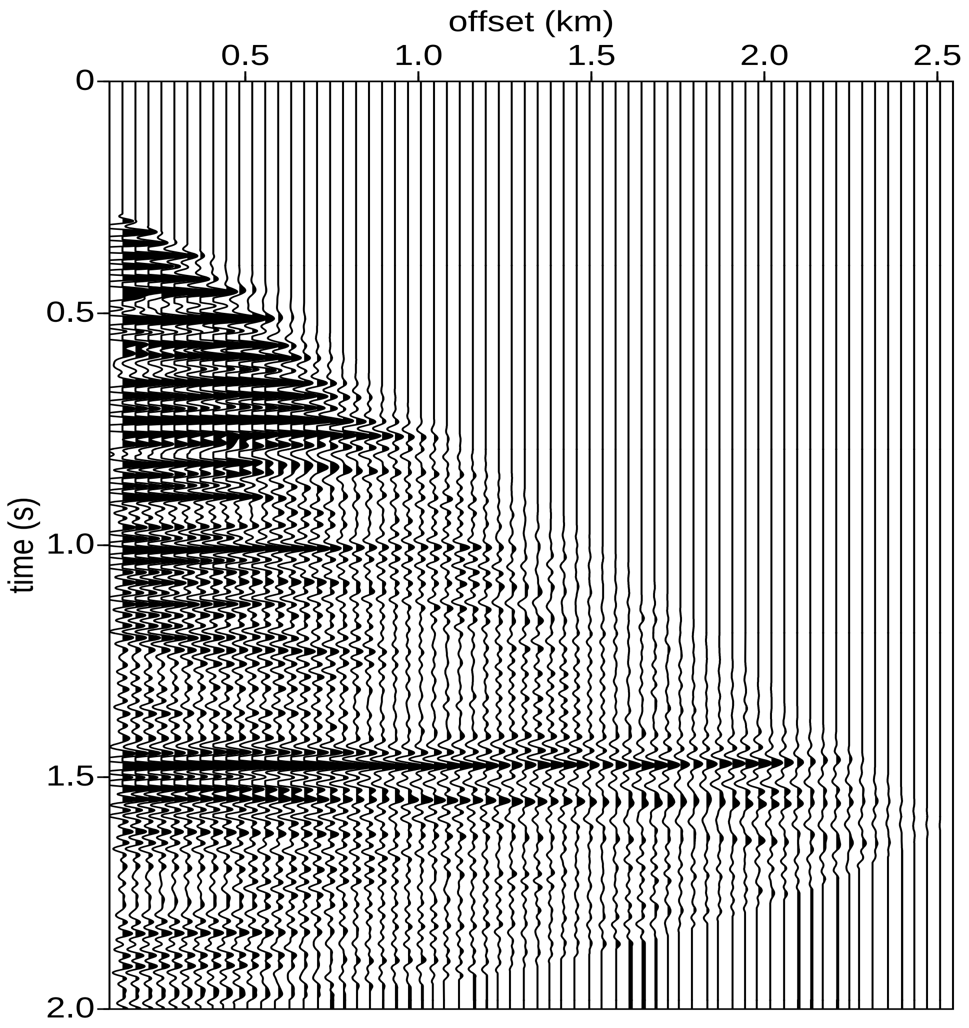
Exxon Mississippi Canyon data, midpoint 1804. V_{RMS} from DS, overplotted on velocity spectrum.



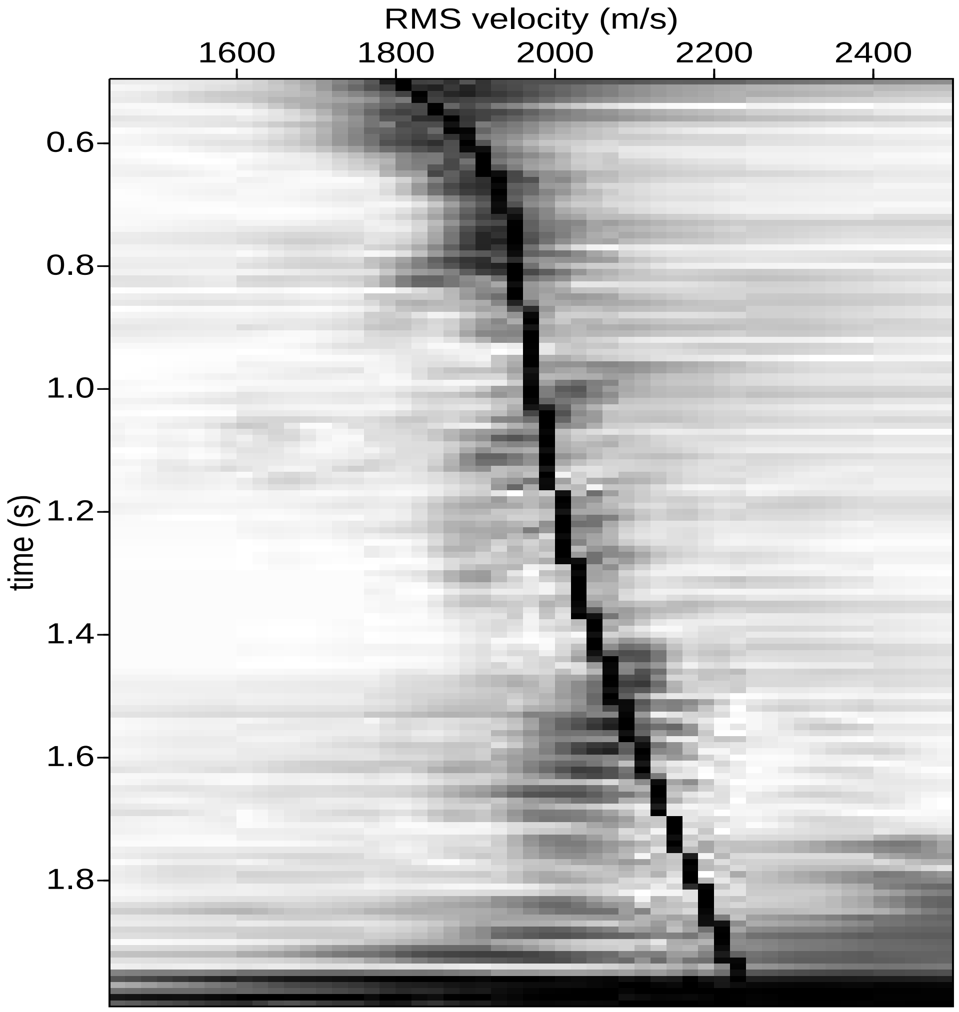
Shell North Sea data, midpoint 11: after predictive deconvolution and NMO with DS velocity.



Shell North Sea data, midpoint 11, after predictive deconvolution. V_{RMS} from DS, overplotted on velocity spectrum.



Shell North Sea data, midpoint 11: before predictive deconvolution (significant multiple energy) NMO with DS velocity.



Shell North Sea data, midpoint 11, before predictive deconvolution. V_{RMS} from DS, overplotted on velocity spectrum.

Defn of differential semblance:

$$D[c;S^{\rm obs}] = < B[c]\mathcal{F}[c]^* S^{\rm obs}, W B[c]\mathcal{F}[c]^* S^{\rm obs} >$$

with *differential* W.

A technical point: in order to (a) maintain uniform estimates over data sets of bounded energy, and (b) maintain symmetry, replace

$$W = -\frac{\partial^2}{\partial h^2}$$

with

$$W = -\frac{\partial}{\partial h}(I - \alpha\nabla^2_{h,z})^{-1}\frac{\partial}{\partial h}$$

i.e. W is *pseudodifferential*, bounded $L^2(\mathbf{R}^2) \rightarrow L^2_{\rm loc}(\mathbf{R}^2)$.

Pseudodifferential operators = operator algebra generated by differential operators, all powers of Laplace.

Conjecture: $c \rightarrow < B[c]\mathcal{F}[c]^* S^{\mathrm{obs}}, W B[c]\mathcal{F}[c]^* S^{\mathrm{obs}} >$ is smooth in $(c, S^{\mathrm{obs}}) \in \mathcal{C} \times L^2(\mathbf{R}^2)$, **if and only if** W is pseudodifferential.

[for verification in simple special case ("plane wave detection"), see Kim & WWS 1998]

Stack normal operator $\Sigma^*\Sigma$ is *not* pseudodifferential.

Convenient reformulation: since (pseudo)differential operators commute modulo lower order terms,

$$D[c; S^{\mathrm{obs}}] \simeq$$

$$< S^{\mathrm{obs}}, \mathcal{F}[c]B[c]\left(-(I-\alpha\nabla^2)^{-1}\frac{\partial^2}{\partial h^1}\right)B[c]\mathcal{F}[c]^* S^{\mathrm{obs}} >$$

using chain rule:

$$=< L^{-1}S^{\mathrm{obs}}, \mathcal{F}[c]B[c]\left(-\frac{\partial^2}{\partial h^1}\right)B[c]\mathcal{F}[c]^* S^{\mathrm{obs}} >$$

where L is some other elliptic operator.

Since one elliptic op is as good as another, might as well replace L with Helmholtz in h,t and redefine

$$D[c;S^{\text{obs}}]\simeq$$

$$=<(I-\alpha\nabla^2)^{-1}S^{\text{obs}},\mathcal{F}[c]B[c]\left(-\frac{\partial^2}{\partial h^1}\right)B[c]\mathcal{F}[c]^*S^{\text{obs}}>$$

Observation (Song, tenKroode): if you choose $B[c]$ so that $\mathcal{F}[c]B[c]$ is *unitary*, then amplitudes play no role in grad D: it is sensitive only to phase mismatch (all of this true to leading order in frequency).

Another technicality: definition of the mute Φ Correctness of adjoint computation for convolutional model hinged on $z\mapsto 2\bar{\tau}(z,h)$ being a change of variable $\Rightarrow$ choose $0<s_{\text{min}}<s_{\text{max}}$, set

$$\Phi(h,t)=1 \text{ where } s_{\text{min}}\leq\frac{\partial\bar{\zeta}}{\partial t}\leq s_{\text{max}}$$

Standard processing meaning (usually phrased in terms of $t_0 = 2\bar{\tau}(0, z)$ rather than in terms of z): points in the trace $S(h, t)$ which are Δt apart are approx. $\frac{\partial \bar{\zeta}}{\partial t} \leq s_{\mathsf{max}} \Delta t$ apart after NMO correction - s_{max} is limit on *stretch*.

Simple fallout from the eikonal equation: stretch bounds depend on bounds on $c \Rightarrow$ uniform choice of Φ over *admissible set* $\mathcal{C} \subset C^2$ characterized by bounds.

Exercise: Show that $\Phi(h, t) = 1$ implies that $(h, \zeta(h, t))$ is safely away from any forward scattering points (Geometric meaning of mute definition).

Thus mute is constant over admissible set of velocities, and plays no role in gradient.

Theory of differential semblance:

- layered acoustic Earth
- convolutional model
- hyperbolic moveout

Main result: for consistent data, all stationary points are global minimizers. Stationary values proportional to data noise.

NB: Uniqueness is not to be expected!

Ref: `www.trip.caam.rice.edu`, 1999 reports

Back to the setup of Lecture 2.

Reflection seismic inverse problem, multidimensional linearized version: from the data $\{p(\mathbf{x}_r, t; \mathbf{x}_s)\}$, estimate $c(\mathbf{x}), \delta c(\mathbf{x})$.

Recall distribution kernel of linearized map $D[c]$:

$$K(\mathbf{x}_r, t, \mathbf{x}_s; \mathbf{x}) = w *_t \int ds \, \frac{\partial G}{\partial t}(\mathbf{x}_r, t-s; \mathbf{x}) \frac{\partial^2 G}{\partial t^2}(\mathbf{x}, s; \mathbf{x}_s)$$

$$\simeq w *_t a(\mathbf{x}, \mathbf{x}_r) a(\mathbf{x}, \mathbf{x}_s) \delta'''(t - \tau(\mathbf{x}, \mathbf{x}_r) - \tau(\mathbf{x}, \mathbf{x}_s))$$

for values of the arguments which satisfy:

- simple geometric optics description of G
- no forward scattering

Chief task: understand structure of normal operator. Main result (Duistermaat 1973, Beylkin 1984, or MGSS '98 notes by WWS): under standing hypotheses, normal operator is *pseudodifferential*

$$DS[c]^*DS[c]\delta c(\mathbf{x}) =$$

$$\int d\xi\, e^{i\mathbf{x}\cdot\xi}\sigma(\mathbf{x},\xi)\widehat{\delta c}(\xi)$$

For any compact $K \subset \mathbf{R}^3$, multiindices α, β, the symbol σ satisfies the estimates

$$|D_\xi^\alpha D_{\mathbf{x}}^\beta \sigma(\mathbf{x},\xi| \le C_{K,\alpha,\beta}(1+|\xi|)^{2-|\alpha|}$$

for $\mathbf{x} \in K, |\xi| \ge 1$.

Consequences: normal operator is

- *order 2*, i.e. maps $H^s \to H^s - 2$
- *pseudolocal*:

$$DS[c]^*DS[c](\chi(\mathbf{x})e^{i\omega\psi(\mathbf{x})})$$

$$= -\omega^2\sigma(\mathbf{x},\nabla\psi(\mathbf{x}))\chi(\mathbf{x})e^{i\omega\psi(\mathbf{x})} + O(\omega)$$

I.e. application of $DS[c]^*DS[c]$ preserves locii of high frequency oscillations (singularities) up to lower order (smooth) error. [refocusing!]

Where does a little "reflector element"

$$\delta c(\mathbf{x}) = \chi(\mathbf{x}) e^{i\omega\psi(\mathbf{x})}$$

go in the data? Apply stationary phase to integral

$$DS[c]\delta c(\mathbf{x}_r, t; \mathbf{x}_s) =$$

$$\int dx\, a(\mathbf{x}, \mathbf{x}_r) a(\mathbf{x}, \mathbf{x}_s) \delta''(t - \tau(\mathbf{x}, \mathbf{x}_r) - \tau(\mathbf{x}, \mathbf{x}_s)) \chi(\mathbf{x}) e^{i\omega\psi(\mathbf{x})}$$

Stationary phase point:

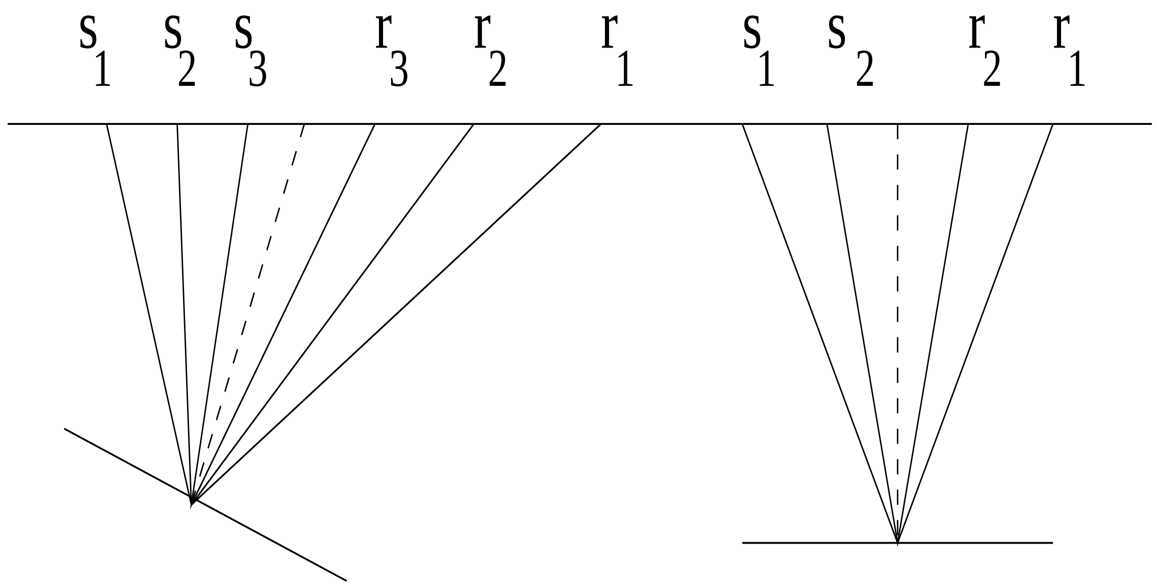
$$\nabla_{\mathbf{x}} \tau(\mathbf{x}, \mathbf{x}_r) + \nabla_{\mathbf{x}} \tau(\mathbf{x}, \mathbf{x}_s) || \nabla\psi(\mathbf{x})$$

Multidimensional imaging and velocity analysis parallels layered medium versions, except that:

- source–receiver midpoints no longer correspond to the horizontal positions of scatterers

- must consider all traces with given offset in forming “virtual well” at given horizontal position, not just those with that midpoint

- must map all traces with given offset at once (“common offset migration”) to produce “common offset section”



- semblance principle: velocity is correct when "virtual wells" at given horizontal position align for all offsets

- no diagonalization: stack power problem is global opt problem over space of $c(\mathbf{x})$

Differential semblance:

$$D[c; S^{\mathrm{obs}}] = < B[c]DS[c]^* S^{\mathrm{obs}}, WB[c]DS[c]^* S^{\mathrm{obs}} >$$

in which $B[c]$ is an elliptic pseudodifferential operator (generalization of mult by pos smooth function) and as before

$$W = -\frac{\partial}{\partial h}(I - \alpha\nabla^2_{h,\mathbf{x}})^{-1}\frac{\partial}{\partial h}$$

- smooth as function of c under standing assumptions

- otherwise little theory

- many implementation tricks

- thorough study by H. Chauris, PhD thesis École des Mines de Paris 2000

Of things not mentioned:

- multiple reflections - big problem, lots of ideas

- multiple reflections - why not use full acoustic (or elastic or...) model?

- multiple refraction (strong forward scattering) *invalidates semblance principle* (Nolan & WWS 1997), also *refocusing fails* - what to do (angle gathers - Prucha et al 1999)

- deterministic theory does not honor complexity of actual geology

- stochastic theory does not incorporate insights of deterministic theory

THE END