Fundamental Issues in Waveform Inversion

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Outline

- Introduction
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- Following Symes and Kern (1994) and Chavent and Jacewitz (1995), Zhou et al. (2008) proposed an alternative migration velocity analysis algorithm that utilizes the extrapolation operators of RTM and removes the square data differences that are used in classic waveform inversion
- Classic waveform inversion consists of minimizing the least-squares misfit between the measured data and the synthetics predicted with the current description of the rock properties (Tarantola, 1987)
- Although it is mathematically attractive, very few success

Method

- Classic waveform inversion, the least-squares waveform data misfit:

$$J_{LS}(p) = \frac{1}{2} < p - p_0, p - p_0 >$$

- Scalar acoustic wave equation:

$$(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2)p = s$$

- The Lagrange multiplier or adjoint state variable:

$$L(p, \lambda; v) = J_{LS}(p) + \left\langle \lambda, (\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2)p - s \right\rangle$$

Issue 1: No single acoustic wave equation is known to represent real wave propagations.

- Acoustic wave equation, elastic wave equation...
  All of these wave equations share one very important property that although these wave equations have different transport equations and therefore different amplitudes, all of them have the same eikonal equation and hence retain the same traveltime information.
- To reduce the dependency of waveform inversion on wave equations, waveform inversion should emphasize on the traveltime information and downplay the role of amplitude information.

Issue 2: RTM is not the first iteration of classic waveform inversion.

- RTM is the first iteration of another form of waveform inversion (maximization problem):

$$J_{CC}(p) = \frac{1}{2} < p, p_0 >$$

- The unconstrained optimization scheme can be described with Lagrange multiplier method by:

$$L(p, \lambda; v) = J_{CC}(p) + \left\langle \lambda, \left(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)p - s \right\rangle$$

Issue 3: Hessian matrix may be necessary for waveform inversion.

- The deconvolution has the effect to remove the source wavelet effect and the inverse of second derivative to time plays the role of smoothing the velocity model and hence helping produce the low-wavenumber velocity model.
- Application of Hessian to the inversion process will also help speed up the convergence rate.

Issue 4: Direct Born inversion and classic waveform inversion are only valid for scatterers or transmission and refraction waves but not for reflection waves.

- Both techniques work well for inversion of velocity with refraction/transmission waves.
- For reflection data, just like standard migration algorithms, only locations of the reflectors have been imaged instead of smooth background velocities.

Issue 5: Waveform inversion for reflection data in data domain should depend on migration and demigration processes.

- The wavefields generated with a standard wave equation will contain only direct/turning waves and refraction waves, and are essentially devoid of reflection waves.
- The low-wavenumber components and the high-wavenumber components of the velocity are fairly well decoupled.

- Demigration relies on the correlation of incident wavefields with migration images to generate the needed reflection waves and does not necessarily need sharp velocity boundaries in the velocity model as required by traditional forward modelling process. One such demigration process was defined by Symes and Kern (1994):

$$(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2)q = s$$

$$(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2)p = r \cdot q$$

the reflection wavefield p, the reference wavefield q, the (smooth) background velocity v.
r, which is the stacked migration image, is responsible for all reflections in this model, it is normally called the (rough) reflectivity.

Lagrange multiplier method

- Lagrange multiplier method is defined as:

$$L(p, q, \lambda_p, \lambda_q; v) = J(p) + \left\langle \lambda_q, (\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2)q - s \right\rangle \quad (1)$$

$$+ \left\langle \lambda_p, (\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2)p - r \cdot q \right\rangle \quad (2)$$

  Here, λ_q and λ_p are the adjoint wavefields that satisfy adjoint-state equations.

- If $J(p) = J_{CC}(p)$, λ_q and λ_p can be solved with following equations:

$$(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\lambda_p = -p_0$$

$$(\frac{1}{v^2}\frac{\partial^2}{\partial t^2} - \nabla^2)\lambda_q = r \cdot \lambda_p$$

The gradient of the objective equation

- The gradient of the objective equation:

$$\nabla J = \left\langle \lambda_q, -\frac{2}{v}\nabla^2 q \right\rangle + \left\langle \lambda_p, -\frac{2}{v}\nabla^2 p \right\rangle$$

  First term corresponding to the incident waves from the source side, while the second-term corresponds to the reflection waves at the receiver side.

Example

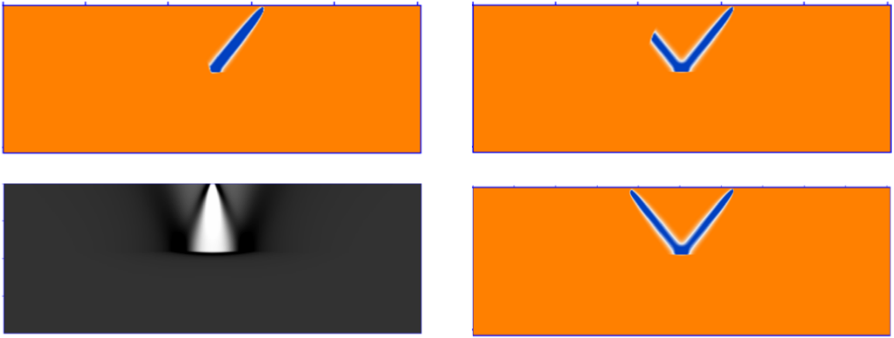

Figure : Partial gradient images at times t=0.87s, 0.39s, respectively for the top two plots (in clockwise). The bottom right plot corresponds to the final complete gradient corresponding to this trace, i.e., at time t=0.0s. The bottom left plot is the gradient after an inversion of a shot gather.

Example

Figure : Gradient image after inversion with the whole 41-shotsof reflection data.

Conclusions

- In this abstract we have discussed some fundamental issues of traditional waveform inversion in data domain.
- We propose to (1) base waveform inversion on optimization functionals, such as cross-correlation, that emphasize more on traveltime information;
- (2) use a demigration approach instead of model-based forward modelling to simulate data.
- With the migration/demigration processes, we can produce the synthetic data that can be used to match the observed data with less worrying about the cycle-skip problem that is intrinsic in classic waveform inversion.